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A Note on Kobayashi's and Yonezawa's "Asynchronous Communication Model Based on Linear Logic"

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Abstract. We explain why the Kobayashi-Yonezawa translation of CCS into linear logic does not work properly. Then we introduce our own translation and prove its correctness.

1. Introduction

In [KoY95, §5.4, pp. 133-134], Kobayashi and Yonezawa introduce a translation of CCS into their own asynchronous communication model, which is based on Girard's linear logic [Gir87]. The difficulty they have to face is to express synchronization within an asynchronous framework. In particular, while the two linear logic formulæ $a^{\perp} \otimes b^{\perp} \otimes C$ and $b^{\perp} \otimes a^{\perp} \otimes C$ are equivalent, the two CCS processes $a.b.C$ and $b.a.C$ are not. To circumvent this problem, the authors propose to translate message receivers and senders as follows:

$$\begin{array}{lcl} Tr_{CCS \to ACL}(a.P) & = & {a_1}^{\perp} \otimes (a_2 \parr ({a_3}^{\perp} \otimes Tr_{CCS \to ACL}(P))) \\ Tr_{CCS \to ACL}(\bar{a}.P) & = & a_1 \parr ({a_2}^{\perp} \otimes (a_3 \parr Tr_{CCS \to ACL}(P))) \end{array}$$

This proposal is based on a double acknowledgement scheme: on the one hand, the sender $(Tr_{CCS \to ACL}(\bar{a}.P))$ sends out a message (a_1), waits for an aknowledgement $({a_2}^{\perp})$, then aknowledges this aknowledgement (a_3); on the other hand, the receiver $(Tr_{CCS \to ACL}(a.P))$ behaves in a dual way. Unfortunately, this is not sufficient to express synchronization, as shown in Section 2. More generally, we show in Section 3 that any translation based on a multiple acknowledgement

Correspondence and offprint requests to: Philippe de Groote, UMR n° 7503 LORIA, Campus Scientifique – BP 239, F-54506 Vandœuvre lès Nancy Cedex – France scheme cannot express synchronization. A possible solution to the problem consists in using first order quantifiers to implement private channels. This solution is explained in Section 4.

2. A counterexample

Consider the following process:

$$\bar{a}.(a.\bar{a}.a.\top \mid \bar{a}.0) \mid a.0 \tag{1}$$

whose translation in linear logic, according to [KoY95] (See Appendix A), must be the following:

$$(a_1 \parr (a_2{}^\perp \otimes (a_3 \parr (A \parr B)))) \parr C \tag{2}$$

where:

$$\begin{array}{rcl} A & = & a_1{}^\perp \otimes (a_2 \parr (a_3{}^\perp \otimes (a_1 \parr (a_2{}^\perp \otimes (a_3 \parr (a_1{}^\perp \otimes (a_2 \parr (a_3{}^\perp \otimes \top)))))))) \\ B & = & a_1 \parr (a_2{}^\perp \otimes (a_3 \parr \bot)) \\ C & = & a_1{}^\perp \otimes (a_2 \parr (a_3{}^\perp \otimes \bot)) \end{array}$$

While (1) does not terminate in the sense of [KoY95, Theorem 5.1., p. 134], (2) is provable in linear logic. This is shown by the following derivation, where A_0 and A_1 abbreviate respectively the following formulæ:

$$\begin{array}{rcl} A_0 & = & a_1{}^\perp \otimes (a_2 \parr (a_3{}^\perp \otimes \top)) \\ A_1 & = & a_1 \parr (a_2{}^\perp \otimes (a_3 \parr A_0)) \end{array}$$

$$\Pi_1 \left\{ \dfrac{\vdash a_1{}^\perp, a_1 \qquad \dfrac{\vdash a_3{}^\perp, a_3 \qquad \dfrac{\dfrac{\dfrac{\vdash a_2{}^\perp, a_2 \qquad \dfrac{\dfrac{\dfrac{\vdash a_3{}^\perp, a_3 \qquad \vdash \top}{\vdash a_3{}^\perp \otimes \top, a_3}}{\vdash a_3{}^\perp \otimes \top, a_3, \bot}}{\vdash a_3{}^\perp \otimes \top, a_3 \parr \bot}}{\vdash a_2, a_3{}^\perp \otimes \top, a_2{}^\perp \otimes (a_3 \parr \bot)}}{\vdash a_2 \parr (a_3{}^\perp \otimes \top), a_2{}^\perp \otimes (a_3 \parr \bot)}}{\vdash a_2 \parr (a_3{}^\perp \otimes \top), a_2{}^\perp \otimes (a_3 \parr \bot), \bot}}{\vdash a_3, a_2 \parr (a_3{}^\perp \otimes \top), a_2{}^\perp \otimes (a_3 \parr \bot), a_3{}^\perp \otimes \bot}}{\vdash a_1, a_3, a_1{}^\perp \otimes (a_2 \parr (a_3{}^\perp \otimes \top)), a_2{}^\perp \otimes (a_3 \parr \bot), a_3{}^\perp \otimes \bot} \right.$$

