On the Feistel counterpart of the BCT

Introduction and Analysis of the FBCT

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Université de Lorraine, INRIA, Loria, CNRS - Nancy, France

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Basic boomerang distinguisher

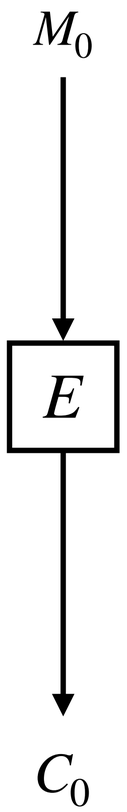
The Boomerang Attack
Wagner, *FSE 1999*

Variant of differential cryptanalysis that considers **quartets** of messages.

Basic boomerang distinguisher

[Wagner, FSE ’99]

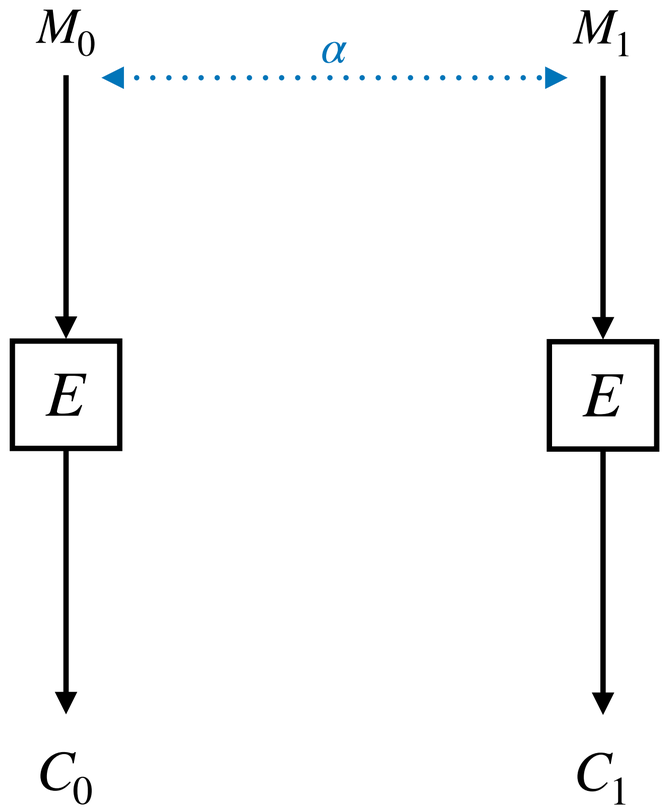
1. Pick M_0 at random, ask for its ciphertext C_0



Basic boomerang distinguisher

[Wagner, FSE ’99]

1. Pick M_0 at random, ask for its ciphertext C_0
2. Ask for C_1, the ciphertext of $M_1 = M_0 \oplus \alpha$



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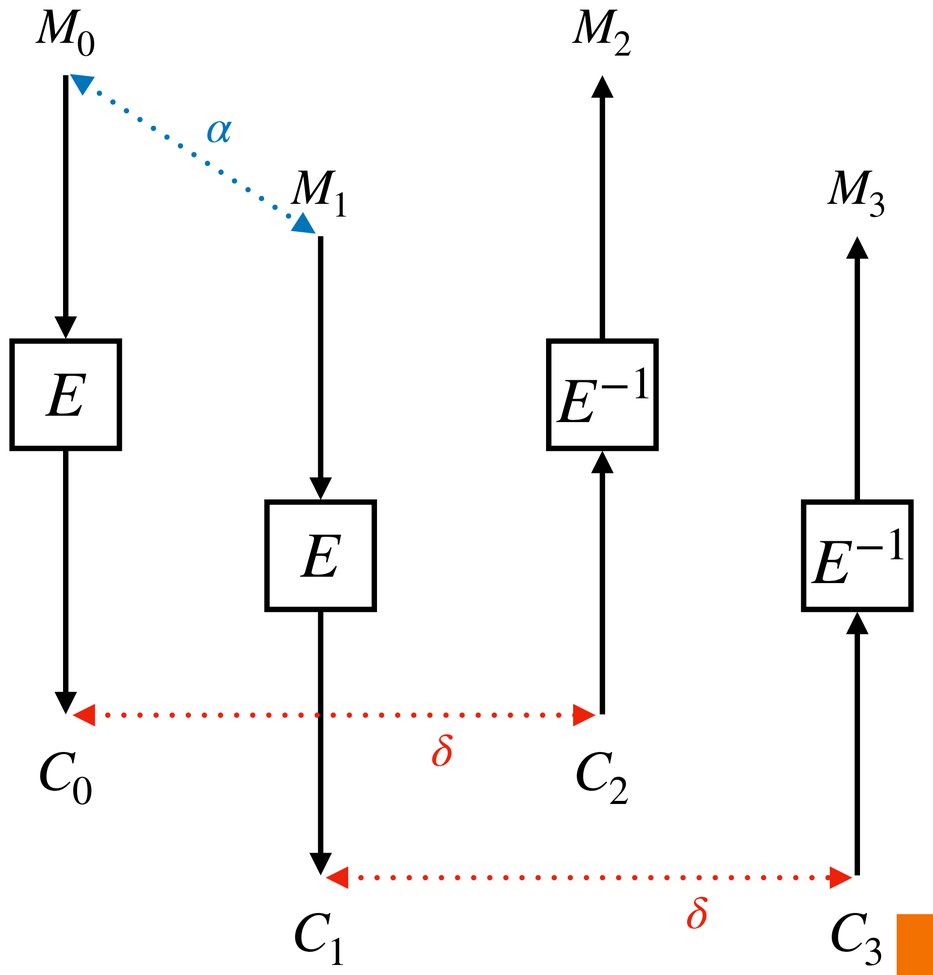
3. Compute $C_2 = C_0 \oplus \delta, C_3 = C_1 \oplus \delta$
4. Ask for their decryption (M_2, M_3)



Basic boomerang distinguisher

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1. Pick M_0 at random, ask for its ciphertext C_0
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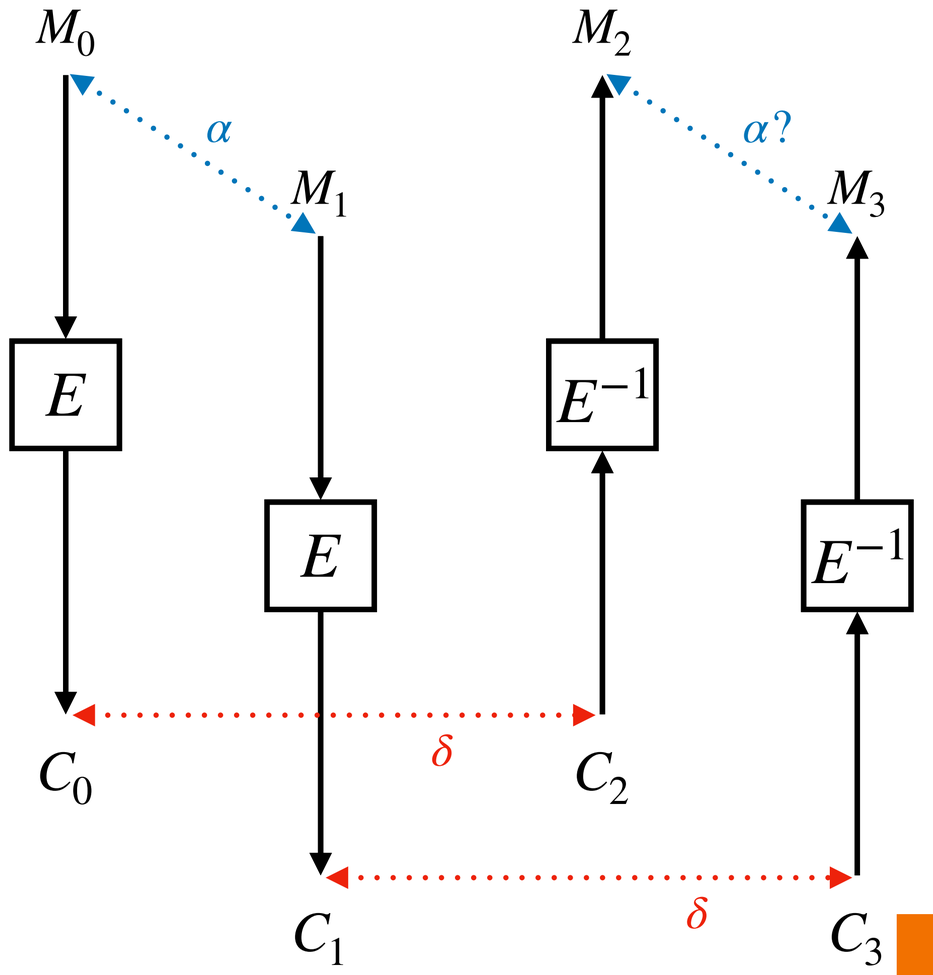
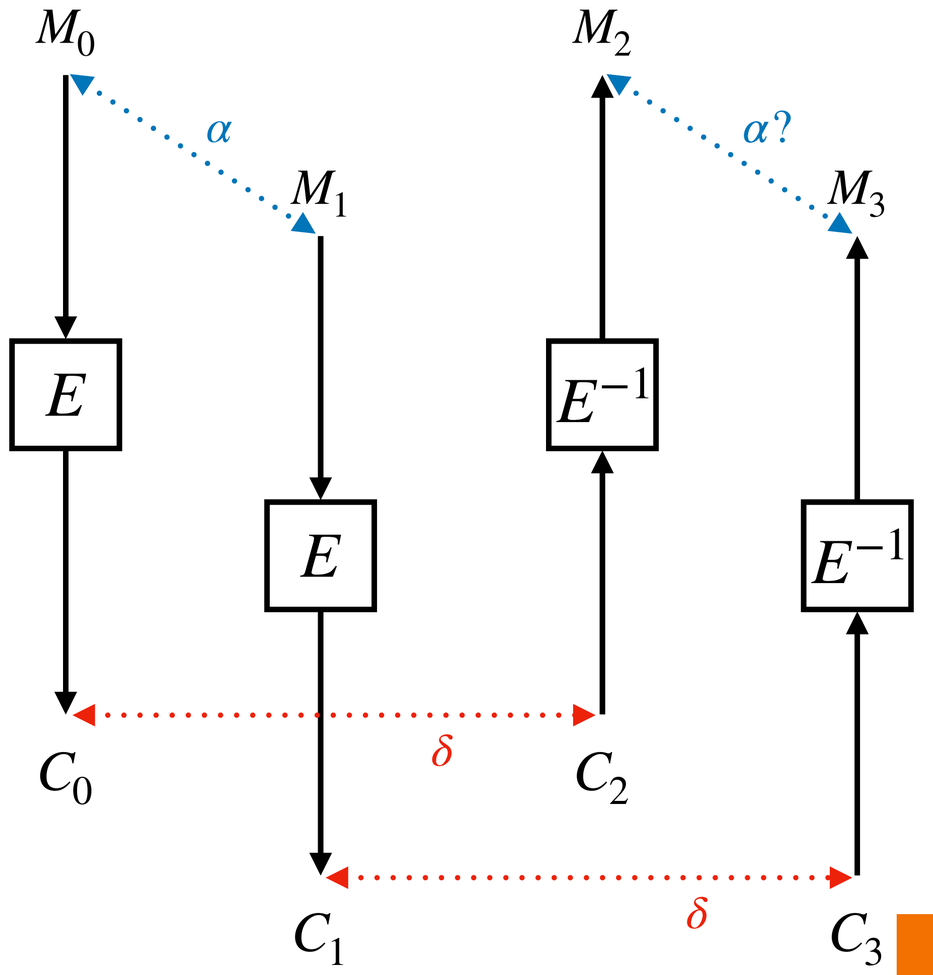
3. Compute $C_2 = C_0 \oplus \delta, C_3 = C_1 \oplus \delta$
4. Ask for their decryption (M_2, M_3)
5. Check if $M_2 \oplus M_3 = \alpha$



Basic boomerang distinguisher

[Wagner, FSE ’99]

