SMT in practice. veriT

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Procédures de décision et vérification de programmes: Lecture 8

SMT = SAT + expressiveness

- SAT solvers

$$\neg\big[\,(p \Rightarrow q) \Rightarrow \big[\,(\neg p \Rightarrow q) \Rightarrow q\big]\big]$$

- Congruence closure (uninterpreted symbols + equality)

$$a = b \wedge \big[f(a) \neq f(b) \vee (p(a) \wedge \neg p(b))\big]$$

- adding arithmetic

$$a \leq b \wedge b \leq a + x \wedge x = 0 \wedge \big[f(a) \neq f(b) \vee (p(a) \wedge \neg p(b + x))\big]$$

- ...

Some examples: Alt-Ergo, Barcelogic, CVC4 (SVC, CVC, CVC-lite, CVC3), MathSAT, OpenSMT, Yices, Z3 ...

The veriT solver

Standard input language: SMT-LIB 2.0

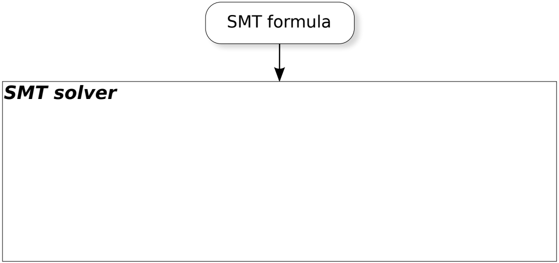
$$a \leq b \wedge b \leq a + x \wedge x = 0 \wedge \left[f(a) \neq f(b) \vee (q(a) \wedge \neg q(b + x)) \right]$$

In SMT-LIB 2.0 format:

```
(set-logic QF_UFLRA)
(set-info :source | Example formula in SMT-LIB 2.0 |)
(set-info :smt-lib-version 2.0)
(declare-fun f (Real) Real)
(declare-fun q (Real) Bool)
(declare-fun a () Real)
(declare-fun b () Real)
(declare-fun x () Real)
(assert (and (<= a b) (<= b (+ a x)) (= x 0)
             (or (not (= (f a) (f b)))
                 (and (q a) (not (q (+ b x)))))))
(check-sat)
(exit)
```

From propositional SAT to SMT

Reducing arbitrary boolean combinations to conjunctions



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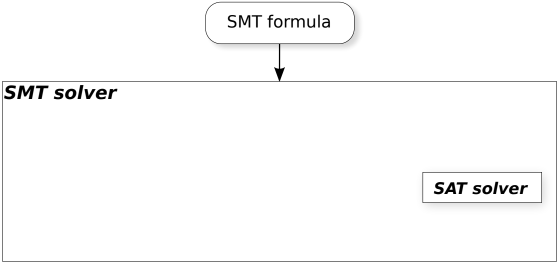

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To SAT solver: $p_{a \leq b} \wedge p_{b \leq a+x} \wedge p_{x=0} \wedge \big[\neg p_{f(a)=f(b)} \vee (p_{q(a)} \wedge \neg p_{q(b+x)})\big]$

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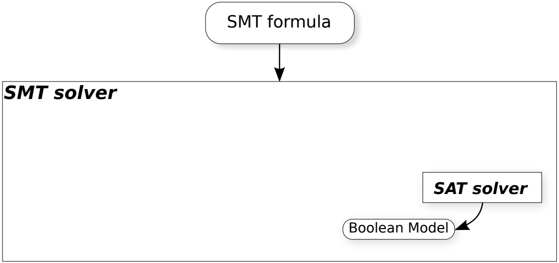

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Boolean model: $p_{a \leq b}, p_{b \leq a+x}, p_{x=0}, \neg p_{f(a)=f(b)}$

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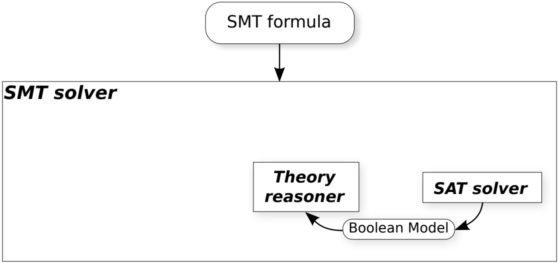

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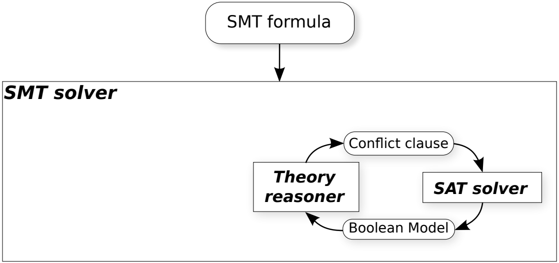