$$
\Pi_2 \left\{ \dfrac{\dfrac{\vdash a_2{}^{\perp}, a_2 \qquad \dfrac{\dfrac{\Pi_1}{\vdash a_1, a_3, A_0, a_2{}^{\perp} \otimes (a_3 \parr \perp), a_3{}^{\perp} \otimes \perp}}{\vdash a_1, a_3 \parr A_0, a_2{}^{\perp} \otimes (a_3 \parr \perp), a_3{}^{\perp} \otimes \perp}}{\vdash a_2, a_1, a_2{}^{\perp} \otimes (a_3 \parr A_0), a_2{}^{\perp} \otimes (a_3 \parr \perp), a_3{}^{\perp} \otimes \perp}}{\vdash a_2, a_1 \parr (a_2{}^{\perp} \otimes (a_3 \parr A_0)), a_2{}^{\perp} \otimes (a_3 \parr \perp), a_3{}^{\perp} \otimes \perp} \right.
$$

$$
\Pi_3 \left\{ \dfrac{\dfrac{\vdash a_1{}^{\perp}, a_1 \qquad \dfrac{\dfrac{\vdash a_3{}^{\perp}, a_3 \qquad \dfrac{\Pi_2}{\vdash a_2, A_1, a_2{}^{\perp} \otimes (a_3 \parr \perp), a_3{}^{\perp} \otimes \perp}}{\vdash a_3, a_2, a_3{}^{\perp} \otimes A_1, a_2{}^{\perp} \otimes (a_3 \parr \perp), a_3{}^{\perp} \otimes \perp}}{\vdash a_3, a_2 \parr (a_3{}^{\perp} \otimes A_1), a_2{}^{\perp} \otimes (a_3 \parr \perp), a_3{}^{\perp} \otimes \perp}}{\vdash a_3, a_1{}^{\perp} \otimes (a_2 \parr (a_3{}^{\perp} \otimes A_1)), a_1, a_2{}^{\perp} \otimes (a_3 \parr \perp), a_3{}^{\perp} \otimes \perp}}{\vdash a_3, a_1{}^{\perp} \otimes (a_2 \parr (a_3{}^{\perp} \otimes A_1)), a_1 \parr (a_2{}^{\perp} \otimes (a_3 \parr \perp)), a_3{}^{\perp} \otimes \perp} \right.
$$

$$
\dfrac{\dfrac{\vdash a_1{}^{\perp}, a_1 \qquad \dfrac{\dfrac{\vdash a_2{}^{\perp}, a_2 \qquad \dfrac{\dfrac{\dfrac{\Pi_3}{\vdash a_3, A, B, a_3{}^{\perp} \otimes \perp}}{\vdash a_3, A \parr B, a_3{}^{\perp} \otimes \perp}}{\vdash a_3 \parr (A \parr B), a_3{}^{\perp} \otimes \perp}}{\vdash a_2{}^{\perp} \otimes (a_3 \parr (A \parr B)), a_2, a_3{}^{\perp} \otimes \perp}}{\vdash a_2{}^{\perp} \otimes (a_3 \parr (A \parr B)), a_2 \parr (a_3{}^{\perp} \otimes \perp)}}{\vdash a_1, a_2{}^{\perp} \otimes (a_3 \parr (A \parr B)), a_1{}^{\perp} \otimes (a_2 \parr (a_3{}^{\perp} \otimes \perp))}}{\vdash (a_1 \parr (a_2{}^{\perp} \otimes (a_3 \parr (A \parr B)))) \parr C}
$$

Consequently, [KoY95, Theorem 5.1., p. 134] does not hold.

3. Generalisation of the counterexample

The idea behind the Kobayashi-Yonezawa translation is "synchronization by acknowledgement": any process which sends out a message must wait for an acknowledgement that witnesses the fact that the message has been captured by some other process. The problem with this scheme is that the acknowledgements are also sent out asynchronously, which possibly gives rise to *acknowledgement clashes*. The Kobayashi-Yonezawa translation tries to solve this problem by using acknowledgements of a second type, whose purpose is to acknowledge the reception of the acknowledgements of the first type. However, as shown in the previous section, such a protocol does not solve the problem.

One may think of using acknowledgements of a third type. This idea kills the counterexample of Section 2 but does not solve the problem in general. Indeed, it is possible to show that any translation based on a multiple acknowledgement scheme (whatever the number of acknowledgements is) cannot express synchronization.