Rewrite $E = E_1 \circ E_0$

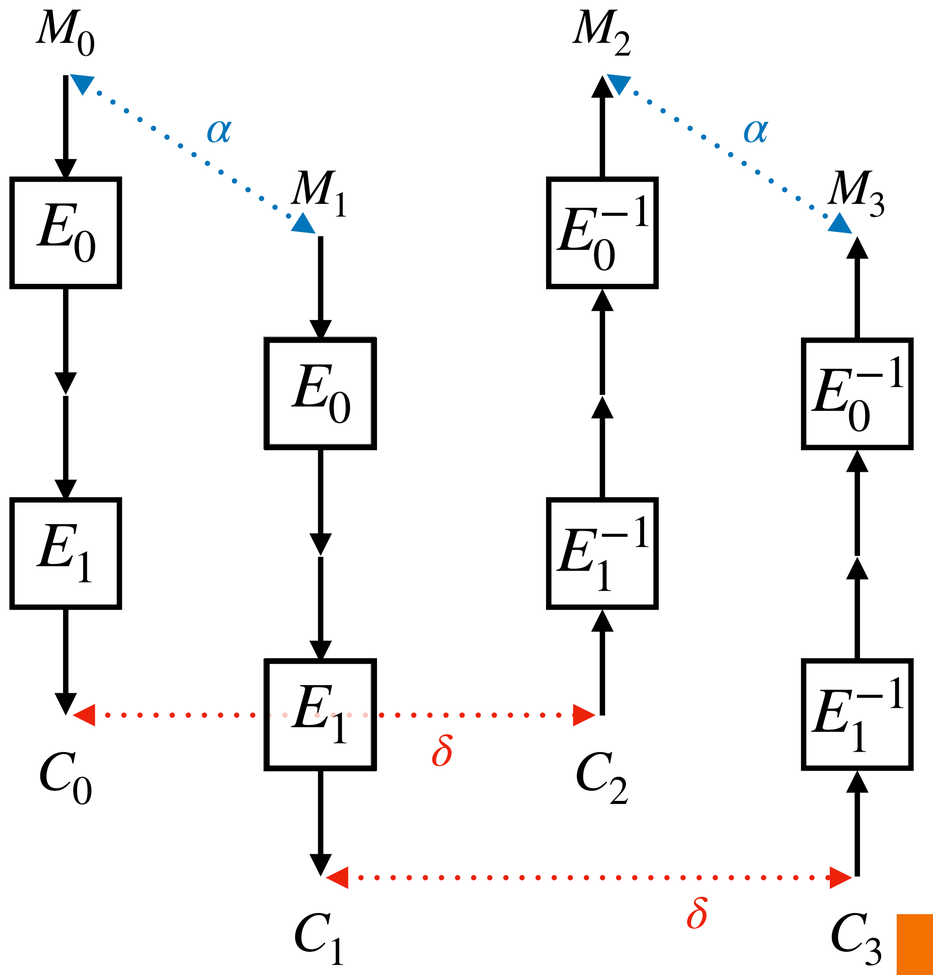


Basic boomerang distinguisher

[Wagner, FSE ’99]

Rewrite $E = E_1 \circ E_0$

Find good differentials:



Basic boomerang distinguisher

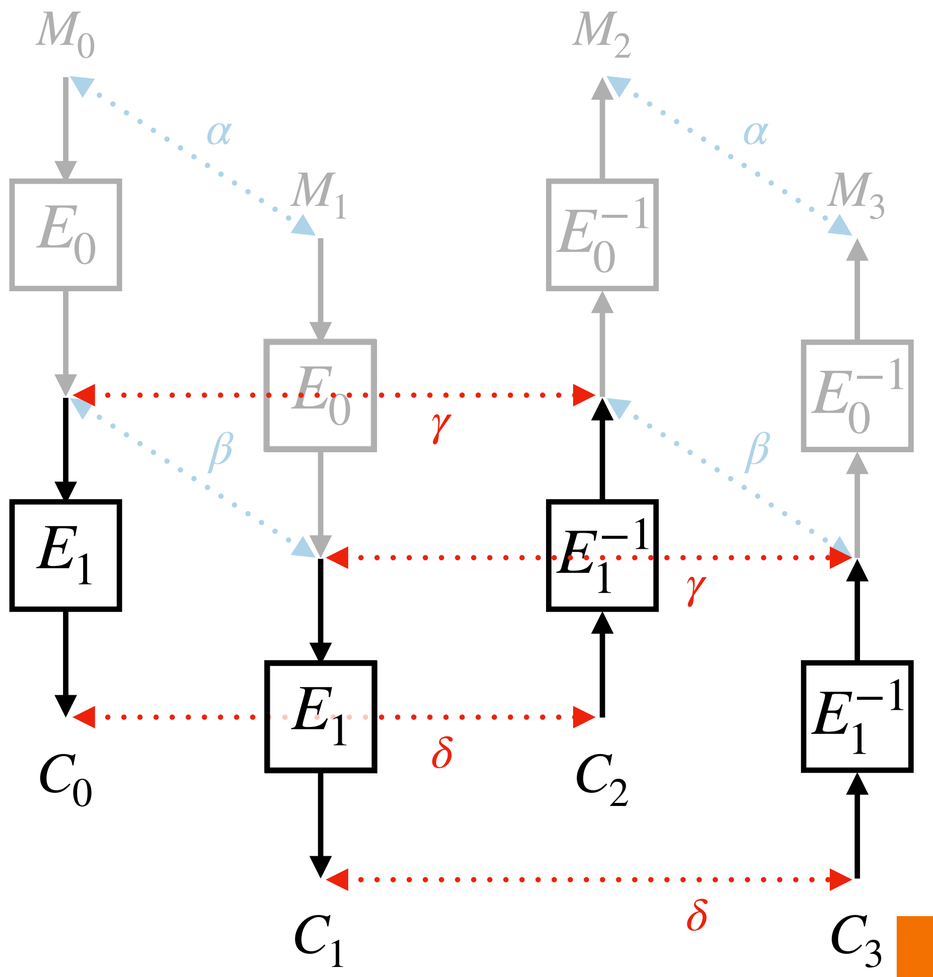
[Wagner, FSE '99]

Rewrite $E = E_1 \circ E_0$

Find good differentials:

$$\mathbb{P}(\alpha \longrightarrow_{E_0} \beta) = p$$
$$\mathbb{P}(\gamma \longrightarrow_{E_1} \delta) = q$$

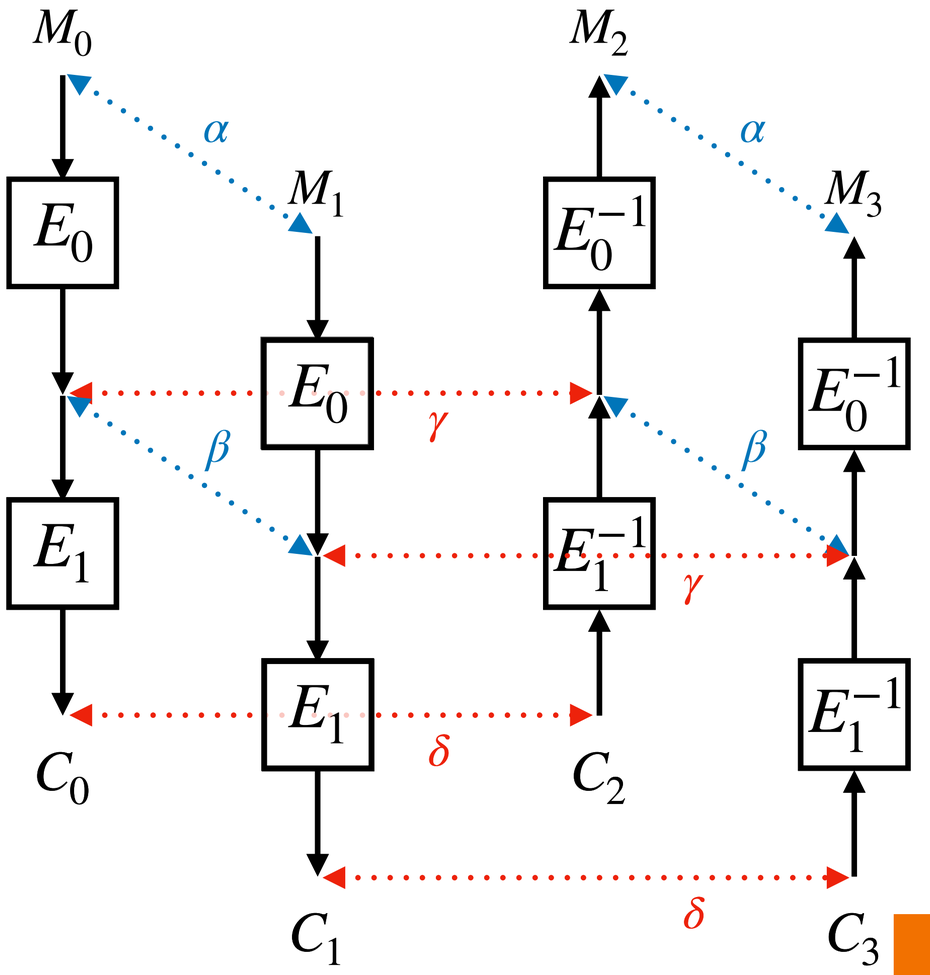
Expected probability of p^2q^2 if the two characteristics are "**independant**".



Basic boomerang distinguisher

[Wagner, FSE ’99]

Incompatibilities are discovered.



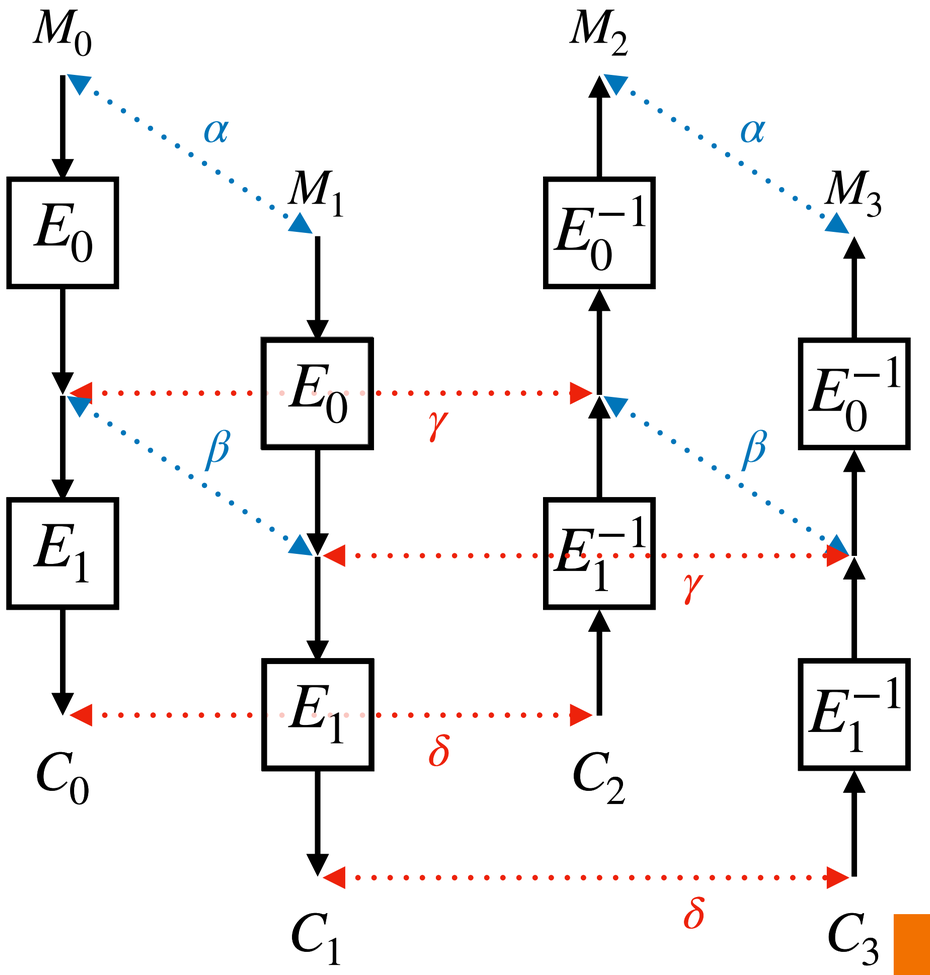
Basic boomerang distinguisher

[Wagner, FSE ’99]

Incompatibilities are discovered.

