Semantics

Consider the following abstract syntax together with its associated Montague-like semantics:

$$
\begin{array}{lcl}
\textsc{alan} & : & \mathrm{NP} \\
\textsc{kurt} & : & \mathrm{NP} \\
\textsc{logician} & : & \mathrm{N} \\
\textsc{british} & : & \mathrm{N} \to \mathrm{N} \\
\textsc{a} & : & \mathrm{N} \to (\mathrm{NP} \to \mathrm{S}) \to \mathrm{S} \\
\textsc{is} & : & \mathrm{NP} \to \mathrm{NP} \to \mathrm{S} \\
\textsc{not} & : & \mathrm{S} \to \mathrm{S}
\end{array}
$$

$$
\begin{array}{lcl}
[\![\mathrm{NP}]\!] & = & \mathrm{e} \\
[\![\mathrm{N}]\!] & = & \mathrm{e} \to \mathrm{t} \\
[\![\mathrm{S}]\!] & = & \mathrm{t}
\end{array}
$$

$$
\begin{array}{lcl}
[\![\textsc{alan}]\!] & = & \mathbf{alan} \\
[\![\textsc{kurt}]\!] & = & \mathbf{kurt} \\
[\![\textsc{logician}]\!] & = & \mathbf{logician} \\
[\![\textsc{british}]\!] & = & \lambda px.\, (\mathbf{british}\, x) \wedge (p\, x) \\
[\![\textsc{a}]\!] & = & \lambda pq.\, \exists y.\, (p\, y) \wedge (q\, y) \\
[\![\textsc{is}]\!] & = & \lambda xy.\, x = y \\
[\![\textsc{not}]\!] & = & \lambda p.\, \neg p
\end{array}
$$

where:

$$
\begin{array}{lcl}
\mathbf{alan} & : & \mathrm{e} \\
\mathbf{kurt} & : & \mathrm{e} \\
\mathbf{logician} & : & \mathrm{e} \to \mathrm{t} \\
\mathbf{british} & : & \mathrm{e} \to \mathrm{t}
\end{array}
$$

1. Compute the semantic representations of the following three sentences:

(1) *Kurt is a logician*
(2) *Alan is a British logician*
(3) *Kurt is not a British logician*

whose abstract syntactic representations are respectively:

(1) $\textsc{a}\ \textsc{logician}\ (\textsc{is}\ \textsc{kurt})$
(2) $\textsc{a}\ (\textsc{british}\ \textsc{logician})\ (\textsc{is}\ \textsc{alan})$
(3) $\textsc{not}\ (\textsc{a}\ (\textsc{british}\ \textsc{logician})\ (\textsc{is}\ \textsc{kurt}))$

2. Consider a model whose set of entities is $\{a, k\}$ and where the object constant **alan** is interpreted by a and the object constant **kurt** is interpreted by k. Complete this model by giving an interpretation of **logician** and **british** that makes the semantic representations of sentences (1), (2), and (3) true.