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New clause: $\neg p_{a \leq b} \vee \neg p_{b \leq a+x} \vee \neg p_{x=0} \vee p_{f(a)=f(b)}$

From propositional SAT to SMT

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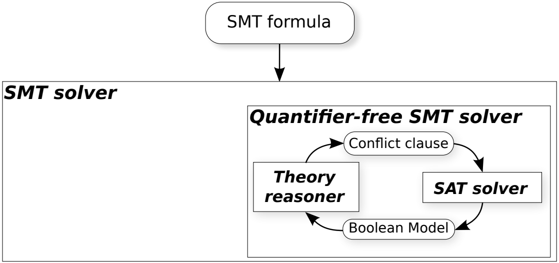

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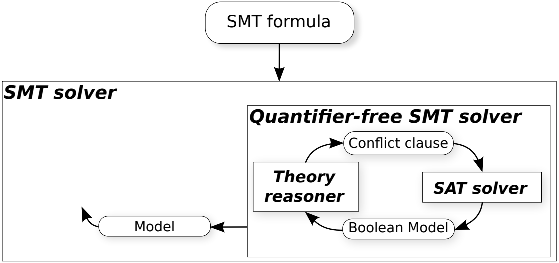

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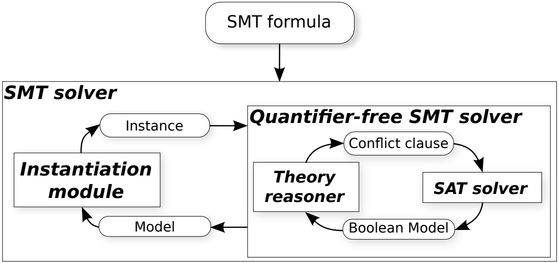

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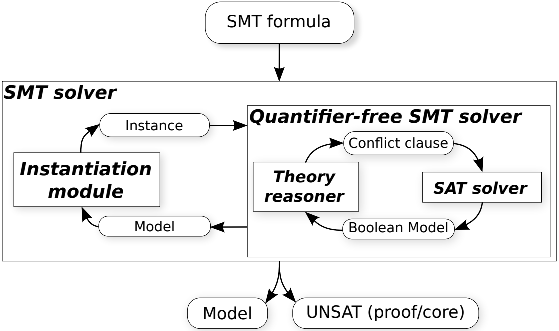

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From propositional SAT to SMT: in practice

- online decision procedures
  theory checks propositional assignment on the fly
- small explanations
  unsat core of propositional assignment
  discard classes of propositional assignments (not one by one)
- theory propagation
  instead of guessing propositional variable assignments, SAT solver assigns theory-entailed literals
- ackermannization, simplifications, and other magic

DPLL: abstract view

Rules handle a data-structure $M \mid\mid F$ where M is a partial assignment of Boolean variables, and F is a set of clauses

Propagate	$M \mid\mid F, C \vee \ell$	$\vdash$	$M\ \ell \mid\mid F, C \vee \ell$ if $M \models \neg C, \ell$ undefined in M
Decide	$M \mid\mid F$	$\vdash$	$M\ \ell^d \mid\mid F$ if ℓ or $\overline{\ell}$ in F, ℓ undefined in M
Fail	$M \mid\mid F, C$	$\vdash$	$\perp$ if $M \models \neg C,$ no decision literals in M
Backtrack	$M\ \ell^d\ N \mid\mid F, C$	$\vdash$	$M\ \bar{\ell} \mid\mid F, C$ if $\begin{cases} M\ \ell^d\ N \models \neg C \\ \text{no decision literals in } N \end{cases}$

CDCL: abstract view

Propagate, Decide, Fail as before

$$\text{Learn} \quad M \mid\mid F \quad \vdash \quad M \mid\mid F, C \quad \text{if } \begin{cases} \text{each atom of } C \text{ in } F \text{ or in } M \\ F \models C \end{cases}$$

$$\text{Backjump} \quad M\ \ell^d\ N \mid\mid F, C \quad \vdash \quad M\ \ell' \mid\mid F, C \quad \text{if } \begin{cases} M\ \ell^d\ N \models \neg C \\ \exists C', \ell' : F, C \models C' \vee \ell' \\ M \models \neg C' \\ \ell' \text{ undefined in } M \\ \ell' \text{ or } \overline{\ell'} \text{ in } F \text{ or in } M\ \ell^d\ N \end{cases}$$

$$\text{Forget} \quad M \mid\mid F, C \quad \vdash \quad M \mid\mid F \quad \text{if } F \models C$$