- Related-key Cryptanalysis of the Full AES-192 and AES-256
  Biryukov & Khovratovich, *ASIACRYPT 2009*
- The Return of the Cryptographic Boomerang
  Murphy, *IEEE Transactions on Information Theory 2011*

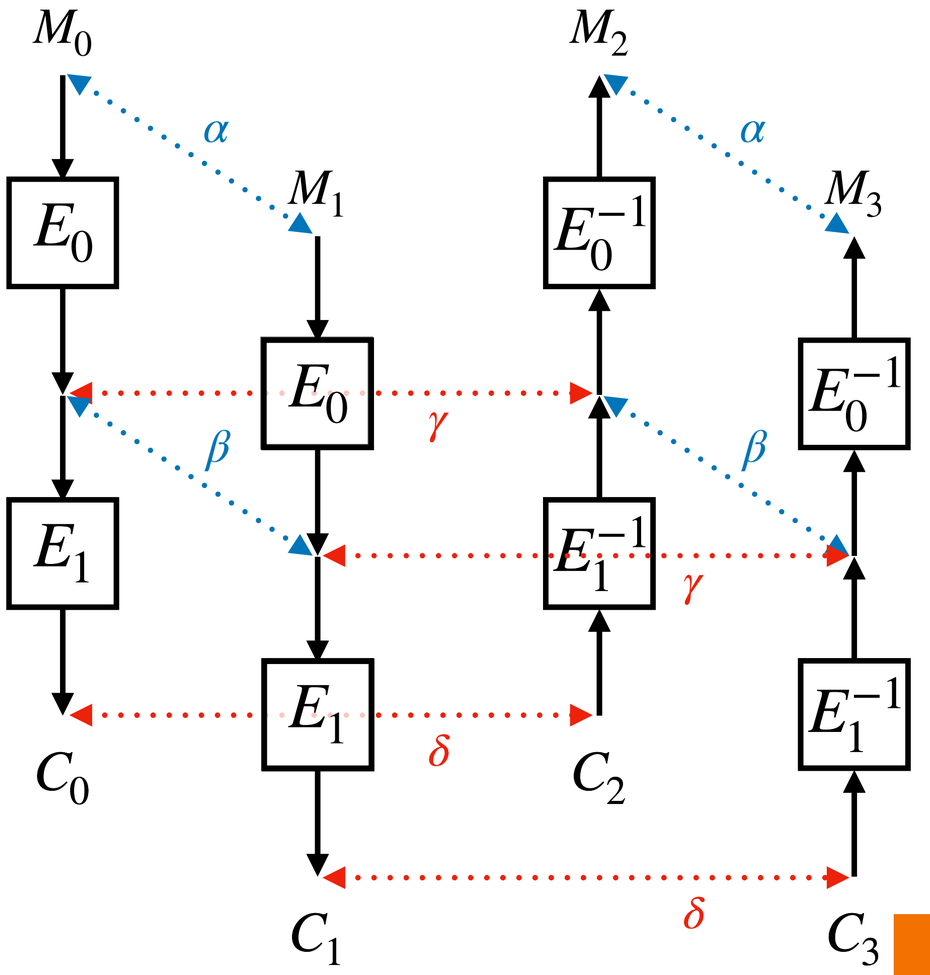
The problems come from interactions at the **junction** of the two trails.



Analysis of the junction

The sandwich attack

A Practical-time Related-key Attack on the KASUMI Cryptosystem Used in GSM and 3G Telephony
Dunkleman, Keller & Shamir, *CRYPTO 2010*



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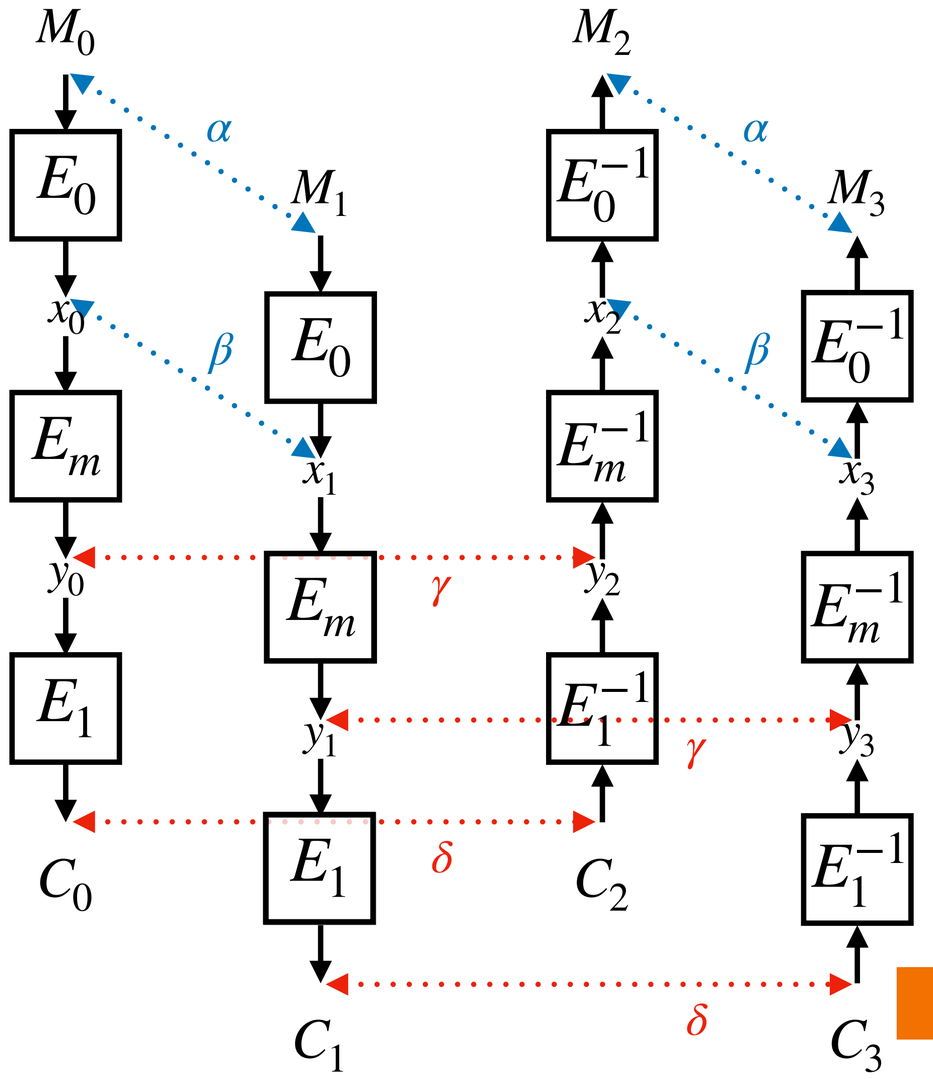
$$E = E_1 \circ E_m \circ E_0$$

E_m is 1 round (**boomerang switch**)

Expected probability of p^2q^2r



The Boomerang Connectivity Table

Automated analysis for a 1-round E_m for SPNs

Boomerang Connectivity Table: a New Cryptanalysis Tool
Cid, Huang, Peyrin, Sasaki & Song, *EUROCRYPT 2018*

The BCT

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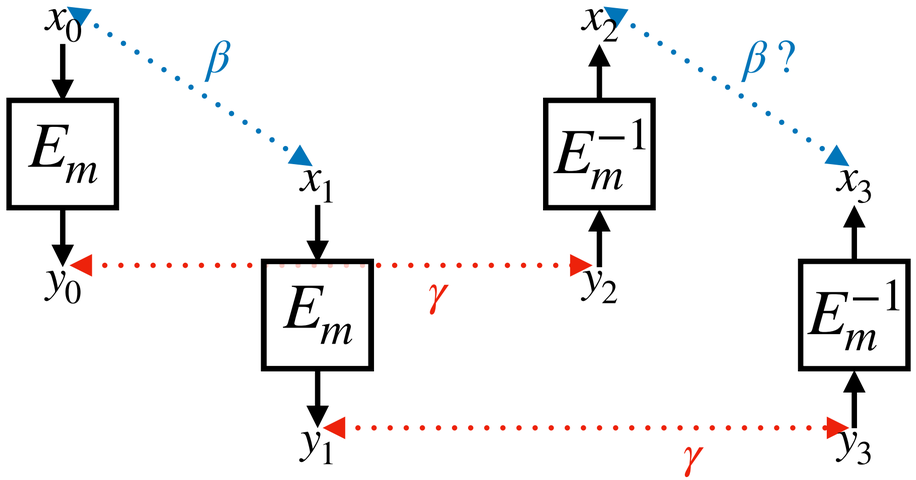

$$E_m^{-1}(E_m(X) \oplus \gamma) \oplus E_m^{-1}(E_m(X \oplus \beta) \oplus \gamma) = \beta$$

The BCT

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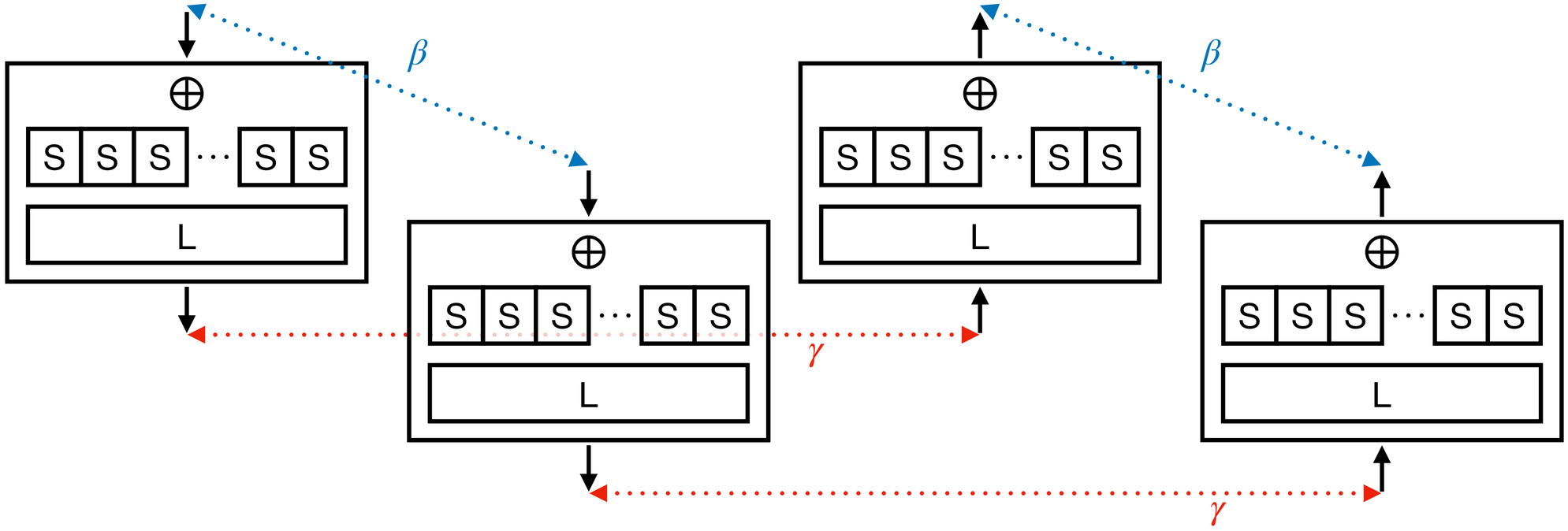

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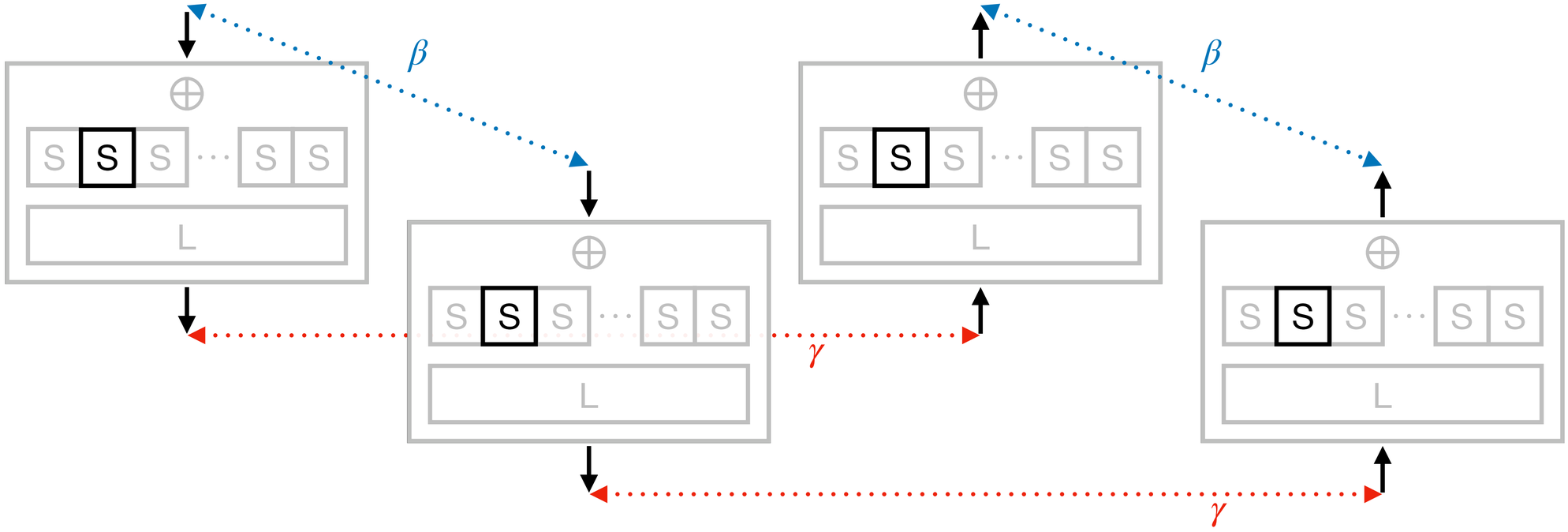