Let a be a CCS *name* and let $a_1, a_2, a_3, \ldots$ be an alphabet of atomic propositions associated to a. Given a formula B, we define two families of formulæ, $\mathcal{F}_n(a, B)$ and $\overline{\mathcal{F}}_n(a, B)$:

$$\begin{array}{rcl} \mathcal{F}_0(a,B) & = & B \\ \mathcal{F}_{n+1}(a,B) & = & {a_{n+1}}^{\perp} \otimes \overline{\mathcal{F}}_n(a,B) \\ \overline{\mathcal{F}}_0(a,B) & = & B \\ \overline{\mathcal{F}}_{n+1}(a,B) & = & a_{n+1} \parr \mathcal{F}_n(a,B) \end{array}$$

This definition allows us to generalise the Kobayashi-Yonezawa translation as follows:

$$\begin{array}{rcl} Tr_n(a.P) & = & \mathcal{F}_n(a, Tr_n(P)) \\ Tr_n(\bar{a}.P) & = & \overline{\mathcal{F}}_n(a, Tr_n(P)) \end{array}$$

Now consider the following family of processes:

$$\begin{array}{rcl} P_1 & = & \bar{a}.a.\top \\ P_{2n} & = & a.P_{2n-1} \mid \bar{a}.0 \\ P_{2n+1} & = & \bar{a}.P_{2n} \mid a.0 \end{array}$$

Clearly, the only possible sequence of transitions starting from a given P_n is the following:

$$P_n \stackrel{\tau}{\longrightarrow} P_{n-1} \stackrel{\tau}{\longrightarrow} \cdots \stackrel{\tau}{\longrightarrow} P_1$$

Since P_1 can be reduced no further, it follows that the above family is a family of non-terminating processes (in the sense of [KoY95]). Consequently, any translation for which [KoY95, Theorem 5.1., p. 134] would hold must translate the above family of processes into non-provable formulæ.

The counterexample of Section 2 shows that $Tr_3(P_3)$ is provable. Therefore, Tr_3 (which corresponds to the Kobayashi-Yonezawa proposal) is not an appropriate translation. This result is generalised by the next proposition, whose proof is not difficult.

Proposition 3.1. For all $n \leq m$, $Tr_n(P_m)$ is provable in linear logic.

4. Forcing synchronization by first-order quantification

A possible solution to the synchronization problem is to use private channels for the transmission of the acknowledgements: a process that sends out a message attaches to this message the *name* of a private channel on which it will be waiting for the acknowledgement. This idea may be implemented in linear logic by means of first-order quantifiers. Consider the following translation (see Appendix B):

$$\begin{array}{rcl} Tr(a.P) & = & \exists x.(a_1[x]^{\perp} \otimes (a_2[x] \parr Tr(P))) \\ Tr(\bar{a}.P) & = & \forall x.(a_1[x] \parr (a_2[x]^{\perp} \otimes Tr(P))) \end{array}$$

where $a_1[x]$ and $a_2[x]$ are monadic relation symbols corresponding to the CCS name a.

The idea behind the above translation is that the eigenvariables of the universal quantifiers play the role of private channels names. Then the usual proviso coming with the ∀-introduction rule ensures that each sender will use a different private channel for the reception of the acknowledgement.

In order to show that the equivalent of [KoY95, Theorem 5.1., p. 134] holds, we introduce two proof-systems: the ACL-like system, which is asynchronous (see Appendix C), and the synchronous system, whose inference rules correspond to the transition rules of CCS (see Appendix D). Then we show that the first system is complete for our modified translation, and that the second system is theorem-equivalent to the first one, in the range of our translation.

Proposition 4.1. Let A be a CCS expression and $\mathbf{P}$ be a CCS process definition. Then $!Tr(\mathbf{P}) \vdash Tr(A)$ is provable in linear logic if and only if it is provable in the ACL-like system.

Proof. A straightforward adaptation of the proof of [KoY95, Theorem 2.1., p. 116]. □

We now consider the ACL-like system augmented with Rule (S) of the synchronous system, that is the union of the two proof-systems. We call this system the hybrid system.

Lemma 4.1. Let $!\mathbf{P} \vdash \Gamma$ be a sequent provable in the hybrid system. Then it is provable in the ACL-like system.

