New Progress in Continuation-Based Dynamic Logic

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Outline

1. A type-theoretic reconstruction of dynamic logic
2. Need for more flexibility
3. A more flexible framework

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The problem of dynamic binding

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An old problem:

A man enters the room. He smiles.

$[\![\text{A man enters the room}]\!] = \exists x.\mathbf{man}(x) \land \mathbf{enters\_the\_room}(x)$. x is bound.

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A well known solution: DRT.

- The reference markers of DRT act as existential quantifiers.
- Nevertheless, from a technical point of view, they must be considered as free variables.

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- We will interpret a sentence according to both its left and right contexts.
- These two kinds of contexts will be abstracted over the meaning of the sentences.

Typing the left and the right contexts

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Montague semantics is based on Church's simple type theory, which provides a full hierarchy of functional types built upon two atomic types:

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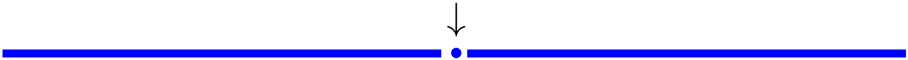

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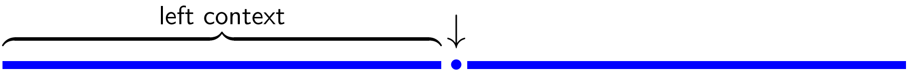

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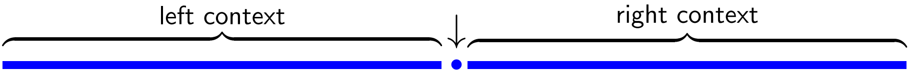

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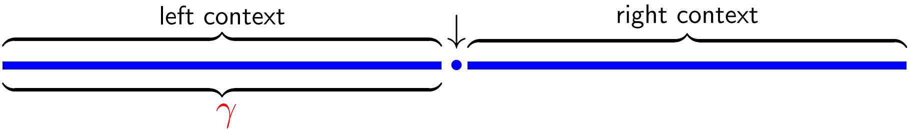

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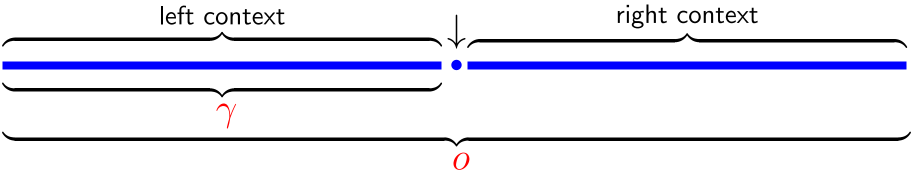

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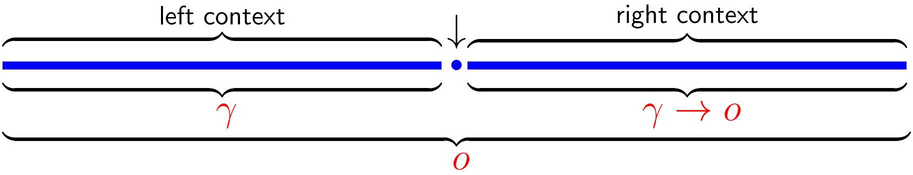

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- Composition of two sentence interpretations:

$$[\![S_1 \,.\, S_2]\!] = \lambda e\phi.\, [\![S_1]\!]\, e\, (\lambda e'.\, [\![S_2]\!]\, e'\, \phi)$$

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$$\mathsf{sel} : \gamma \to \iota$$

- *A man enters the room*. (S_1)
  He smiles. (S_2)

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- $[\![S_1 \,.\, S_2]\!] = \lambda e_1\phi.\, [\![S_1]\!]\, e_1\, (\lambda e_2.\, [\![S_2]\!]\, e_2\, \phi)$

$[\![$A man enters the room. He smiles $]\!]$

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$$= \quad [\![S_1 . S_2]\!]$$