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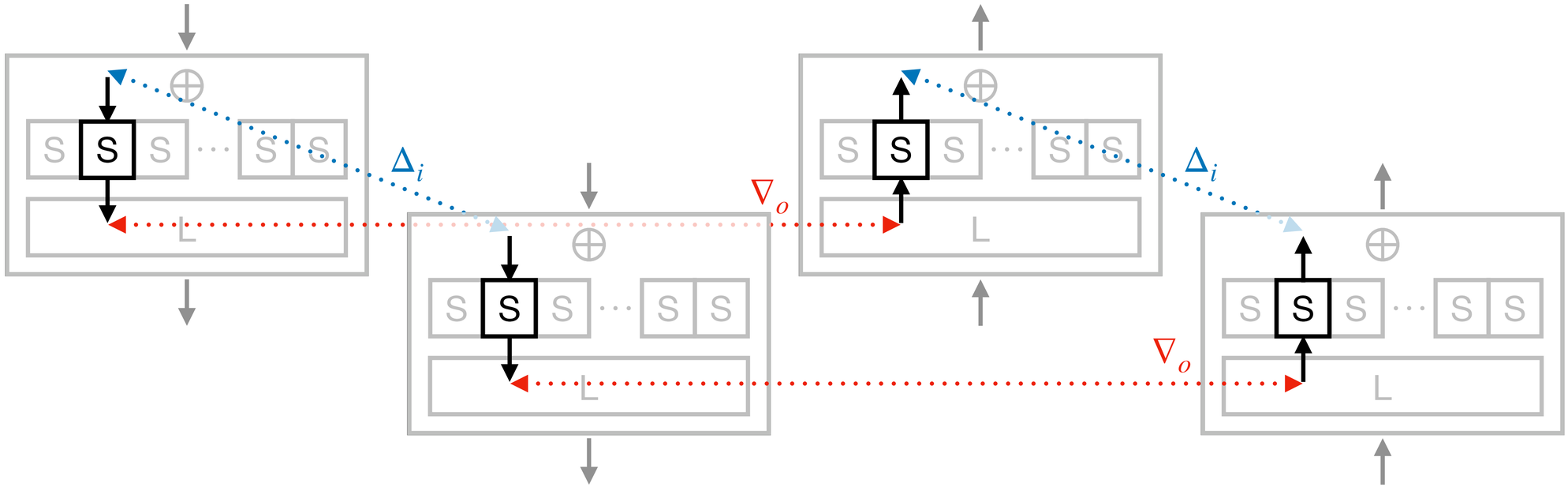

$$SE_m^{-1}(SE_m(X) \oplus \nabla_o) \oplus SE_m^{-1}(SE_m(X \oplus \Delta_i) \oplus \nabla_o) = \Delta_i$$

The BCT

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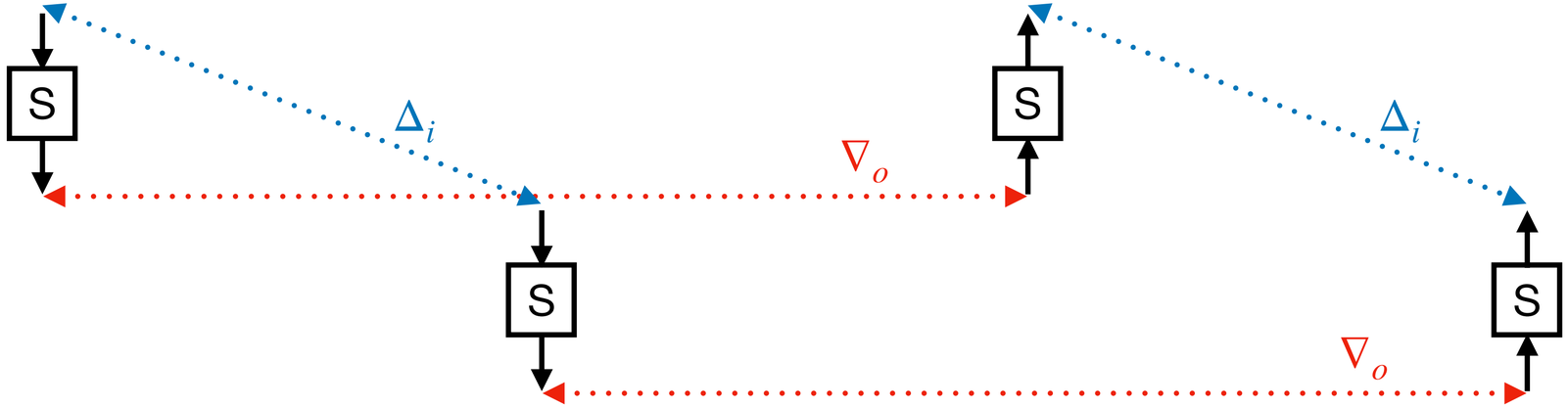

Probability over 1 round = product of the probabilities over each S-box

$$S^{-1}(S(x) \oplus \nabla_o) \oplus S^{-1}(S(x \oplus \Delta_i) \oplus \nabla_o) = \Delta_i$$

The BCT

$$\mathrm{BCT}(\Delta_i, \nabla_o) = \#\{x \mid S^{-1}(S(x) \oplus \nabla_o) \oplus S^{-1}(S(x \oplus \Delta_i) \oplus \nabla_o\}) = \Delta_i\}$$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**
1	**16**	0	**16**	0	0	0	0	0	8	8	8	8	0	0	0	0
2	**16**	8	0	8	8	**16**	8	0	0	0	0	0	0	0	0	0
3	**16**	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2
4	**16**	0	8	0	0	0	2	2	4	4	4	4	2	2	0	0
5	**16**	0	8	0	0	0	2	2	4	4	4	4	2	2	0	0
6	**16**	2	0	2	2	0	0	2	2	0	2	0	0	2	2	0
7	**16**	2	0	2	2	0	0	2	0	2	0	2	2	0	0	2
8	**16**	4	0	4	4	8	4	0	0	0	0	0	2	2	2	2
9	**16**	4	0	4	4	8	4	0	0	0	0	0	2	2	2	2
a	**16**	4	0	4	4	8	4	0	2	2	2	2	0	0	0	0
b	**16**	4	0	4	4	8	4	0	0	0	0	0	2	2	2	2
c	**16**	0	8	0	0	0	2	2	4	4	4	4	0	0	2	2
d	**16**	0	8	0	0	0	2	2	4	4	4	4	0	0	2	2
e	**16**	2	0	2	2	0	0	2	0	2	0	2	0	2	2	0
f	**16**	2	0	2	2	0	0	2	2	0	2	0	2	0	0	2

The BCT

Recap

- Reduces the problem of computing the boomerang switch over 1 round of SPN to the one of computing it over each S-box of its nonlinear layer
- Easily gives incompatibility, Ladder switch
- Gives a new criteria for S-boxes

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	0	**16**	0	0	0	0	0	8	8	8	8	0	0	0	0
2	16	8	0	8	8	**16**	8	0	0	0	0	0	0	0	0	0
3	16	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2
4	16	0	8	0	0	0	2	2	4	4	4	4	2	2	0	0
5	16	0	8	0	0	0	2	2	4	4	4	4	2	2	0	0
6	16	2	0	2	2	0	0	2	2	0	2	0	0	2	2	0
7	16	2	0	2	2	0	0	2	0	2	0	2	2	0	0	2
8	16	4	0	4	4	8	4	0	0	0	0	0	2	2	2	2
9	16	4	0	4	4	8	4	0	0	0	0	0	2	2	2	2
a	16	4	0	4	4	8	4	0	2	2	2	2	0	0	0	0
b	16	4	0	4	4	8	4	0	0	0	0	0	2	2	2	2
c	16	0	8	0	0	0	2	2	4	4	4	4	0	0	2	2
d	16	0	8	0	0	0	2	2	4	4	4	4	0	0	2	2
e	16	2	0	2	2	0	0	2	0	2	0	2	0	2	2	0
f	16	2	0	2	2	0	0	2	2	0	2	0	2	0	0	2

What about Feistel ciphers ?

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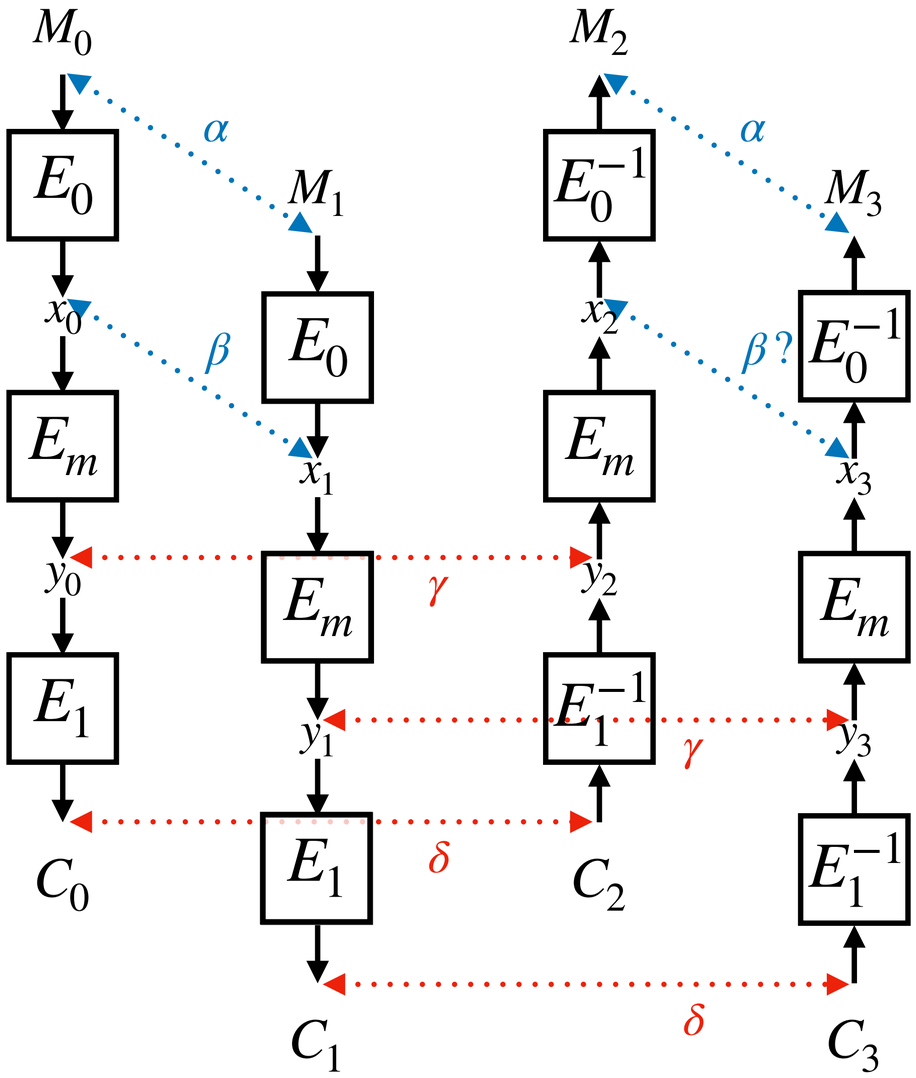
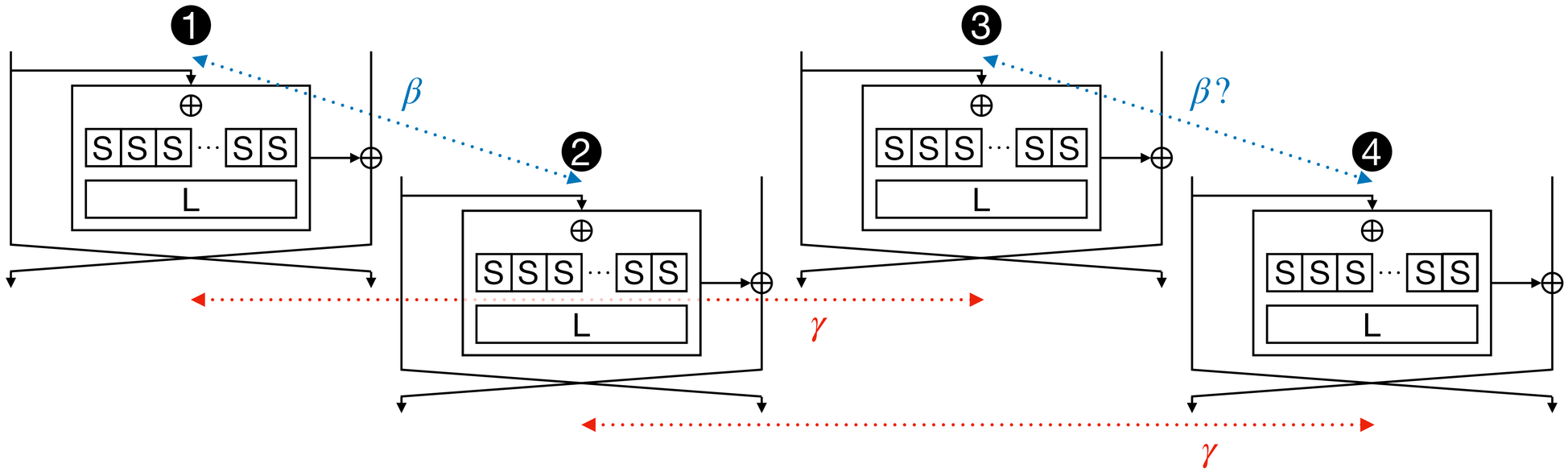
- Definition of the Feistel Boomerang Connectivity Table
- Properties of the FBCT & comparison with the BCT
- Multiple-round case