Proof. The following derivation shows that Rule (S) is an admissible rule of the ACL-like system:

$$\dfrac{\dfrac{\dfrac{\dfrac{!\mathbf{P} \vdash P, Q, \Gamma}{!\mathbf{P} \vdash a_2[\xi], a_2[\xi]^\perp \otimes P, Q, \Gamma}\,(\mathrm{C}_2)}{!\mathbf{P} \vdash a_1[\xi], a_2[\xi]^\perp \otimes P, a_1[\xi]^\perp \otimes (a_2[\xi] \parr Q), \Gamma}\,(\mathrm{C}_1)}{!\mathbf{P} \vdash a_1[\xi], a_2[\xi]^\perp \otimes P, \exists x.(a_1[x]^\perp \otimes (a_2[x] \parr Q)), \Gamma}\,(\exists)}{!\mathbf{P} \vdash \forall x.(a_1[x] \parr (a_2[x]^\perp \otimes P)), \exists x.(a_1[x]^\perp \otimes (a_2[x] \parr Q)), \Gamma}\,(\forall)$$

□

Lemma 4.2. Let $!\mathbf{P} \vdash a_2[\xi], a_2[\xi]^\perp \otimes P, \Gamma$ be a sequent provable in the hybrid system, such that $a_2[\xi]^\perp$ does not occur in Γ. Then $!\mathbf{P} \vdash P, \Gamma$ is provable with the same number of occurrences of Rule (∀).

Proof. A straightforward induction on the length of the derivation. The two interesting cases are when the last rule of the derivation is either (C_2) or (∀). In the first case, the non-occurrence hypothesis ensures that the preceeding sequent in the derivation is indeed $!\mathbf{P} \vdash P, \Gamma$. In the second case, the eigenvariable proviso ensures that the non-occurrence hypothesis is maintained. □

Lemma 4.3. Let $!\mathbf{P} \vdash \Gamma$ be a sequent provable in the hybrid system. Then it is provable in the hybrid system without any use of Rule (∀).

Proof. The proof is by induction on the number of occurrences of Rule (∀) in the derivation of $!\mathbf{P} \vdash \Gamma$.

If there is no such occurrence, then we are done. Otherwise, consider the last occurrence:

$$\dfrac{\begin{array}{c}\Pi \\ \vdots \\ \mathbf{!P} \vdash a_1[\xi],\, a_2[\xi]^\perp \otimes P,\, \Gamma'\end{array}}{\mathbf{!P} \vdash \forall x.(a_1[x] \parr (a_2[x]^\perp \otimes P)),\, \Gamma'}\ (\forall)$$

By induction hypothesis, there exists a derivation Π' without any use of Rule $(\forall)$. Then, the remaining occurrence may be moved up in Π' by applying the permutation schemes of Appendix E.1. This process results either in the elimanation of the remaining occurence or in the following configuration:

$$\dfrac{\dfrac{\dfrac{\begin{array}{c}\Pi'' \\ \vdots \\ \mathbf{!P} \vdash a_2[\xi]^\perp \otimes P,\, a_2[\xi],\, Q,\, \Delta[\xi],\, \Gamma''\end{array}}{\mathbf{!P} \vdash a_1[\xi],\, a_2[\xi]^\perp \otimes P,\, a_1[\xi]^\perp \otimes (a_2[\xi] \parr Q),\, \Delta[\xi],\, \Gamma''}\ (\mathrm{C}_1)}{\mathbf{!P} \vdash a_1[\xi],\, a_2[\xi]^\perp \otimes P,\, \exists x.(a_1[x]^\perp \otimes (a_2[x] \parr Q)),\, \exists x.\Delta[x],\, \Gamma''}\ (\exists)}{\begin{array}{c}\mathbf{!P} \vdash \forall x.(a_1[x] \parr (a_2[x]^\perp \otimes P)),\, \exists x.(a_1[x]^\perp \otimes (a_2[x] \parr Q)),\, \exists x.\Delta[x],\, \Gamma'' \\ \vdots\end{array}}\ (\forall)$$

where $\exists x.\Delta[x]$ stands for a (possibly empty) sequence of existential formulæ, and the double inference bar stands for a sequence of applications of Rule $(\exists)$.

Then, by Lemma 4.2, there exists a derivation Π''' of $\mathbf{!P} \vdash P,\, Q,\, \Delta[\xi],\, \Gamma''$, which allows the above configuration to be replaced by the following one:

$$\dfrac{\dfrac{\begin{array}{c}\Pi''' \\ \vdots \\ \mathbf{!P} \vdash P,\, Q,\, \Delta[\xi],\, \Gamma''\end{array}}{\mathbf{!P} \vdash P,\, Q,\, \exists x.\Delta[x],\, \Gamma''}\ (\exists)}{\begin{array}{c}\mathbf{!P} \vdash \forall x.(a_1[x] \parr (a_2[x]^\perp \otimes P)),\, \exists x.(a_1[x]^\perp \otimes (a_2[x] \parr Q)),\, \exists x.\Delta[x],\, \Gamma'' \\ \vdots\end{array}}\ (\mathrm{S})$$

□

Lemma 4.4. Let A be a CCS expression and $\mathbf{P}$ be a CCS process definition such that $!Tr(\mathbf{P}) \vdash Tr(A)$ is provable in the hybrid system. Let $\#(\mathrm{C}_2)$, $\#(\mathrm{C}_1)$, $\#(\forall)$ be respectively the numbers of occurrences of Rules (C_2), (C_1), $(\forall)$ in the derivation of $!Tr(\mathbf{P}) \vdash Tr(A)$. Then, $\#(\mathrm{C}_2) \leq \#(\mathrm{C}_1) \leq \#(\forall)$.