$[\![$A man enters the room. He smiles $]\!]$

$$= \; [\![S_1 . \, S_2]\!]$$

$$= \; \lambda e_1 \phi_1 . \, [\![S_1]\!] \, e_1 \, (\lambda e_2 . \, [\![S_2]\!] \, e_2 \, \phi_1)$$

$[\![$A man enters the room. He smiles $]\!]$

$$
\begin{aligned}
&= [\![S_1 \,.\, S_2]\!] \\
&= \lambda e_1 \phi_1 .\, [\![S_1]\!]\, e_1\, (\lambda e_2 .\, [\![S_2]\!]\, e_2\, \phi_1) \\
&= \lambda e_1 \phi_1 .\, (\lambda e \phi .\, \exists x .\, (\mathbf{man}\, x) \wedge (\mathbf{enters}\, x) \wedge (\phi\, (x :: e)))\, e_1\, (\lambda e_2 .\, [\![S_2]\!]\, e_2\, \phi_1)
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$$

$$
[\![n]\!] = \iota \rightarrow [\![s]\!] \tag{2}
$$

$$
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$$[\![s]\!] = o \quad (1)$$
$$[\![n]\!] = \iota \rightarrow [\![s]\!] \quad (2)$$
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Replacing (1) with:

$$[\![s]\!] = \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o$$

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Replacing (1) with:

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we obtain:

$$\begin{array}{rcl} [\![n]\!] & = & \iota \to \gamma \to (\gamma \to o) \to o \\ [\![np]\!] & = & (\iota \to \gamma \to (\gamma \to o) \to o) \to \gamma \to (\gamma \to o) \to o \end{array}$$

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[\![\text{donkey}]\!] & = & \lambda x e \phi.\, \mathbf{donkey}\, x \wedge \phi\, e \\
[\![\text{owns}]\!] & = & \lambda o s.\, s\, (\lambda x.\, o\, (\lambda y e \phi.\, \mathbf{own}\, x\, y \wedge \phi\, e)) \\
[\![\text{beats}]\!] & = & \lambda o s.\, s\, (\lambda x.\, o\, (\lambda y e \phi.\, \mathbf{beat}\, x\, y \wedge \phi\, e)) \\
[\![\text{who}]\!] & = & \lambda r n x e \phi.\, n\, x\, e\, (\lambda e.\, r\, (\lambda \psi.\, \psi\, x)\, e\, \phi) \\
[\![\text{a}]\!] & = & \lambda n \psi e \phi.\, \exists x.\, n\, x\, e\, (\lambda e.\, \psi\, x\, (x{::}e)\, \phi) \\
[\![\text{every}]\!] & = & \lambda n \psi e \phi.\, (\forall x.\, \neg(n\, x\, e\, (\lambda e.\, \neg(\psi\, x\, (x{::}e)\, (\lambda e.\, \top))))) \wedge \phi\, e \\
[\![\text{it}]\!] & = & \lambda \psi e \phi.\, \psi\, (\mathtt{sel}\, e)\, e\, \phi
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[\![\text{who}]\!] &= \lambda rnxe\phi.\, n\, x\, e\, (\lambda e.\, r\, (\lambda\psi.\, \psi\, x)\, e\, \phi) \\
[\![\text{a}]\!] &= \lambda n\psi e\phi.\, \exists x.\, n\, x\, e\, (\lambda e.\, \psi\, x\, (x{::}e)\, \phi) \\
[\![\text{every}]\!] &= \lambda n\psi e\phi.\, (\forall x.\, \neg(n\, x\, e\, (\lambda e.\, \neg(\psi\, x\, (x{::}e)\, (\lambda e.\, \top))))) \wedge \phi\, e \\
[\![\text{it}]\!] &= \lambda\psi e\phi.\, \psi\, (\mathtt{sel}\, e)\, e\, \phi
\end{aligned}
$$

...which might seem a little bit involved.