The Feistel Boomerang Connectivity Table

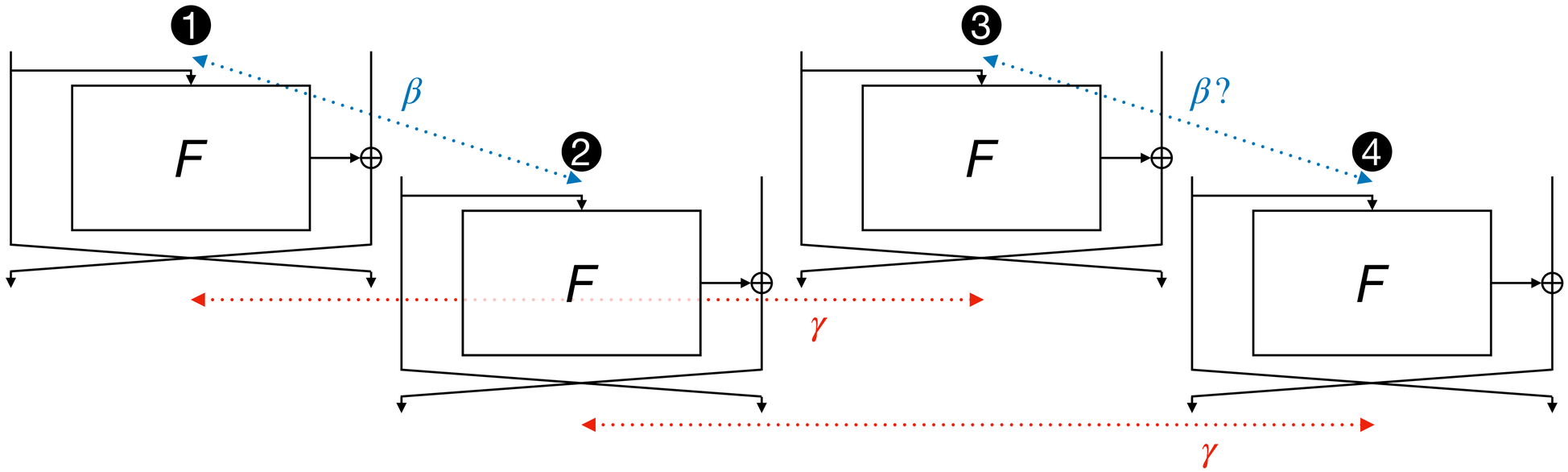
The Feistel counterpart of the BCT



The FBCT



The FBCT



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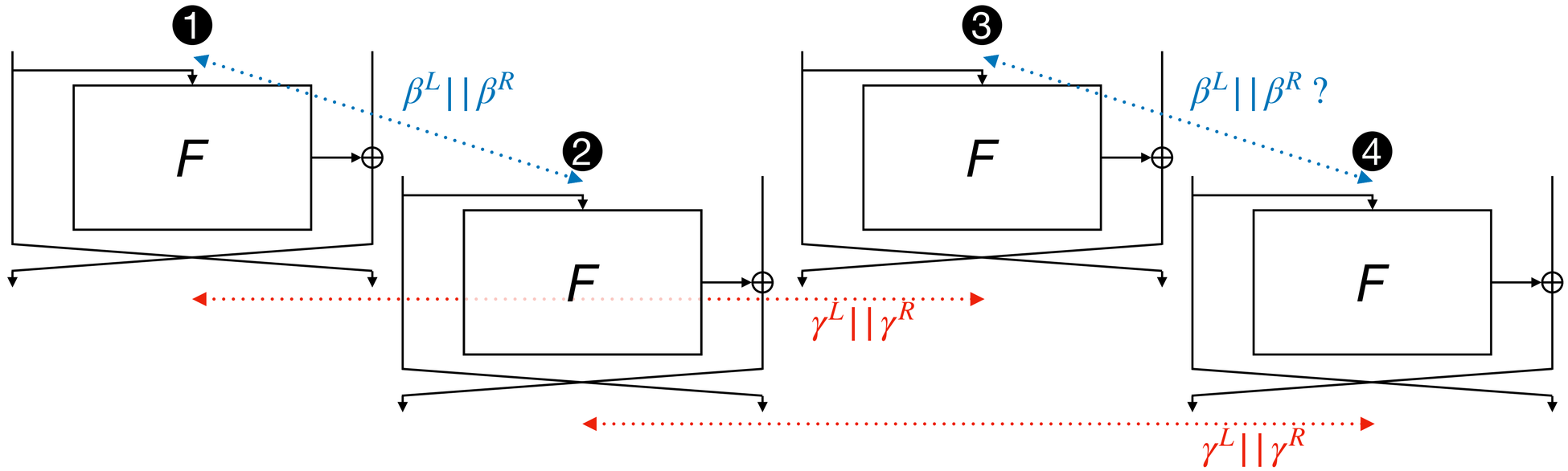
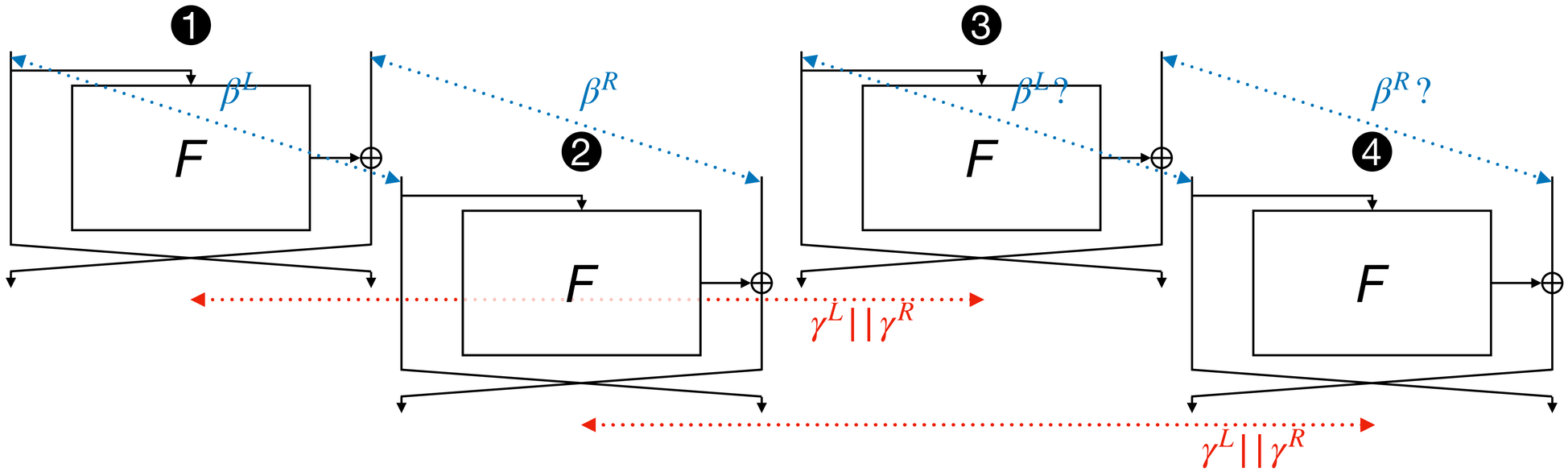
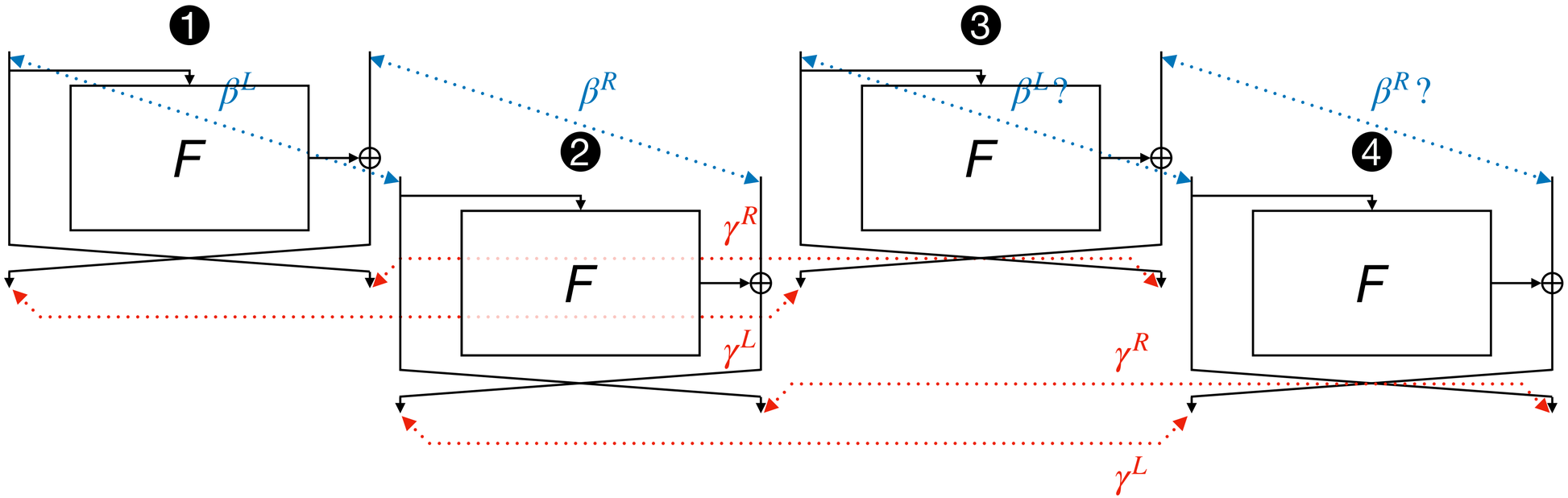

1
2
3
4
F
F
F
F
$\beta^L || \beta^R$
$\beta^L || \beta^R$?
$\gamma^L || \gamma^R$
$\gamma^L || \gamma^R$

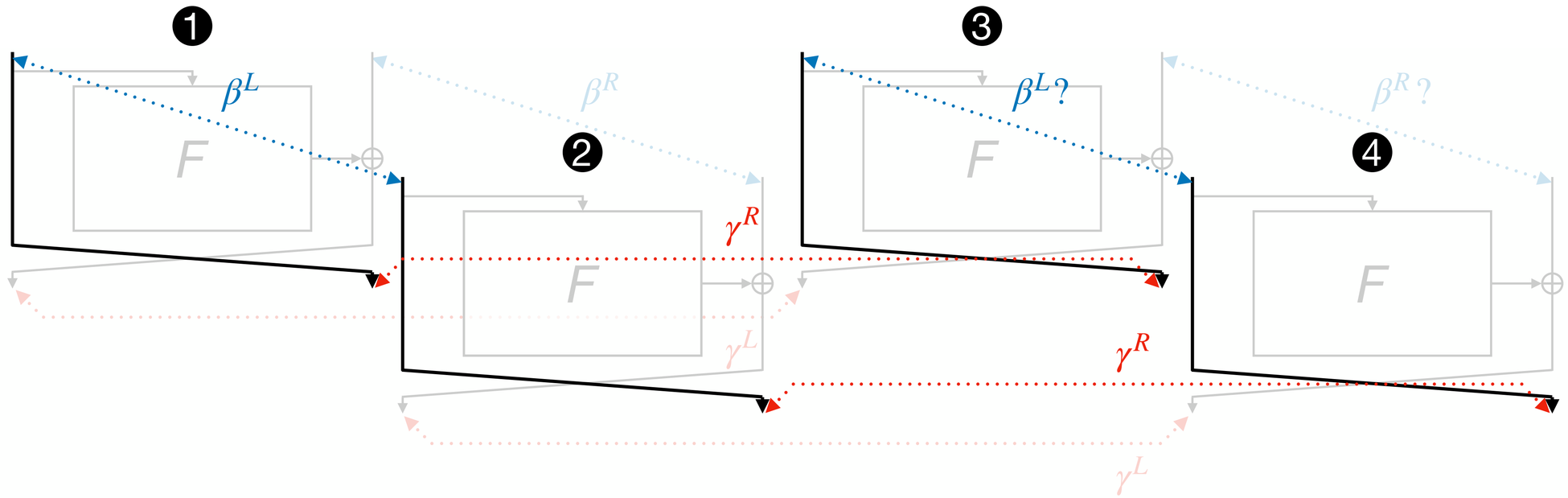
The FBCT


❶
❷
❸
❹
β^L
β^R
β^L ?
β^R ?
F
F
F
F
$\gamma^L || \gamma^R$
$\gamma^L || \gamma^R$