Proof. Reading the derivation backwardly, each application of Rule (C_2) consumes an occurrence of an atomic formula of the form $a_2[\xi]$ that has been introduced by an application of Rule (C_1). Similarly, each application of Rule (C_1) consumes an occurrence of an atomic formula of the form $a_1[\xi]$ that has been introduced by an application of Rule $(\forall)$. □

Lemma 4.5. Let $\mathbf{!P} \vdash \Gamma$ be a sequent provable in the synchronous system augmented with Rule $(\exists)$ of the ACL-like system. Then it is provable in the synchronous system.

Proof. Any application of Rule $(\exists)$ may be moved up in the derivation and eventually eliminated, by applying the permutation schemes of Appendix E.2. □

Proposition 4.2. Let A be a CCS expression and $\mathbf{P}$ be a CCS process definition. Then $!Tr(\mathbf{P}) \vdash Tr(A)$ is provable in the ACL-like system if and only if it is provable in the synchronous system.

Proof. The if-part of the proposition is given by Lemma 4.1. For the only-if-part, consider some ACL-like derivation (say Π) of $!Tr(\mathbf{P}) \vdash Tr(A)$. By Lemma 4.3, Π may be turned into a hybrid derivation (say Π') that does not use Rule ($\forall$). Then, by Lemma 4.4, Π' does not use Rules (C_1) and (C_2) either. Consequently, by Lemma 4.5, Π' may be turned into a synchronous derivation. □

As a direct consequence of Propositions 4.1 and 4.2, we have that the equivalent of [KoY95, Theorem 5.1., p. 134] holds for our modified translation.

References

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A. Kobayashi-Yonezawa translation

$$
\begin{aligned}
Tr_{CCS\to ACL}(a.P) &= {a_1}^{\perp} \otimes (a_2 \parr ({a_3}^{\perp} \otimes Tr_{CCS\to ACL}(P)))\\
Tr_{CCS\to ACL}(\bar{a}.P) &= a_1 \parr ({a_2}^{\perp} \otimes (a_3 \parr Tr_{CCS\to ACL}(P)))\\
Tr_{CCS\to ACL}(P \mid Q) &= Tr_{CCS\to ACL}(P) \parr Tr_{CCS\to ACL}(Q)\\
Tr_{CCS\to ACL}(P + Q) &= Tr_{CCS\to ACL}(P) \oplus Tr_{CCS\to ACL}(Q)\\
Tr_{CCS\to ACL}(\mathbf{0}) &= \perp\\
Tr_{CCS\to ACL}(\top) &= \top\\
Tr_{CCS\to ACL}(A) &= A \quad (A,\text{ a constant})\\
Tr_{CCS\to ACL}(A \stackrel{\text{def}}{=} P) &= A \multimapinv Tr_{CCS\to ACL}(P)
\end{aligned}
$$

B. Modified translation

$$
\begin{aligned}
Tr(a.P) &= \exists x.(a_1[x]^{\perp} \otimes (a_2[x] \parr Tr(P)))\\
Tr(\bar{a}.P) &= \forall x.(a_1[x] \parr (a_2[x]^{\perp} \otimes Tr(P)))\\
Tr(P \mid Q) &= Tr(P) \parr Tr(Q)\\
Tr(P + Q) &= Tr(P) \oplus Tr(Q)\\
Tr(\mathbf{0}) &= \perp\\
Tr(\top) &= \top\\
Tr(A) &= A \quad (A,\text{ a constant})\\
Tr(A \stackrel{\text{def}}{=} P) &= A \multimapinv Tr(P)
\end{aligned}
$$

C. ACL-like system for the modified translation

$$
!\mathbf{P} \vdash \top, \Gamma \quad (\top) \qquad \frac{!\mathbf{P} \vdash \Gamma}{!\mathbf{P} \vdash \perp, \Gamma} \ (\perp) \qquad \frac{!\mathbf{P}, !(A \multimapinv B) \vdash B, \Gamma}{!\mathbf{P}, !(A \multimapinv B) \vdash A, \Gamma} \ (\mathrm{U})
$$

$$\frac{!\mathbf{P} \vdash P, \Gamma}{!\mathbf{P} \vdash P \oplus Q, \Gamma} (\oplus_1) \qquad \frac{!\mathbf{P} \vdash Q, \Gamma}{!\mathbf{P} \vdash P \oplus Q, \Gamma} (\oplus_2) \qquad \frac{!\mathbf{P} \vdash P, Q, \Gamma}{!\mathbf{P} \vdash P \parr Q, \Gamma} (\parr)$$