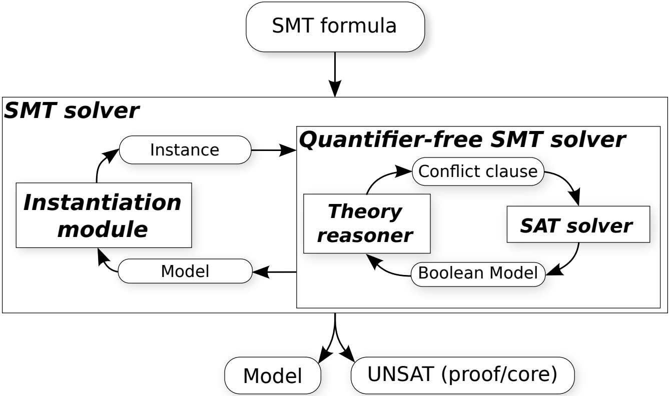
CDCL: SMT level

$$T\text{–Learn} \quad M \mid\mid F \vdash M \mid\mid F, C \quad \text{if } \begin{cases} \text{each atom of } C \text{ in } F \text{ or in } M \\ F \models_T C \end{cases}$$

$$T\text{–Forget} \quad M \mid\mid F, C \vdash M \mid\mid F \quad \text{if } F \models_T C$$

$$T\text{–Backjump} \quad M\ l^d\ N \mid\mid F, C \vdash M\ l' \mid\mid F, C \quad \text{if } \begin{cases} M\ l^d\ N \models \neg C \\ \exists C', l' : F, C \models_T C' \vee l' \\ M \models \neg C' \\ l' \text{ undefined in } M \\ l' \text{ or } \neg l' \text{ in } F \text{ or in } M\ l^d\ N \end{cases}$$

Theory reasoning and combinations in SMT



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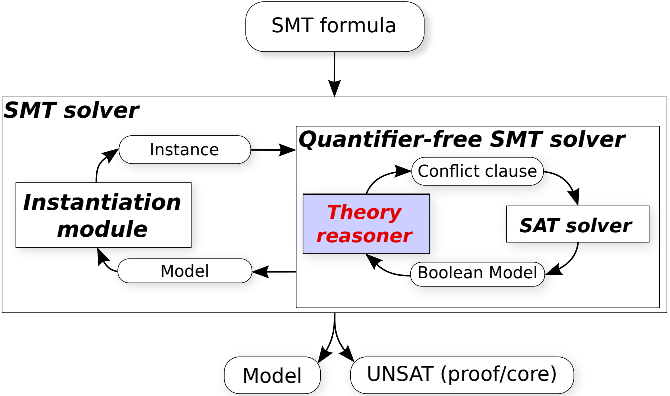

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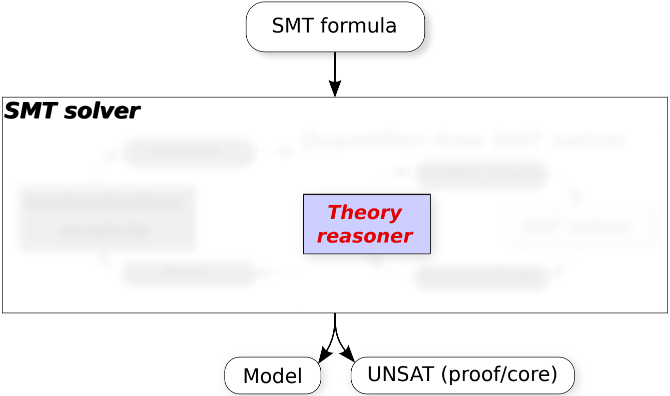

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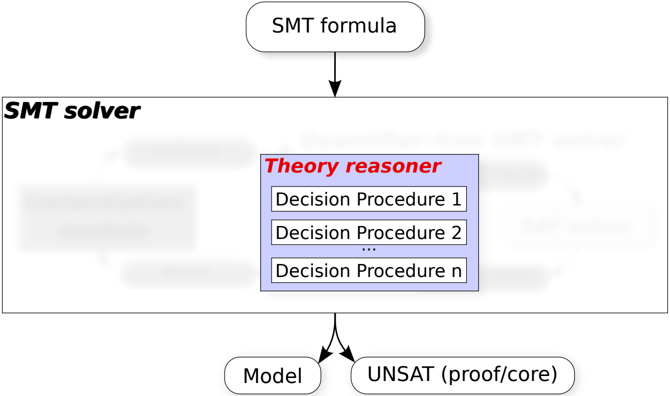

