# $\mathrm{Ty}_2$ revisited


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# Montague's Intentional Logic

## (Montague, 1973)

$$
\begin{aligned}
[\![x^{\alpha}]\!]_{g,i} &= g(x) \\
[\![c^{\alpha}]\!]_{g,i} &= V(c)(i) \\
[\![\lambda x^{\alpha}.\, t^{\beta}]\!]_{g,i} &= (d \in D_{\alpha}) \mapsto [\![t^{\beta}]\!]_{g[x:=d],i} \\
[\![t^{\alpha \to \beta}\, u^{\alpha}]\!]_{g,i} &= [\![t^{\alpha \to \beta}]\!]_{g,i}\, [\![u^{\alpha}]\!]_{g,i} \\
[\![\hat{}\, t^{\alpha}]\!]_{g,i} &= (i \in I) \mapsto [\![t^{\alpha}]\!]_{g,i} \\
[\![\check{}\, t^{\mathbf{s} \to \alpha}]\!]_{g,i} &= [\![t^{\mathbf{s} \to \alpha}]\!]_{g,i}(i)
\end{aligned}
$$

# Gallin's $\mathrm{Ty}_2$

## Translation: $\mathrm{IL} \to \mathrm{Ty}_2$ (Gallin, 1975)

$$
\begin{aligned}
(x^{\alpha})^* &= x^{\alpha} \\
(c^{\alpha})^* &= c^{\mathbf{s}\to\alpha}\,\mathbf{x}^{\mathbf{s}} \\
(\lambda x^{\alpha}.\, t^{\beta})^* &= \lambda x^{\alpha}.\,(t^{\beta})^* \\
(t^{\alpha\to\beta}\,u^{\alpha})^* &= (t^{\alpha\to\beta})^*\,(u^{\alpha})^* \\
(\hat{}\,t^{\alpha})^* &= \lambda \mathbf{x}^{\mathbf{s}}.\,(t^{\alpha})^* \\
(\check{}\,t^{\mathbf{s}\to\alpha})^* &= (t^{\mathbf{s}\to\alpha})^*\,\mathbf{x}^{\mathbf{s}}
\end{aligned}
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(t^{\alpha\to\beta}\,u^{\alpha})^{*} &= (t^{\alpha\to\beta})^{*}\,(u^{\alpha})^{*} \\
({}^{\wedge}t^{\alpha})^{*} &= \lambda \mathbf{x}^{\mathbf{s}}.\,(t^{\alpha})^{*} \\
({}^{\vee}t^{\mathbf{s}\to\alpha})^{*} &= (t^{\mathbf{s}\to\alpha})^{*}\,\mathbf{x}^{\mathbf{s}}
\end{aligned}
$$

$\models_{\mathrm{IL}} \varphi$ iff $\models_{\mathrm{Ty}_2} \varphi^{*}$

# Alternative translation

## Translation: $\mathrm{IL} \rightarrow \mathrm{Ty}_2$ (de Groote-Kanazawa, 2013)

$$
\begin{aligned}
(x^{\alpha})^{\dagger} &= \lambda i^{\mathbf{s}}.\, x^{\alpha} \\
(c^{\alpha})^{\dagger} &= \lambda i^{\mathbf{s}}.\, c^{\mathbf{s}\rightarrow\alpha}\, i^{\mathbf{s}} \\
(\lambda x^{\alpha}.\, t^{\beta})^{\dagger} &= \lambda i^{\mathbf{s}} x^{\alpha}.\, (t^{\beta})^{\dagger}\, i^{\mathbf{s}} \\
(t^{\alpha\rightarrow\beta}\, u^{\alpha})^{\dagger} &= \lambda i^{\mathbf{s}}.\, (t^{\alpha\rightarrow\beta})^{\dagger}\, i^{\mathbf{s}}\, ((u^{\alpha})^{\dagger}\, i^{\mathbf{s}}) \\
({}^{\wedge}t^{\alpha})^{\dagger} &= \lambda i^{\mathbf{s}}.\, (t^{\alpha})^{\dagger} \\
({}^{\vee}t^{\mathbf{s}\rightarrow\alpha})^{\dagger} &= \lambda i^{\mathbf{s}}.\, (t^{\mathbf{s}\rightarrow\alpha})^{\dagger}\, i^{\mathbf{s}}\, i^{\mathbf{s}}
\end{aligned}
$$

# Alternative translation

## Translation: $\mathrm{IL} \to \mathrm{Ty}_2$ (de Groote-Kanazawa, 2013)

$$
\begin{aligned}
(x^\alpha)^\dagger &= \mathsf{K}\, x^\alpha \\
(c^\alpha)^\dagger &= \mathsf{C}\,\mathsf{B}\,\mathsf{I}\, c^{\mathbf{s}\to\alpha} \\
(\lambda x^\alpha.\, t^\beta)^\dagger &= \mathsf{C}\,(\lambda x^\alpha.\,(t^\beta)^\dagger) \\
(t^{\alpha\to\beta}\, u^\alpha)^\dagger &= \mathsf{S}\,(t^{\alpha\to\beta})^\dagger\,(u^\alpha)^\dagger \\
({}^\wedge t^\alpha)^\dagger &= \mathsf{K}\,(t^\alpha)^\dagger \\
({}^\vee t^{\mathbf{s}\to\alpha})^\dagger &= \mathsf{W}\,(t^{\mathbf{s}\to\alpha})^\dagger
\end{aligned}
$$