$$\frac{!\mathbf{P} \vdash a_1[\xi]^\perp \otimes (a_2[\xi] \parr P), \Gamma}{!\mathbf{P} \vdash \exists x.(a_1[x]^\perp \otimes (a_2[x] \parr P)), \Gamma} (\exists) \qquad \frac{!\mathbf{P} \vdash a_1[\xi], a_2[\xi]^\perp \otimes P, \Gamma}{!\mathbf{P} \vdash \forall x.(a_1[x] \parr (a_2[x]^\perp \otimes P)), \Gamma} (\forall)$$

$$\frac{!\mathbf{P} \vdash a_2[\xi], P, \Gamma}{!\mathbf{P} \vdash a_1[\xi], a_1[\xi]^\perp \otimes (a_2[\xi] \parr P), \Gamma} (\mathrm{C}_1) \qquad \frac{!\mathbf{P} \vdash P, \Gamma}{!\mathbf{P} \vdash a_2[\xi], a_2[\xi]^\perp \otimes P, \Gamma} (\mathrm{C}_2)$$

In Rule (U), A is an atomic proposition. In Rule ($\forall$), ξ cannot be free in Γ.

D. Synchronous system for the modified translation

$$!\mathbf{P} \vdash \top, \Gamma \quad (\top) \qquad \frac{!\mathbf{P} \vdash \Gamma}{!\mathbf{P} \vdash \bot, \Gamma} (\bot) \qquad \frac{!\mathbf{P}, !(A \multimapinv B) \vdash B, \Gamma}{!\mathbf{P}, !(A \multimapinv B) \vdash A, \Gamma} (\mathrm{U})$$

$$\frac{!\mathbf{P} \vdash P, \Gamma}{!\mathbf{P} \vdash P \oplus Q, \Gamma} (\oplus_1) \qquad \frac{!\mathbf{P} \vdash Q, \Gamma}{!\mathbf{P} \vdash P \oplus Q, \Gamma} (\oplus_2) \qquad \frac{!\mathbf{P} \vdash P, Q, \Gamma}{!\mathbf{P} \vdash P \parr Q, \Gamma} (\parr)$$

$$\frac{!\mathbf{P} \vdash P, Q, \Gamma}{!\mathbf{P} \vdash \forall x.(a_1[x] \parr (a_2[x]^\perp \otimes P)), \exists x.(a_1[x]^\perp \otimes (a_2[x] \parr Q)), \Gamma} (\mathrm{S})$$

In Rule (U), A is an atomic proposition.

E. Permutation schemes

E.1. $\forall$-schemes

Let $S[x]$ stand for the formula $a_1[x] \parr (a_2[x]^\perp \otimes P)$, and $\mathcal{S}[\xi]$ for the sequence of formulæ $a_1[\xi]$, $a_2[\xi]^\perp \otimes Q$. Let $\mathcal{R}[\xi]$ stand for a (possibly empty) sequence of formulæ $b_{01}[\xi]^\perp \otimes (b_{02}[\xi] \parr Q_0), \ldots, b_{n1}[\xi]^\perp \otimes (b_{n2}[\xi] \parr Q_n)$, and let $\exists x.\mathcal{R}[x]$ stand for the (possibly empty) corresponding sequence of formulæ $\exists x.(b_{01}[x]^\perp \otimes (b_{02}[x] \parr Q_0)), \ldots, \exists x.(b_{n1}[x]^\perp \otimes (b_{n2}[x] \parr Q_n))$. Let $\zeta \neq \xi$. In the proof schemes that follow, a double inference bar stand for a (possibly empty) sequence of applications of Rule ($\exists$).

($\top/\forall$)

$$\dfrac{\dfrac{!\mathbf{P} \vdash \top, \mathcal{S}[\xi], \mathcal{R}[\xi], \Gamma}{!\mathbf{P} \vdash \top, \mathcal{S}[\xi], \exists x.\mathcal{R}[x], \Gamma}}{!\mathbf{P} \vdash \top, \forall x.S[x], \exists x.\mathcal{R}[x], \Gamma} \quad \leadsto \quad !\mathbf{P} \vdash \top, \forall x.S[x], \exists x.\mathcal{R}[x], \Gamma$$

(⊥/∀)

$$
\begin{array}{c}
\mathbf{!P} \vdash \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline
\mathbf{!P} \vdash \bot,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P} \vdash \bot,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash \bot,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
\quad \leadsto \quad
\begin{array}{c}
\mathbf{!P} \vdash \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P} \vdash \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash \bot,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
$$