Dynamic propositions

Dynamic propositions

- Let $\Omega \triangleq \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o$.

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- Ω may be seen as the type of dynamic propositions.
- We therefore intend to design a logic acting on propositions of type Ω

Formal framework

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First-order logic

$$\begin{array}{lcll} \top & : & o & \textit{(truth)} \\ \neg & : & o \to o & \textit{(negation)} \\ \wedge & : & o \to o \to o & \textit{(conjunction)} \\ \exists & : & (\iota \to o) \to o & \textit{(existential quantification)} \end{array}$$

Formal framework

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$$\top \;:\; o \qquad \textit{(truth)}$$
$$\neg \;:\; o \to o \qquad \textit{(negation)}$$
$$\wedge \;:\; o \to o \to o \qquad \textit{(conjunction)}$$
$$\exists \;:\; (\iota \to o) \to o \qquad \textit{(existential quantification)}$$

Dynamic primitives

$$:: \;:\; \iota \to \gamma \to \gamma \qquad \textit{(context updating)}$$
$$\mathsf{sel} \;:\; \gamma \to \iota \qquad \textit{(choice operator)}$$

Dynamic connectives

Dynamic connectives

Conjunction

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Conjunction

Conjunction is nothing but sentence composition. We therefore define:

$$P \mathbin{/\!\!\!\wedge} Q \triangleq \lambda e\phi.\, P\, e\, (\lambda e.\, Q\, e\, \phi)$$

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Existential quantification

Existential quantification introduces "reference markers". It is therefore responsible for context updating:

$$\exists\!\!\mathsf{l} x.\, P \triangleq \lambda e\phi.\, \exists x.\, P\, x\, (x :: e)\, \phi$$

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$$\exists\!\!\!\mid x.\, P \triangleq \lambda e\phi.\, \exists x.\, P\, x\, (x :: e)\, \phi$$

Negation

We do not want the continuation of the discourse to fall into the scope of the negation. Consequently, negation must be defined as follows:

$$\neg\!\!\!\mid P \triangleq (\lambda e\phi.\, \neg(P\, e\, (\lambda e.\, \top))) \wedge (\phi\, e)$$

Disjunction, Implication, and Universal Quantification

Disjunction, Implication, and Universal Quantification

These are defined using de Morgan's laws:

$$\begin{aligned} P \mathbin{\mathbb{V}} Q &\triangleq \neg\!\!\!\mid (\neg\!\!\!\mid P \mathbin{/\!\!\!\wedge} \neg\!\!\!\mid Q) \\ P \Rightarrow Q &\triangleq \neg\!\!\!\mid (P \mathbin{/\!\!\!\wedge} \neg\!\!\!\mid Q) \\ \mathbb{V}x.\, P &\triangleq \neg\!\!\!\mid (\exists\!\!\mid x.\, \neg\!\!\!\mid P) \end{aligned}$$

Translation of first-order logic

Translation of first-order logic

Embedding of first-order logic into dynamic logic

Translation of first-order logic

Embedding of first-order logic into dynamic logic

$$\overline{A} \;=\; \lambda e\phi.\, A \wedge (\phi\, e)$$

Translation of first-order logic

Embedding of first-order logic into dynamic logic

$$\begin{aligned}
\overline{A} &= \lambda e\phi.\, A \wedge (\phi\, e) \\
\overline{\neg P} &= \neg\!\!\!\neg\, \overline{P} \\
\overline{P \wedge Q} &= \overline{P} \mathbin{/\!\!\!\wedge} \overline{Q} \\
\overline{P \vee Q} &= \overline{P} \mathbin{\vee\!\!\!\vee} \overline{Q} \\
\overline{P \rightarrow Q} &= \overline{P} \Rrightarrow \overline{Q} \\
\overline{\exists x.\, P} &= \exists\!\!\!\exists x.\, \overline{P} \\
\overline{\forall x.\, P} &= \forall\!\!\!\forall x.\, \overline{P}
\end{aligned}$$

