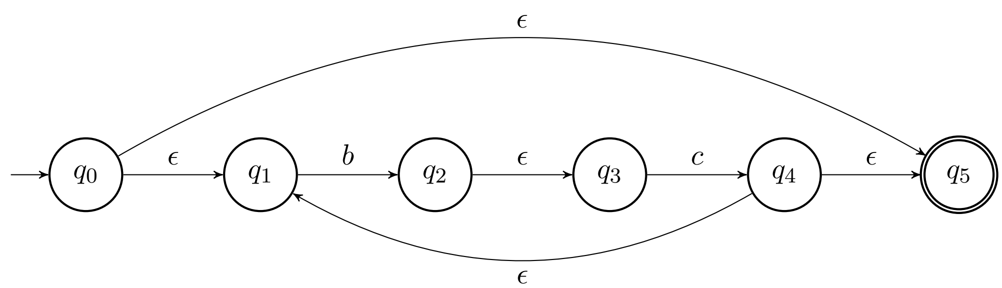
Formal Languages

I. Give a finite state automaton that accepts the language specified by the following regular expression:

$$(a + (b \cdot c))^*$$

II. Consider the following automaton:



II.A. Remember that $Cl_\epsilon(q)$, the ϵ-closure of a state q, is inductively defined as follows :

1. $q \in Cl_\epsilon(q)$;
2. if $p \in Cl_\epsilon(q)$ and $s \in \delta(p, \epsilon)$ then $s \in Cl_\epsilon(q)$.

Compute the ϵ-closures of the six states of the above automaton.

II.B. Eliminate the ϵ-transitions from the above automaton.

III. Let $P = \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\rangle$ be the pushdown automaton such that:

- $Q = \{q_0, q_1\}$;
- $\Sigma = \{a, b\}$;
- $\Gamma = \{Z_0, Z_1\}$;
- $F = \{q_1\}$;
- $\delta(q_0, a, Z_0) = \{(q_0, Z_1 Z_0)\}$ $\quad$ $\delta(q_0, a, Z_1) = \{(q_0, Z_1 Z_1)\}$
  $\delta(q_0, b, Z_1) = \{(q_1, Z_1)\}$ $\quad$ $\delta(q_1, a, Z_1) = \{(q_1, \epsilon)\}$
  $\delta(q_1, \epsilon, Z_0) = \{(q_1, \epsilon)\}$ $\quad$ $\delta(\_, \_, \_) = \varnothing$ (in all the other cases).

III.A. Give a context-free grammar that generates the language recognized by P according to the empty-stack criterion.

Hint: it is sufficient to consider only the following non-terminal symbols: $[q_0 Z_0 q_1]$, $[q_1 Z_0 q_1]$, $[q_0 Z_1 q_1]$, $[q_1 Z_1 q_1]$.

III.B. What is the language recognized by P ?

IV. Let $G = \langle N, \Sigma, P, S \rangle$ be a context-free grammar. Define a context-free grammar G_k such that $L(G_k) = (L(G))^*$.