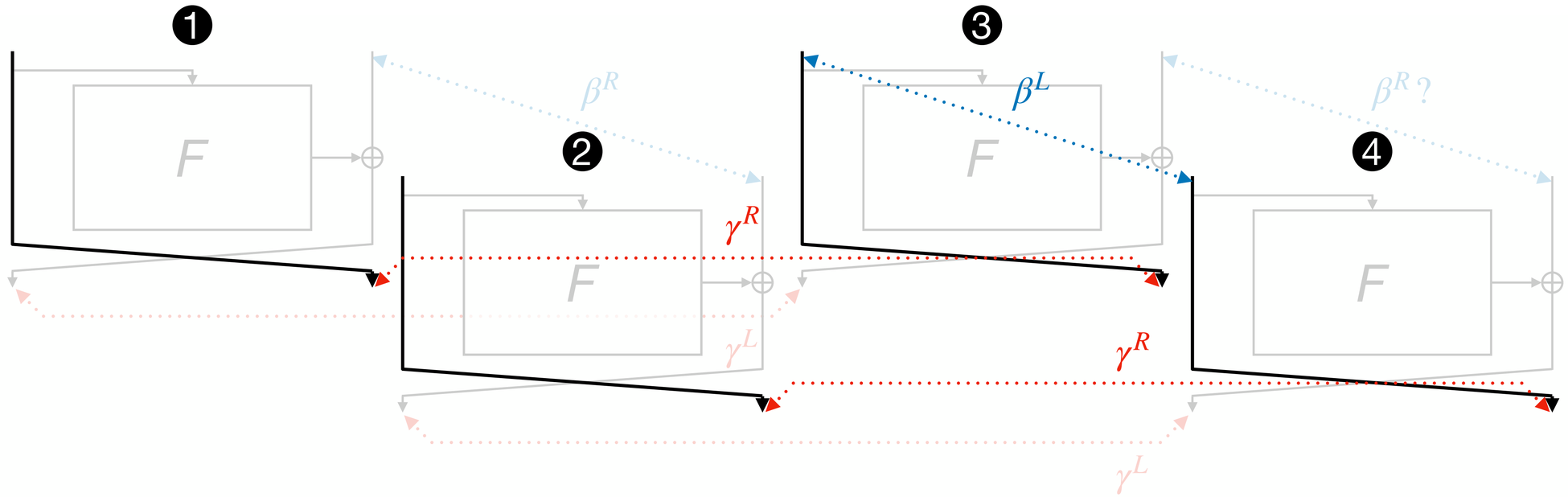
The FBCT


❶
❷
❸
❹
F
F
F
F
β^L
β^R
β^L ?
β^R ?
γ^R
γ^L
γ^R
γ^L

The FBCT (left part)

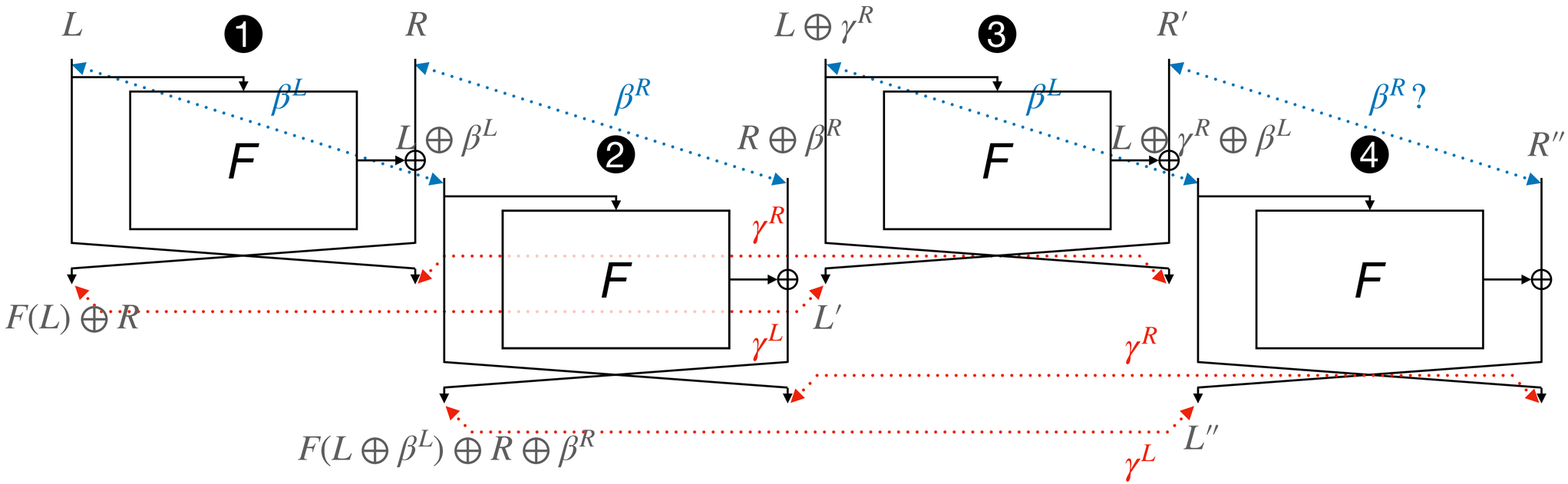


The FBCT (left part)



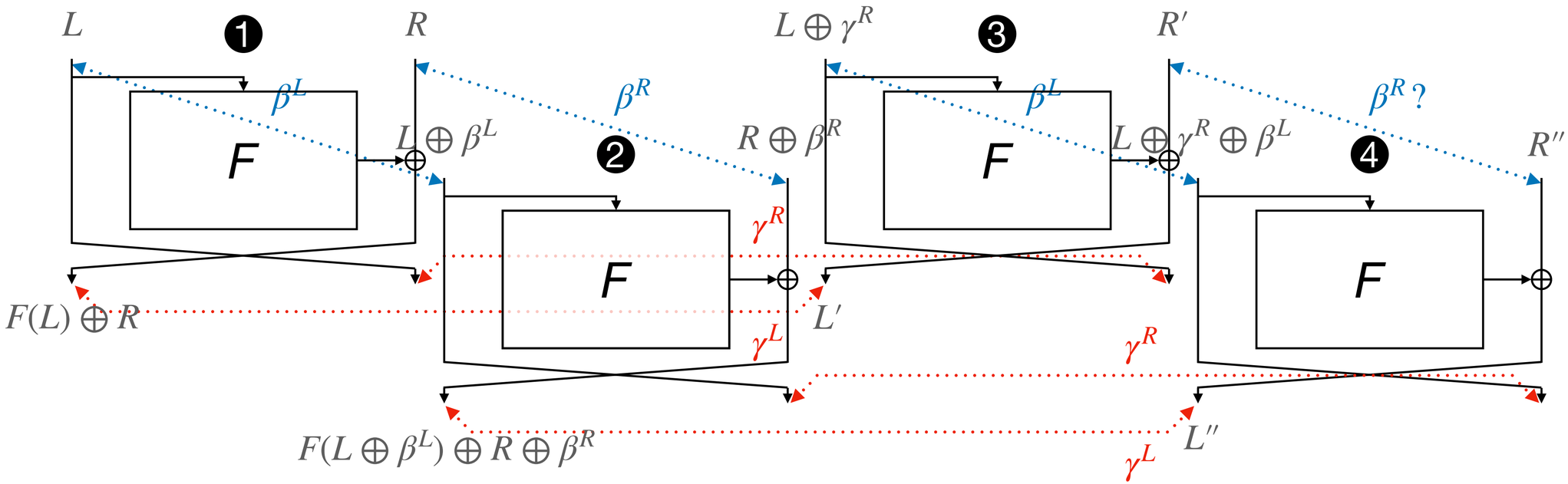
The left part of the difference comes for free.

The FBCT (right part)



We want that $R' \oplus R'' = \beta^R$

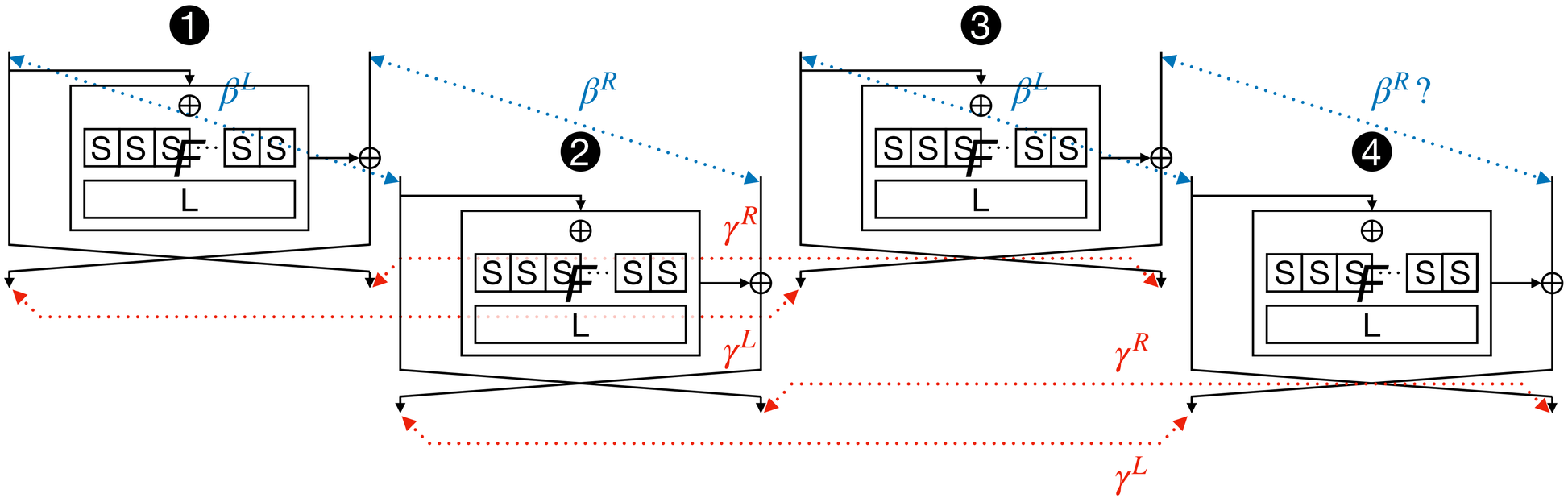
The FBCT (right part)



We want that $R' \oplus R'' = \beta^R$

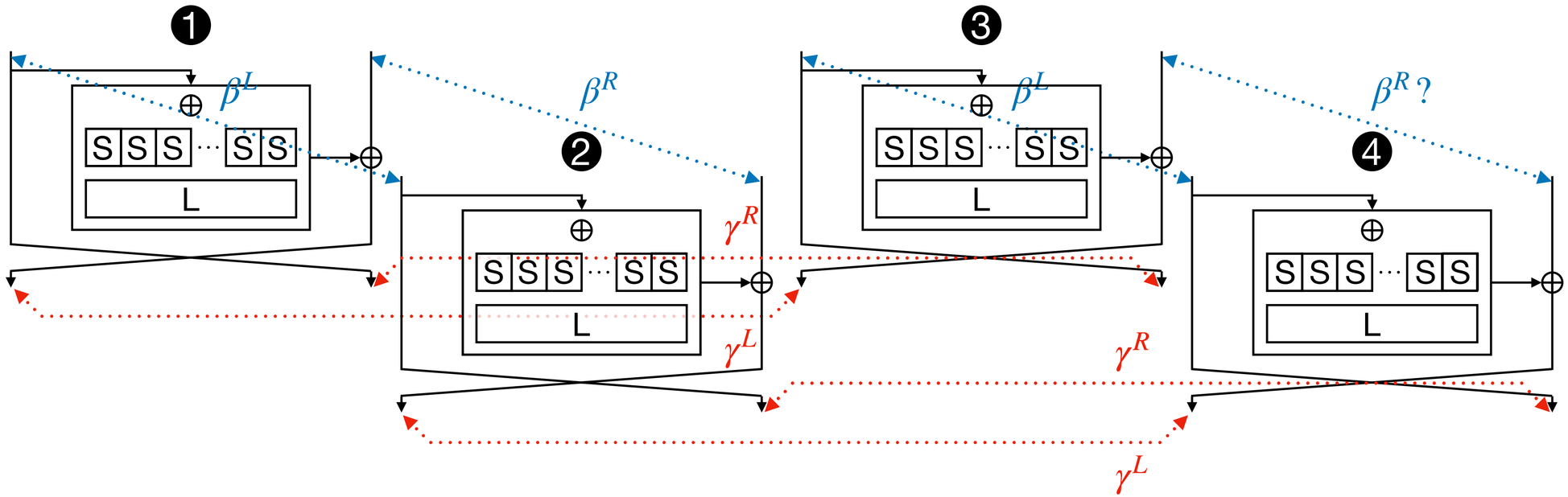
$$R' \oplus R'' = \underbrace{F(L \oplus \gamma^R) \oplus F(L) \oplus F(L \oplus \gamma^R \oplus \beta^L) \oplus F(L \oplus \beta^L)}_{0} \oplus \beta^R$$