Translation of first-order logic

Embedding of first-order logic into dynamic logic

$$
\begin{aligned}
\overline{A} &= \lambda e\phi.\, A \wedge (\phi\, e) \\
\overline{\neg P} &= \neg\!\!\!\neg\, \overline{P} \\
\overline{P \wedge Q} &= \overline{P} \mathbin{/\!\!\!\wedge} \overline{Q} \\
\overline{P \vee Q} &= \overline{P} \mathbin{\vee\!\!\!\vee} \overline{Q} \\
\overline{P \to Q} &= \overline{P} \Rightarrow \overline{Q} \\
\overline{\exists x.\, P} &= \exists\!\!\exists x.\, \overline{P} \\
\overline{\forall x.\, P} &= \forall\!\!\!\forall x.\, \overline{P}
\end{aligned}
$$

This embedding is such that, for every term e of type γ:

$$P \equiv \overline{P}\, e\, (\lambda e.\, \top)$$

Application to donkey sentences

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Montague-like semantic interpretation:

Application to donkey sentences

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$$
\begin{aligned}
[\![\text{farmer}]\!] &= \lambda x.\, \mathbf{farmer}\, x \\
[\![\text{donkey}]\!] &= \lambda x.\, \mathbf{donkey}\, x \\
[\![\text{owns}]\!] &= \lambda OS.\, S\, (\lambda x.\, O\, (\lambda y.\, \mathbf{own}\, x\, y)) \\
[\![\text{beats}]\!] &= \lambda OS.\, S\, (\lambda x.\, O\, (\lambda y.\, \mathbf{beat}\, x\, y)) \\
[\![\text{who}]\!] &= \lambda RQx.\, (Q\, x) \wedge (R\, (\lambda P.\, P\, x)) \\
[\![\text{a}]\!] &= \lambda PQ.\, \exists x.\, (P\, x) \wedge (Q\, x) \\
[\![\text{every}]\!] &= \lambda PQ.\, \forall x.\, (P\, x) \rightarrow (Q\, x) \\
[\![\text{it}]\!] &= ???
\end{aligned}
$$

Dynamic interpretation:

Dynamic interpretation:

$$\begin{aligned}
[\![\text{farmer}]\!] &= \lambda x.\, \overline{\mathbf{farmer}\, x} \\
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[\![\text{who}]\!] &= \lambda RQx.\, (Q\, x) \mathbin{/\!\!\!\wedge} (R\,(\lambda P.\, P\, x)) \\
[\![\text{a}]\!] &= \lambda PQ.\, \exists\!\!\!\exists x.\, (P\, x) \mathbin{/\!\!\!\wedge} (Q\, x) \\
[\![\text{every}]\!] &= \lambda PQ.\, \forall\!\!\!\forall x.\, (P\, x) \Rrightarrow (Q\, x) \\
[\![\text{it}]\!] &= \lambda Pe\phi.\, P\,(\mathsf{sel}\, e)\, e\, \phi
\end{aligned}$$

Outline

2. Need for more flexibility
   - Accessibility constraints
   - Limitations

Accessibility constraints

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Bill doesn't have a car.
**It is black.*
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Bill doesn't have a car.
**It is black.*
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Harvey courts a girl at every convention. (non specific reading)
**She is very pretty.*
(Karttunen 1976)

You must write a letter to your parents.
**They are expecting the letter.*
(Karttunen 1976)

But...

But...