(U/∀)

$$
\begin{array}{c}
\mathbf{!P},\, !(A \multimapinv B) \vdash B,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline
\mathbf{!P},\, !(A \multimapinv B) \vdash A,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P},\, !(A \multimapinv B) \vdash A,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P},\, !(A \multimapinv B) \vdash A,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
$$

$$
\leadsto \quad
\begin{array}{c}
\mathbf{!P},\, !(A \multimapinv B) \vdash B,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P},\, !(A \multimapinv B) \vdash B,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P},\, !(A \multimapinv B) \vdash B,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P},\, !(A \multimapinv B) \vdash A,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
$$

($\oplus_i$/∀)

$$
\begin{array}{c}
\mathbf{!P} \vdash A_i,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline
\mathbf{!P} \vdash A_1 \oplus A_2,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P} \vdash A_1 \oplus A_2,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash A_1 \oplus A_2,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
\quad \leadsto \quad
\begin{array}{c}
\mathbf{!P} \vdash A_i,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P} \vdash A_i,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash A_i,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash A_1 \oplus A_2,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
$$

(⅋/∀)

$$
\begin{array}{c}
\mathbf{!P} \vdash A,\, B,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline
\mathbf{!P} \vdash A \parr B,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P} \vdash A \parr B,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash A \parr B,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
\quad \leadsto \quad
\begin{array}{c}
\mathbf{!P} \vdash A,\, B,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P} \vdash A,\, B,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash A,\, B,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash A \parr B,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
$$

(∃/∀)

$$
\begin{array}{c}
\mathbf{!P} \vdash b_1[\zeta]^\perp \otimes (b_2[\zeta] \parr A),\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline
\mathbf{!P} \vdash \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr A)),\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P} \vdash \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr A)),\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr A)),\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
$$

$$
\leadsto \quad
\begin{array}{c}
\mathbf{!P} \vdash b_1[\zeta]^\perp \otimes (b_2[\zeta] \parr A),\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
\mathbf{!P} \vdash b_1[\zeta]^\perp \otimes (b_2[\zeta] \parr A),\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash b_1[\zeta]^\perp \otimes (b_2[\zeta] \parr A),\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
\mathbf{!P} \vdash \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr A)),\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}
$$

($C_1/\forall$)

$$\begin{array}{c}
!\mathbf{P} \vdash b_2[\zeta],\, A,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline
!\mathbf{P} \vdash b_1[\zeta],\, b_1[\zeta]^\perp \otimes (b_2[\zeta] \parr A),\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
!\mathbf{P} \vdash b_1[\zeta],\, b_1[\zeta]^\perp \otimes (b_2[\zeta] \parr A),\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
!\mathbf{P} \vdash b_1[\zeta],\, b_1[\zeta]^\perp \otimes (b_2[\zeta] \parr A),\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}$$

$$\rightsquigarrow \quad \begin{array}{c}
!\mathbf{P} \vdash b_2[\zeta],\, A,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
!\mathbf{P} \vdash b_2[\zeta],\, A,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
!\mathbf{P} \vdash b_2[\zeta],\, A,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
!\mathbf{P} \vdash b_1[\zeta],\, b_1[\zeta]^\perp \otimes (b_2[\zeta] \parr A),\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}$$

($C_2/\forall$)

$$\begin{array}{c}
!\mathbf{P} \vdash A,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline
!\mathbf{P} \vdash b_2[\zeta],\, b_2[\zeta]^\perp \otimes A,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
!\mathbf{P} \vdash b_2[\zeta],\, b_2[\zeta]^\perp \otimes A,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
!\mathbf{P} \vdash b_2[\zeta],\, b_2[\zeta]^\perp \otimes A,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}$$

$$\rightsquigarrow \quad \begin{array}{c}
!\mathbf{P} \vdash A,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
!\mathbf{P} \vdash A,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
!\mathbf{P} \vdash A,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
!\mathbf{P} \vdash b_2[\zeta],\, b_2[\zeta]^\perp \otimes A,\, \forall x.S[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}$$

($S/\forall$)

$$\begin{array}{c}
!\mathbf{P} \vdash A,\, B,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline
!\mathbf{P} \vdash \forall x.(b_1[x] \parr (b_2[x]^\perp \otimes A)),\, \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr B)),\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
!\mathbf{P} \vdash \forall x.(b_1[x] \parr (b_2[x]^\perp \otimes A)),\, \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr B)),\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
!\mathbf{P} \vdash \forall x.(b_1[x] \parr (b_2[x]^\perp \otimes A)),\, \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr B)),\, \forall x.\mathcal{S}[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}$$

$\rightsquigarrow$

$$\begin{array}{c}
!\mathbf{P} \vdash A,\, B,\, \mathcal{S}[\xi],\, \mathcal{R}[\xi],\, \Gamma \\ \hline\hline
!\mathbf{P} \vdash A,\, B,\, \mathcal{S}[\xi],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
!\mathbf{P} \vdash A,\, B,\, \forall x.\mathcal{S}[x],\, \exists x.\mathcal{R}[x],\, \Gamma \\ \hline
!\mathbf{P} \vdash \forall x.(b_1[x] \parr (b_2[x]^\perp \otimes A)),\, \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr B)),\, \forall x.\mathcal{S}[x],\, \exists x.\mathcal{R}[x],\, \Gamma
\end{array}$$

E.2. $\exists$-schemes

Let $R[\xi]$ stand for the formula $a_1[\xi]^\perp \otimes (a_2[\xi] \parr P)$, and $\exists x.R[x]$ for the formula $\exists x.a_1[x]^\perp \otimes (a_2[x] \parr P)$.