The FBCT (right part)



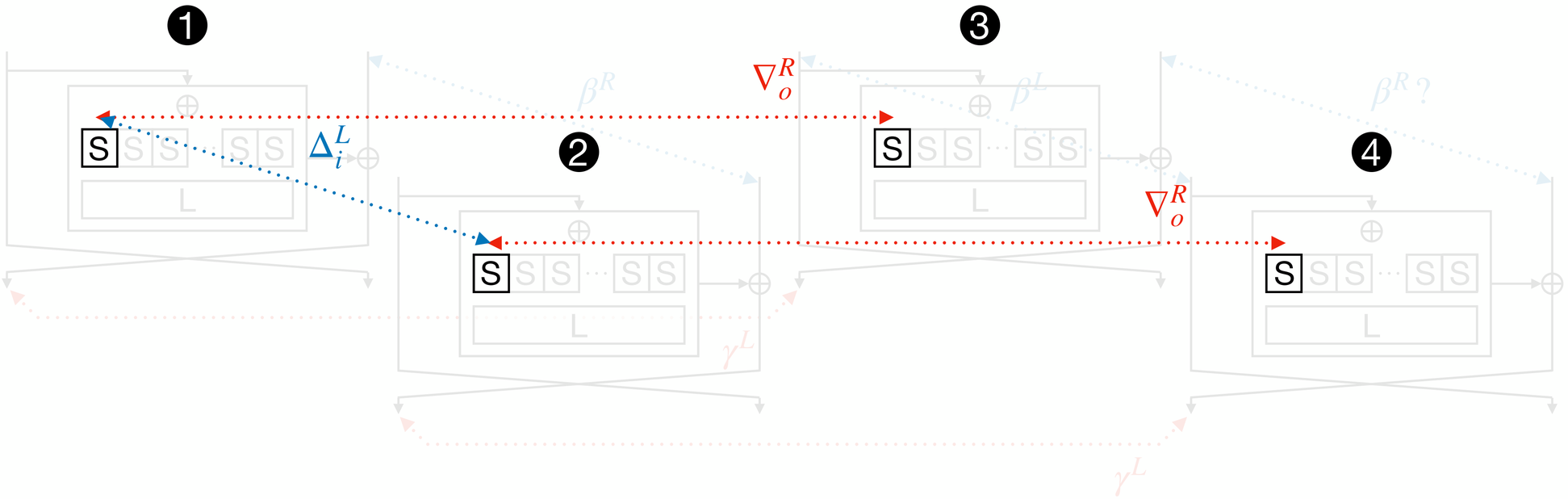
$$F(L \oplus \gamma^R) \oplus F(L) \oplus F(L \oplus \gamma^R \oplus \beta^L) \oplus F(L \oplus \beta^L) = 0$$

The FBCT (right part)



$$F(L \oplus \gamma^R) \oplus F(L) \oplus F(L \oplus \gamma^R \oplus \beta^L) \oplus F(L \oplus \beta^L) = 0$$

The FBCT (right part)



$$F(L \oplus \gamma^R) \oplus F(L) \oplus F(L \oplus \gamma^R \oplus \beta^L) \oplus F(L \oplus \beta^L) = 0$$

$$S(x \oplus \nabla_o^R) \oplus S(x) \oplus S(x \oplus \nabla_o^R \oplus \Delta_i^L) \oplus S(x \oplus \Delta_i^L) = 0$$

second derivative canceling out

Properties of the FBCT

$$\mathrm{FBCT}_S(\Delta_i, \nabla_o) = \#\{x \in \mathbb{F}_2^n \mid S(x) \oplus S(x \oplus \Delta_i) \oplus S(x \oplus \nabla_o) \oplus S(x \oplus \Delta_i \oplus \nabla_o) = 0\}$$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**
1	**16**	**16**	0	0	0	0	0	0	0	0	8	8	0	0	0	0
2	**16**	0	**16**	0	0	0	0	0	0	0	0	8	0	0	0	0
3	**16**	0	0	**16**	8	8	8	8	0	0	0	0	0	0	0	0
4	**16**	0	0	8	**16**	0	0	8	0	0	0	0	0	0	0	0
5	**16**	0	0	8	0	**16**	8	0	0	0	0	0	0	0	0	0
6	**16**	0	0	8	0	8	**16**	0	0	0	0	0	0	0	0	0
7	**16**	0	0	8	8	0	0	**16**	0	0	0	0	0	0	0	0
8	**16**	0	0	0	0	0	0	0	**16**	0	0	0	0	0	0	0
9	**16**	0	8	0	0	0	0	0	0	**16**	0	8	0	0	0	0
a	**16**	8	0	0	0	0	0	0	0	0	**16**	8	0	0	0	0
b	**16**	8	8	0	0	0	0	0	0	8	8	**16**	0	0	0	0
c	**16**	0	0	0	0	0	0	0	0	0	0	0	**16**	0	0	0
d	**16**	0	0	0	0	0	0	0	0	0	0	0	0	**16**	0	0
e	**16**	0	0	0	0	0	0	0	0	0	0	0	0	0	**16**	0
f	**16**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**16**

Properties of the FBCT

$$\mathrm{FBCT}_S(\Delta_i, \nabla_o) = \#\{x \in \mathbb{F}_2^n \mid S(x) \oplus S(x \oplus \Delta_i) \oplus S(x \oplus \nabla_o) \oplus S(x \oplus \Delta_i \oplus \nabla_o) = 0\}$$

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**
1	**16**	**16**	0	0	0	0	0	0	0	0	8	8	0	0	0	0
2	**16**	0	**16**	0	0	0	0	0	0	0	0	8	0	0	0	0
3	**16**	0	0	**16**	8	8	8	8	0	0	0	0	0	0	0	0
4	**16**	0	0	8	**16**	0	0	8	0	0	0	0	0	0	0	0
5	**16**	0	0	8	0	**16**	8	0	0	0	0	0	0	0	0	0
6	**16**	0	0	8	0	8	**16**	0	0	0	0	0	0	0	0	0
7	**16**	0	0	8	8	0	0	**16**	0	0	0	0	0	0	0	0
8	**16**	0	0	0	0	0	0	0	**16**	0	0	0	0	0	0	0
9	**16**	0	8	0	0	0	0	0	0	**16**	0	8	0	0	0	0
a	**16**	8	0	0	0	0	0	0	0	0	**16**	8	0	0	0	0
b	**16**	8	8	0	0	0	0	0	0	8	8	**16**	0	0	0	0
c	**16**	0	0	0	0	0	0	0	0	0	0	0	**16**	0	0	0
d	**16**	0	0	0	0	0	0	0	0	0	0	0	0	**16**	0	0
e	**16**	0	0	0	0	0	0	0	0	0	0	0	0	0	**16**	0
f	**16**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**16**

Symmetry: $\mathrm{FBCT}(\Delta_i, \nabla_o) = \mathrm{FBCT}(\nabla_o, \Delta_i)$

Diagonal: $\mathrm{FBCT}(\Delta_i, \Delta_i) = 2^n$

Multiplicity: $\mathrm{FBCT}(\Delta_i, \nabla_o) \equiv 0 \pmod 4$

Equalities: $\mathrm{FBCT}(\Delta_i, \nabla_o) = \mathrm{FBCT}(\Delta_i, \Delta_i \oplus \nabla_o)$

Properties of the FBCT

Theorem

S is APN if and only if its FBCT verifies $\mathrm{FBCT}(\Delta_i, \nabla_o) = 0 \ \forall \ 1 \leq \Delta_i \neq \nabla_o \leq 2^n - 1$

Properties of the FBCT

Theorem

S is APN if and only if its FBCT verifies $\text{FBCT}(\Delta_i, \nabla_o) = 0 \ \forall \ 1 \leq \Delta_i \neq \nabla_o \leq 2^n - 1$

e.g. S = [1, 3, 6, 5, 2, 4, 7, 0]

8	0	0	0	0	0	0	0
0	0	2	2	0	0	2	2
0	0	0	0	2	2	2	2
0	0	2	2	2	2	0	0
0	2	0	2	0	2	0	2
0	2	2	0	0	2	2	0
0	2	0	2	2	0	2	0
0	2	0	0	2	0	0	2

DDT

8	8	8	8	8	8	8	8
8	8	0	0	0	0	0	0
8	0	8	0	0	0	0	0
8	0	0	8	0	0	0	0
8	0	0	0	8	0	0	0
8	0	0	0	0	8	0	0
8	0	0	0	0	0	8	0
8	0	0	0	0	0	0	8

FBCT

8	8	8	8	8	8	8	8
8	0	2	2	0	0	2	2
8	0	0	0	2	2	2	2
8	0	2	2	2	2	0	0
8	2	0	2	0	2	0	2
8	2	2	0	0	2	2	0
8	2	0	2	2	0	2	0
8	2	0	0	2	0	0	2

BCT

Comparing the BCT and the FBCT

Boomerang uniformity for the **SPN** case:
$\max_{\Delta_i \neq 0, \nabla_o \neq 0} BCT(\Delta_i, \nabla_o)$

On the Boomerang Uniformity of Cryptographic Sboxes
Boura & Canteaut, *ToSC 2018*

Comparing the BCT and the FBCT

Boomerang uniformity for the **SPN** case:

$\max_{\Delta_i \neq 0, \nabla_o \neq 0} BCT(\Delta_i, \nabla_o)$

Boomerang uniformity for the **Feistel** case:

$\max_{\Delta_i \neq 0, \nabla_o \neq 0, \Delta_i \neq \nabla_o} \mathrm{FBCT}(\Delta_i, \nabla_o)$

Comparing the BCT and the FBCT

Boomerang uniformity for the **SPN** case:
$\max_{\Delta_i \neq 0, \nabla_o \neq 0} BCT(\Delta_i, \nabla_o)$

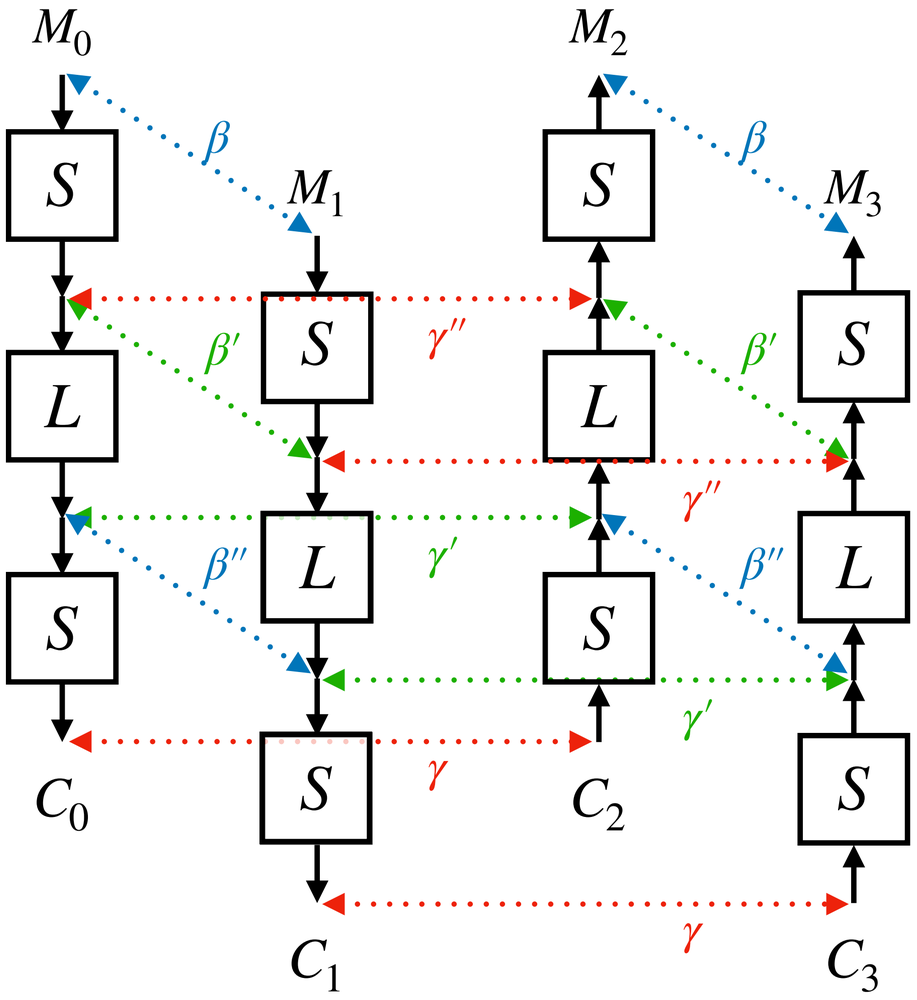
Boomerang uniformity for the **Feistel** case:
$\max_{\Delta_i \neq 0, \nabla_o \neq 0, \Delta_i \neq \nabla_o} \mathrm{FBCT}(\Delta_i, \nabla_o)$

Boomerang uniformity preserved under	BCT	FBCT
Affine equivalence	✓	✓
Extended-affine equivalence	✗	✓
CCZ equivalence	✗	✗
Inversion (if S is invertible)	✓	✗

A good S-box for an SPN is a good S-box for a Feistel regarding many usual criteria **(differential, linear, algebraic degree)**
But its behavior can be different regarding boomerang switches if we use it in an SPN or a Feistel.