I don't have a microwave oven. I wouldn't know what to do with it.
(Frank 1996)

Harvey courts a girl at every convention. She always comes to the banquet with him. The girl is usually also very pretty.
(Karttunen 1976)

You must write a letter to your parents. It has to be sent by airmail. The letter must get there by tomorrow
(Karttunen 1976)

A thief might break into the house. He would take the silver.
(Roberts 1989)

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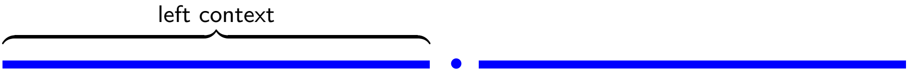
- Only one mode of composition.
- No systematic way of defining the logical connectives.
- The obtained logic relies heavily on classical logic.
- No methodology for extending the frameork (generalized quantifiers, plurality, temporality, intentionality, ...)

Outline

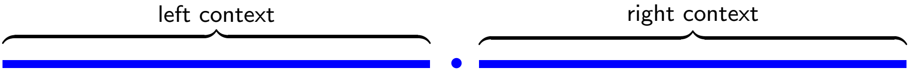
3. A more flexible framework
   - Revisiting the picture
   - Accomodating the connectives
   - Specifying accessibility constraints
   - Taking polarity into account
   - Putting everything together

Revisiting the picture

Revisiting the picture



Revisiting the picture



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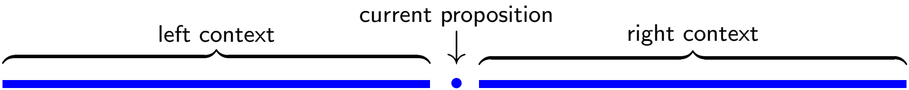

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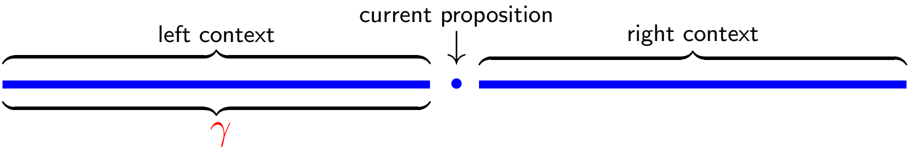

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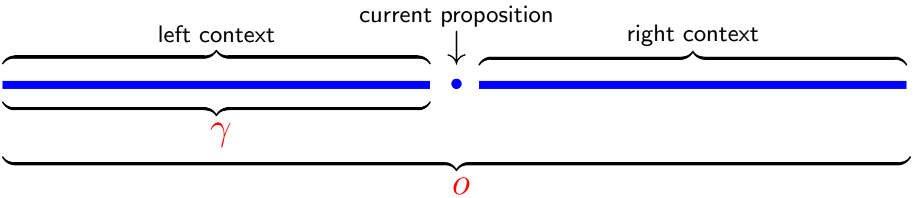

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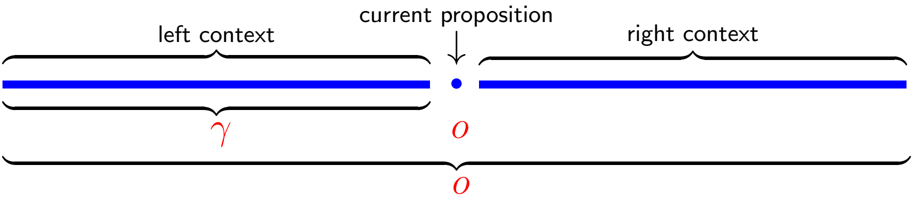

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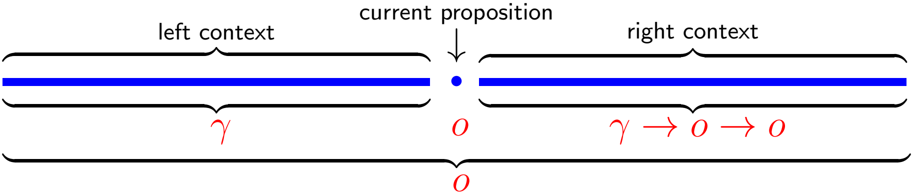