($\top/\exists$)

$$\frac{!\mathbf{P} \vdash \top,\, R[\xi],\, \Gamma}{!\mathbf{P} \vdash \top,\, \exists x.R[x],\, \Gamma} \quad\leadsto\quad !\mathbf{P} \vdash \top,\, \exists x.R[x],\, \Gamma$$

($\bot/\exists$)

$$\frac{\dfrac{!\mathbf{P} \vdash R[\xi],\, \Gamma}{!\mathbf{P} \vdash \bot,\, R[\xi],\, \Gamma}}{!\mathbf{P} \vdash \bot,\, \exists x.R[x],\, \Gamma} \quad\leadsto\quad \frac{\dfrac{!\mathbf{P} \vdash R[\xi],\, \Gamma}{!\mathbf{P} \vdash \exists x.R[x],\, \Gamma}}{!\mathbf{P} \vdash \bot,\, \exists x.R[x],\, \Gamma}$$

(U$/\exists$)

$$\frac{\dfrac{!\mathbf{P},\, !(A \circ\!\!- B) \vdash B,\, R[\xi],\, \Gamma}{!\mathbf{P},\, !(A \circ\!\!- B) \vdash A,\, R[\xi],\, \Gamma}}{!\mathbf{P},\, !(A \circ\!\!- B) \vdash A,\, \exists x.R[x],\, \Gamma} \quad\leadsto\quad \frac{\dfrac{!\mathbf{P},\, !(A \circ\!\!- B) \vdash A,\, R[\xi],\, \Gamma}{!\mathbf{P},\, !(A \circ\!\!- B) \vdash B,\, \exists x.R[x],\, \Gamma}}{!\mathbf{P},\, !(A \circ\!\!- B) \vdash B,\, \exists x.R[x],\, \Gamma}$$

($\oplus_i/\exists$)

$$\frac{\dfrac{!\mathbf{P} \vdash A_i,\, R[\xi],\, \Gamma}{!\mathbf{P} \vdash A_1 \oplus A_2,\, R[\xi],\, \Gamma}}{!\mathbf{P} \vdash A_1 \oplus A_2,\, \exists x.R[x],\, \Gamma} \quad\leadsto\quad \frac{\dfrac{!\mathbf{P} \vdash A_i,\, R[\xi],\, \Gamma}{!\mathbf{P} \vdash A_i,\, \exists x.R[x],\, \Gamma}}{!\mathbf{P} \vdash A_1 \oplus A_2,\, \exists x.R[x],\, \Gamma}$$

($\parr/\exists$)

$$\frac{\dfrac{!\mathbf{P} \vdash A,\, B,\, R[\xi],\, \Gamma}{!\mathbf{P} \vdash A \parr B,\, R[\xi],\, \Gamma}}{!\mathbf{P} \vdash A \parr B,\, \exists x.R[x],\, \Gamma} \quad\leadsto\quad \frac{\dfrac{!\mathbf{P} \vdash A,\, B,\, R[\xi],\, \Gamma}{!\mathbf{P} \vdash A,\, B,\, \exists x.R[x],\, \Gamma}}{!\mathbf{P} \vdash A \parr B,\, \exists x.R[x],\, \Gamma}$$

(S$/\exists$)

$$\frac{\dfrac{!\mathbf{P} \vdash A,\, B,\, R[\xi],\, \Gamma}{!\mathbf{P} \vdash \forall x.(b_1[x] \parr (b_2[x]^\perp \otimes A)),\, \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr B)),\, R[\xi],\, \Gamma}}{!\mathbf{P} \vdash \forall x.(b_1[x] \parr (b_2[x]^\perp \otimes A)),\, \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr B)),\, \exists x.R[x],\, \Gamma}$$

$$\leadsto\quad \frac{\dfrac{!\mathbf{P} \vdash A,\, B,\, R[\xi],\, \Gamma}{!\mathbf{P} \vdash A,\, B,\, \exists x.R[x],\, \Gamma}}{!\mathbf{P} \vdash \forall x.(b_1[x] \parr (b_2[x]^\perp \otimes A)),\, \exists x.(b_1[x]^\perp \otimes (b_2[x] \parr B)),\, \exists x.R[x],\, \Gamma}$$