Boomerang switches over more rounds

Two-round case



Boomerang Switch in Multiple Rounds

Wang & Peyrin, *ToSC 2019*

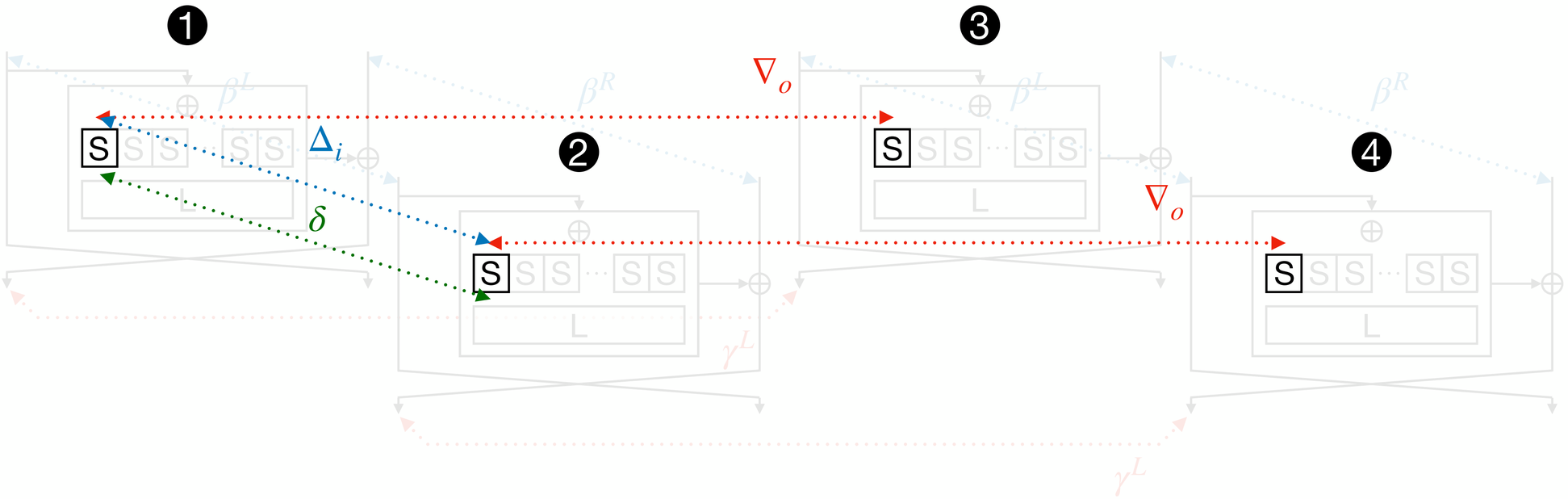
Boomerang Connectivity Table Revisited. Application to SKINNY and AES

Song, Qin & Hu, *ToSC 2019*

$$BDT(\beta, \beta', \gamma'') = \#\{x \mid S^{-1}(S(x) \oplus \gamma'') \oplus S^{-1}(S(x \oplus \beta) \oplus \gamma'') = \beta, \\ S(x) \oplus S(x \oplus \beta) = \beta'\}$$

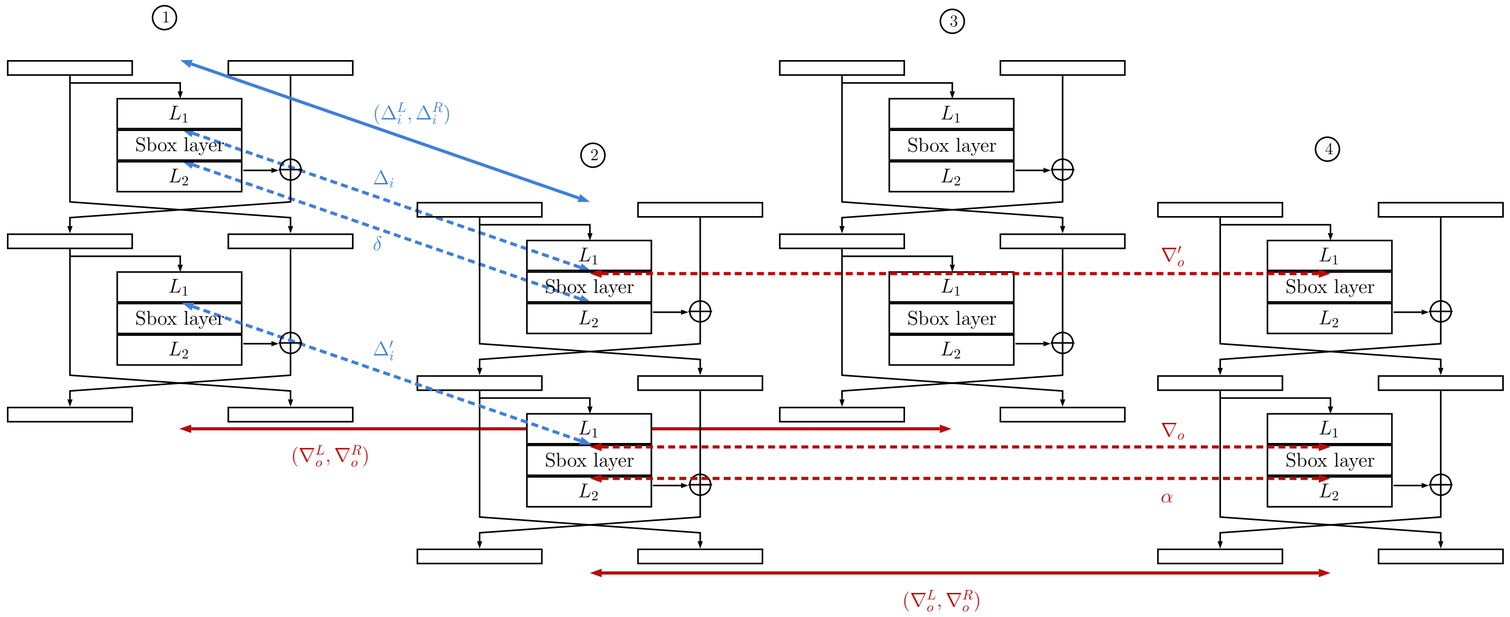
$$BDT'(\gamma, \gamma', \beta'') = \#\{x \mid S(S^{-1}(x) \oplus \beta'') \oplus S(S^{-1}(x \oplus \gamma) \oplus \beta'') = \gamma, \\ S^{-1}(x) \oplus S^{-1}(x \oplus \gamma) = \gamma'\}$$

Two-round case



$$FBDT(\Delta_i, \delta, \nabla_o) = \#\{x \in \mathbb{F}_2^n \mid S(x) \oplus S(x \oplus \Delta_i) \oplus S(x \oplus \nabla_o) \oplus S(x \oplus \Delta_i \oplus \nabla_o) = 0$$
$$\text{and } S(x) \oplus S(x \oplus \Delta_i) = \delta\}$$

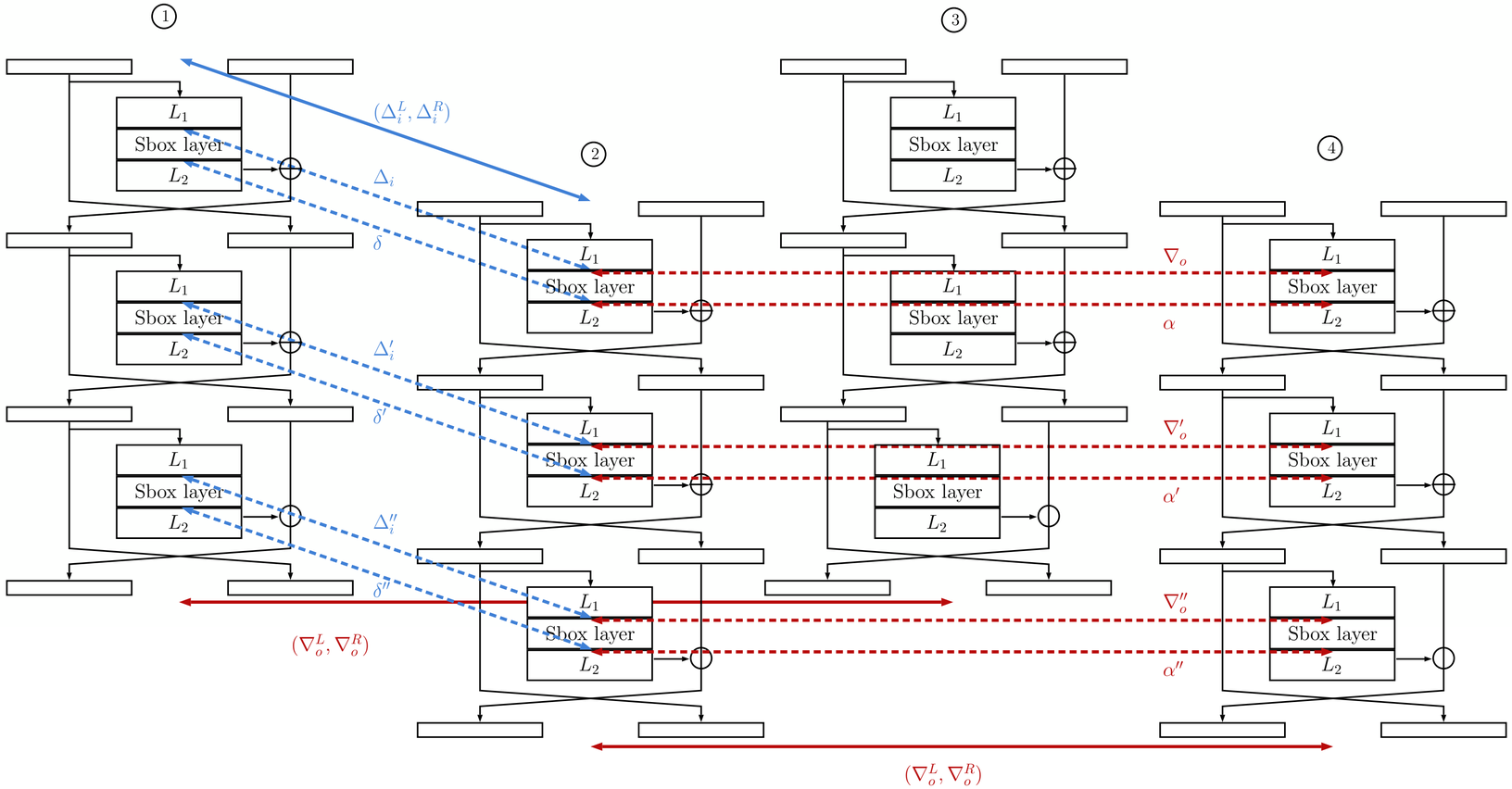
Two-round case



$$2^{-2tn} \times \sum_{0 \le \delta, \alpha < 2^n} FBDT(\Delta_i, \delta, \nabla_o') \times FBDT(\nabla_o, \alpha, \Delta_i')$$

$$FBDT(\Delta_i, \delta, \nabla_o) = \#\{x \in \mathbb{F}_2^n \mid S(x) \oplus S(x \oplus \Delta_i) \oplus S(x \oplus \nabla_o) \oplus S(x \oplus \Delta_i \oplus \nabla_o) = 0$$
$$\text{and } S(x) \oplus S(x \oplus \Delta_i) = \delta\}$$

Switches over 3 rounds and more…



FBET table:

$$\#\{x \in \mathbb{F}_2^n \mid S(x) \oplus S(x \oplus \Delta_i) \oplus S(x \oplus \nabla_o) \oplus S(x \oplus \Delta_i \oplus \nabla_o) = 0,$$
$$S(x) \oplus S(x \oplus \Delta_i) = \delta,$$
$$S(x \oplus \Delta_i) \oplus S(x \oplus \Delta_i \oplus \nabla_o) = \alpha\}$$

Conclusion

- Introduction of the **FBCT**, a new tool that:
  - easily evaluates the probability of a 1-round boomerang switch
  - gives a new criterion when choosing an S-box for a Feistel cipher
- Proposal of a **generic formula** for a switch over many rounds:
  - evaluation is computationally expensive if E_m covers many rounds with many active S-boxes
  - might be preferable to experimentally evaluate it

Thank you for your attention.