Accomodating the connectives

Accomodating the connectives

Conjunction

$$P \mathbin{/\!\!\!\wedge} Q \triangleq \lambda e_0 \phi.\, P\, e_0\, (\lambda e_1 p.\, Q\, e_1\, (\lambda e_2 q.\, \phi\, e_2\, (p \wedge q)))$$

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Conjunction

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Variants

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$$\lambda e_0 \phi.\, P\, e_0\, (\lambda e_1 p.\, Q\, e_1\, (\lambda e_2 q.\, p \wedge (\phi\, e_2\, q)))$$

Accomodating the connectives

Conjunction

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Variants

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$$\vdots$$

Generalizing

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Let $* : o \to o \to o$ be a (static) binary connective. We define:

$$[*]^{++} \triangleq \lambda PQe_0\phi.\, P\, e_0\, (\lambda e_1 p.\, Q\, e_1\, (\lambda e_2 q.\, \phi\, e_2\, (p * q)))$$

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Let $* : o \to o \to o$ be a (static) binary connective. We define:

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Then, we have:

$$P \mathbin{/\!\!\!\wedge} Q \triangleq P\, [\wedge]^{++}\, Q$$

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Let $* : o \to o \to o$ be a (static) binary connective. We define:

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Then, we have:

$$P \mathbin{/\!\!\!\wedge} Q \triangleq P\, [\wedge]^{++}\, Q$$

Similarly, let $\triangleright : o \to o$ be a unary connective. We define:

$$[\triangleright]^{+} \triangleq \lambda Pe_0\phi.\, P\, e_0\, (\lambda e_1 p.\, \phi\, e_1\, (\triangleright p))$$

Finally, let $\mathcal{Q} : (\iota \to o) \to o$ be a first-order quantifier. We define:

$$[\mathcal{Q}] \triangleq \lambda P e_0 \phi.\, \mathcal{Q} x.\, P\, x\, e_0\, (\lambda e_1 p.\, \phi\, (x :: e_1)\, p)$$

Specifying accessibility constraints

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The following lambda-term:

$$\lambda e p.\, p : \gamma \rightarrow o \rightarrow o$$

discards the current left context and returns the current proposition. It corresponds to the empty continuation.

Specifying accessibility constraints

The following lambda-term:

$$\lambda ep.\, p : \gamma \rightarrow o \rightarrow o$$

discards the current left context and returns the current proposition. It corresponds to the empty continuation.

We may then define the following closure operator:

$$\diamond P \triangleq \lambda e\phi.\, \phi\, e\, (P\, e\, (\lambda ep.\, p))$$

Specifying accessibility constraints

The following lambda-term:

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discards the current left context and returns the current proposition. It corresponds to the empty continuation.

We may then define the following closure operator:

$$\diamond P \triangleq \lambda e\phi.\, \phi\, e\, (P\, e\, (\lambda ep.\, p))$$

This allows, for instance, the "usual" dynamic disjunction to be defined as follows:

$$P \Vvee Q \triangleq \diamond (\diamond P\, [\vee]^{++} \diamond Q)$$

Taking polarity into account

Taking polarity into account

The part played by classical negation

Taking polarity into account

The part played by classical negation

$$\forall x. (\exists y. A[x, y]) \rightarrow B[x]$$

Taking polarity into account

The part played by classical negation

$$\forall x.\, (\exists y.\, A[x,y]) \rightarrow B[x] \equiv \forall x.\, \neg(\exists y.\, A[x,y]) \vee B[x]$$

Taking polarity into account

The part played by classical negation

$$\begin{aligned}\forall x.\,(\exists y.\,A[x,y]) \to B[x] &\equiv \forall x.\,\neg(\exists y.\,A[x,y]) \lor B[x]\\ &\equiv \forall x.\,(\forall y.\,\neg A[x,y]) \lor B[x]\end{aligned}$$