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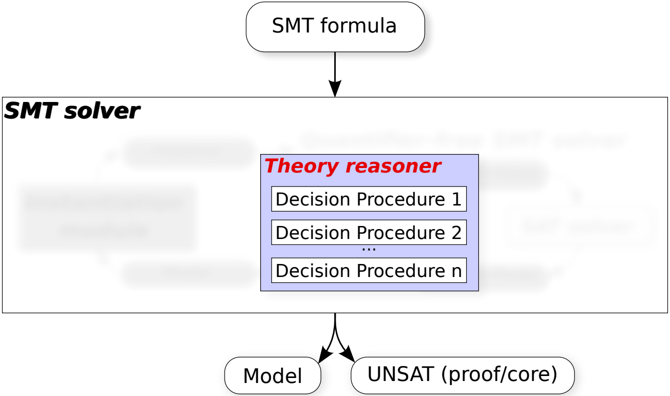

Some examples:

- uninterpreted symbols and equality: congruence closure
- linear arithmetic: mostly simplex
- non-linear arithmetic: CAD, Virtual Substitution, Gröbner Bases, Interval Propagation
- arrays: based on uninterpreted symbols

Combinations of theories

Nelson-Oppen

Combining theories: uninterpreted symbols and arithmetic.

$$x \leq y,\ y \leq x + f(x),\ P(h(x) - h(y)), \neg P(0),\ f(x) = 0$$

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$$x \leq y,\; y \leq x + f(x),\; P(h(x) - h(y)), \neg P(0),\; f(x) = 0$$

Separate into pure literals

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Exchange equalities until unsatisfiability is deduced

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UNSAT

Combinations of theories

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UNSAT

Sound: deduce only logical consequences

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UNSAT

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Complete: decidable theories with cardinality restrictions

Combinations of theories (1/2)

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UNSAT

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Combinations of theories (2/2)

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Non linear arithmetic is also stably infinite.

Uninterpreted symbols and non linear arithmetic:

$$x^2 = 1, P(x), \neg P(-1), \neg P(1)$$

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Exchange disjunctions of equalities

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For non-convex theories, disjunctions have to be exchanged
Even deducing equalities is unpractical with a black box

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get "model" if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$

On SAT, get a model from NLRA

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
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$x = 1, v_1 = 1, v_2 = -1$	

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Pretend equalities in the model were in the input

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$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x = 1, v_1 = 1, v_2 = -1$	$x = v_1$

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get "model" if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$

Compute conflict clause

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x = 1, v_1 = 1, v_2 = -1$	$x = v_1$

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get "model" if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$
$\neg P(x) \vee x \neq 1 \vee P(1)$

Add conflict clause to underlying SAT solver

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x = 1, v_1 = 1, v_2 = -1$	$x = v_1$

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get "model" if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$
$\neg P(x) \vee x \neq 1 \vee P(1)$

Update literals

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x \neq 1$	

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get "model" if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$
$\neg P(x) \vee x \neq 1 \vee P(1)$

Get a model from NLRA (again)

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x \neq 1$	
$x = -1, v_1 = 1, v_2 = -1$	

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get "model" if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$
$\neg P(x) \vee x \neq 1 \vee P(1)$

Pretend equalities in the model were in the input (again)

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x \neq 1$	$x = v_2$
$x = -1, v_1 = 1, v_2 = -1$	

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get "model" if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$
$\neg P(x) \vee x \neq 1 \vee P(1)$

Compute conflict clause (again)

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x \neq 1$	$x = v_2$
$x = -1, v_1 = 1, v_2 = -1$	

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get “model” if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$
$\neg P(x) \vee x \neq 1 \vee P(1)$
$\neg P(x) \vee x \neq -1 \vee P(-1)$

Add conflict clause to underlying SAT solver (again)

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x \neq 1$	$x = v_2$
$x = -1, v_1 = 1, v_2 = -1$	

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get "model" if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$
$\neg P(x) \vee x \neq 1 \vee P(1)$
$\neg P(x) \vee x \neq -1 \vee P(-1)$

Update literals (again)

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x \neq 1$	$x = v_2$
$x \neq -1$	

Combining theories: using model equalities

Open the box a bit: besides (un)sat, get "model" if sat

$x^2 = 1, P(x), \neg P(-1), \neg P(1)$
$\neg P(x) \vee x \neq 1 \vee P(1)$
$\neg P(x) \vee x \neq -1 \vee P(-1)$
$x = 1 \vee x = -1 \vee x^2 = 1$

Conclude unsatisfiability (finally)

Arithmetic	Uninterpreted
$x^2 = 1$	$P(x)$
$v_1 = 1$	$\neg P(v_1)$
$v_2 = -1$	$\neg P(v_2)$
$x \neq 1$	$x = v_2$
$x \neq -1$	

Perspectives

- Quantifiers: better instantiations, superposition+SMT
- Higher-order
- More theories: data-structures, floating points. . .
- Higher efficiency
- Parallelism