Taking polarity into account

The part played by classical negation

$$\begin{aligned}\forall x.\,(\exists y.\,A[x,y]) \to B[x] &\equiv \forall x.\,\neg(\exists y.\,A[x,y]) \vee B[x]\\ &\equiv \forall x.\,(\forall y.\,\neg A[x,y]) \vee B[x]\\ &\equiv \forall x.\,\forall y.\,\neg A[x,y] \vee B[x]\end{aligned}$$

Taking polarity into account

The part played by classical negation

$$
\begin{aligned}
\forall x.\,(\exists y.\,A[x,y]) \rightarrow B[x] &\equiv \forall x.\,\neg(\exists y.\,A[x,y]) \vee B[x] \\
&\equiv \forall x.\,(\forall y.\,\neg A[x,y]) \vee B[x] \\
&\equiv \forall x.\,\forall y.\,\neg A[x,y] \vee B[x] \\
&\equiv \forall x.\,\forall y.\,A[x,y] \rightarrow B[x]
\end{aligned}
$$

Taking polarity into account

The part played by classical negation

$$\begin{aligned}\forall x.\,(\exists y.\,A[x,y]) \rightarrow B[x] &\equiv \forall x.\,\neg(\exists y.\,A[x,y]) \vee B[x]\\ &\equiv \forall x.\,(\forall y.\,\neg A[x,y]) \vee B[x]\\ &\equiv \forall x.\,\forall y.\,\neg A[x,y] \vee B[x]\\ &\equiv \forall x.\,\forall y.\,A[x,y] \rightarrow B[x]\end{aligned}$$

Connective polarity

Taking polarity into account

The part played by classical negation

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Connective polarity

$$(\neg P^{-})^{+}$$

$$(P^{+} \vee Q^{+})^{+}$$

$$(P^{+} \wedge Q^{+})^{+}$$

$$(P^{-} \rightarrow Q^{+})^{+}$$

Adding a dualizing operator

$$\cdot^{\perp} : o \rightarrow o$$

Adding a dualizing operator

$$.^{\perp} : o \rightarrow o$$

such that:

$$\begin{aligned} P^{\perp\perp} &= P \\ (\neg P)^{\perp} &= \neg\, P^{\perp} \\ (P \vee Q)^{\perp} &= P^{\perp} \wedge Q^{\perp} \\ (P \wedge Q)^{\perp} &= P^{\perp} \vee Q^{\perp} \\ (\exists x.\, P)^{\perp} &= \forall x.\, P^{\perp} \\ (\forall x.\, P)^{\perp} &= \exists x.\, P^{\perp} \end{aligned}$$

Remeber that we defined:

$$[\triangleright]^{+} \triangleq \lambda P e_0 \phi.\, P\, e_0\, (\lambda e_1 p.\, \phi\, e_1\, (\triangleright p))$$

and

$$[*]^{++} \triangleq \lambda P Q e_0 \phi.\, P\, e_0\, (\lambda e_1 p.\, Q\, e_1\, (\lambda e_2 q.\, \phi\, e_2\, (p * q)))$$

Remeber that we defined:

$$[\triangleright]^{+} \triangleq \lambda P e_0 \phi.\, P\, e_0\, (\lambda e_1 p.\, \phi\, e_1\, (\triangleright\, p))$$

and

$$[*]^{++} \triangleq \lambda P Q e_0 \phi.\, P\, e_0\, (\lambda e_1 p.\, Q\, e_1\, (\lambda e_2 q.\, \phi\, e_2\, (p * q)))$$

Similarly, we define:

$$[\triangleright]^{-} \triangleq \lambda P e_0 \phi.\, (P\, e_0\, (\lambda e_1 p.\, (\phi\, e_1\, (\triangleright\, p))^{\perp}))^{\perp}$$

and

$$[*]^{-+} \triangleq \lambda P Q e_0 \phi.\, (P\, e_0\, (\lambda e_1 p.\, (Q\, e_1\, (\lambda e_2 q.\, \phi\, e_2\, (p * q)))^{\perp}))^{\perp}$$

Putting everything together

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Definition of the dynamic connectives

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$$P \mathbin{/\!\!\wedge} Q \triangleq P\,[\wedge]^{++}\,Q$$

Putting everything together

Definition of the dynamic connectives

$$P \mathbin{/\!\!\!\wedge} Q \triangleq P\,[\wedge]^{++}\,Q$$
$$P \mathbin{\backslash\!\!\!\vee} Q \triangleq \diamond(\diamond P\,[\vee]^{++} \diamond Q)$$

Putting everything together

Definition of the dynamic connectives

$$P \mathbin{/\!\!\!\wedge} Q \triangleq P\,[\wedge]^{++}\,Q$$
$$P \mathbin{\backslash\!\!\!\vee} Q \triangleq \diamond(\diamond P\,[\vee]^{++} \diamond Q)$$
$$P \Rightarrow Q \triangleq \diamond(P\,[\rightarrow]^{-+}\,Q)$$

Putting everything together

Definition of the dynamic connectives

$$
\begin{aligned}
P \mathbin{/\!\!\!\wedge} Q &\triangleq P\,[\wedge]^{++}\,Q \\
P \mathbin{\backslash\!\!\!\vee} Q &\triangleq \diamond(\diamond P\,[\vee]^{++}\diamond Q) \\
P \Rightarrow Q &\triangleq \diamond(P\,[\rightarrow]^{-+}\,Q) \\
\neg\!\shortmid P &\triangleq \diamond([\neg]^{-}\,P)
\end{aligned}
$$

Putting everything together

Definition of the dynamic connectives

$$
\begin{aligned}
P \mathbin{/\!\!\!\wedge} Q &\triangleq P\,[\wedge]^{++}\,Q \\
P \mathbin{\setminus\!\!\!\vee} Q &\triangleq \diamond(\diamond P\,[\vee]^{++} \diamond Q) \\
P \Rightarrow Q &\triangleq \diamond(P\,[\rightarrow]^{-+}\,Q) \\
\neg P &\triangleq \diamond([\neg]^{-}\,P) \\
\exists x.\,P &\triangleq [\exists]\,(\lambda x.\,P)
\end{aligned}
$$

Putting everything together

Definition of the dynamic connectives

$$
\begin{aligned}
P \mathbin{/\!\!\!\wedge} Q &\triangleq P\,[\wedge]^{++}\,Q \\
P \mathbin{\backslash\!\!\!\vee} Q &\triangleq \diamond(\diamond P\,[\vee]^{++} \diamond Q) \\
P \Rightarrow Q &\triangleq \diamond(P\,[\rightarrow]^{-+}\,Q) \\
\neg\!\!\shortmid P &\triangleq \diamond([\neg]^{-}\,P) \\
\exists\!\!\shortmid x.\,P &\triangleq [\exists]\,(\lambda x.\,P) \\
\forall\!\!\!\forall x.\,P &\triangleq \diamond([\forall]\,(\lambda x.\,P))
\end{aligned}
$$

Embedding of first-order logic into dynamic logic

Embedding of first-order logic into dynamic logic

$$\overline{A} \;=\; \lambda e\phi.\, \phi\, e\, A$$

Embedding of first-order logic into dynamic logic

$$\begin{aligned}
\overline{A} &= \lambda e\phi.\, \phi\, e\, A \\
\overline{\neg P} &= \neg\!\!\shortmid\, \overline{P} \\
\overline{P \wedge Q} &= \overline{P} \mathbin{/\!\!\wedge} \overline{Q} \\
\overline{P \vee Q} &= \overline{P} \mathbin{\vee\!\!\!\vee} \overline{Q} \\
\overline{P \to Q} &= \overline{P} \Rightarrow \overline{Q} \\
\overline{\exists x.\, P} &= \exists\!\!\shortmid x.\, \overline{P} \\
\overline{\forall x.\, P} &= \forall\!\!\!\forall x.\, \overline{P}
\end{aligned}$$