# Alternative translation

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(t^{\alpha\rightarrow\beta}\, u^\alpha)^\dagger &= \mathsf{S}\,(t^{\alpha\rightarrow\beta})^\dagger\,(u^\alpha)^\dagger \\
({}^\wedge t^\alpha)^\dagger &= \mathsf{K}\,(t^\alpha)^\dagger \\
({}^\vee t^{\mathbf{s}\rightarrow\alpha})^\dagger &= \mathsf{W}\,(t^{\mathbf{s}\rightarrow\alpha})^\dagger
\end{aligned}
$$

$$\varphi^\dagger\, \mathbf{x}^{\mathbf{s}} = \varphi^*$$

# Hybrid Logic

## Translation: $\mathrm{HyLo} \to \mathrm{Ty}_2$

$$
\begin{aligned}
(p)^{\ddagger} &= \lambda i^{\mathbf{s}}.\, p^{\mathbf{s}\to\mathbf{t}}\; i^{\mathbf{s}} \\
(j)^{\ddagger} &= \lambda i^{\mathbf{s}}.\, j^{\mathbf{s}} = i^{\mathbf{s}} \\
(\neg\varphi)^{\ddagger} &= \lambda i^{\mathbf{s}}.\, \neg((\varphi)^{\ddagger}\; i^{\mathbf{s}}) \\
(\varphi \wedge \psi)^{\ddagger} &= \lambda i^{\mathbf{s}}.\, ((\varphi)^{\ddagger}\; i^{\mathbf{s}}) \wedge ((\psi)^{\ddagger}\; i^{\mathbf{s}}) \\
(@_j\, \varphi)^{\ddagger} &= \lambda i^{\mathbf{s}}.\, (\varphi)^{\ddagger}\; j^{\mathbf{s}} \\
(\downarrow_j .\, \varphi)^{\ddagger} &= \lambda j^{\mathbf{s}}.\, (\varphi)^{\ddagger}\; j^{\mathbf{s}}
\end{aligned}
$$

# Hybrid Logic

## HyLo interpretation

$\mathfrak{M} = \langle W, R, V \rangle$

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$$
\begin{array}{lll}
\mathfrak{M}, g, w \models p & \text{iff} & w \in V(p) \\
\mathfrak{M}, g, w \models j & \text{iff} & g(j) = w \\
\mathfrak{M}, g, w \models \neg\varphi & \text{iff} & \mathfrak{M}, g, w \not\models \varphi \\
\mathfrak{M}, g, w \models \varphi \wedge \psi & \text{iff} & \mathfrak{M}, g, w \models \varphi \text{ and } \mathfrak{M}, g, w \models \psi \\
\mathfrak{M}, g, w \models @_j\, \varphi & \text{iff} & \mathfrak{M}, g, g(j) \models \varphi \\
\mathfrak{M}, g, w \models \downarrow_j .\, \varphi & \text{iff} & \mathfrak{M}, g[j := w], w \models \varphi
\end{array}
$$

# Hybrid Logic

## $\mathrm{Ty}_2$ model

$\mathfrak{M} = \langle (D_\alpha)_{\alpha \in \mathscr{T}}, (V_\alpha)_{\alpha \in \mathscr{T}} \rangle$

- $D_{\mathbf{t}} = \{0, 1\}$
- $D_{\mathbf{s}} = W$
- $D_{\alpha \to \beta} \subset {D_\beta}^{D_\alpha}$
- $V_\alpha : \mathscr{C}_\alpha \to D_\alpha$

# Hybrid Logic

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- $V_\alpha : \mathscr{C}_\alpha \to D_\alpha$

## Ty$_2$ interpretation

$$
\begin{aligned}
[\![ x^\alpha ]\!]_g^{\mathfrak{M}} &= g(x) \\
[\![ c^\alpha ]\!]_g^{\mathfrak{M}} &= V_\alpha(c) \\
[\![ \lambda x^\alpha .\, t^\beta ]\!]_g^{\mathfrak{M}} &= (d \in D_\alpha) \mapsto [\![ t^\beta ]\!]_{g[x:=d]}^{\mathfrak{M}} \\
[\![ t^{\alpha \to \beta}\, u^\alpha ]\!]_g^{\mathfrak{M}} &= [\![ t^{\alpha \to \beta} ]\!]_g^{\mathfrak{M}}\, [\![ u^\alpha ]\!]_g^{\mathfrak{M}}
\end{aligned}
$$

# Hybrid Logic

$\mathfrak{M} = \langle W, R, V \rangle \quad \mathfrak{M}' = \langle (D_\alpha)_{\alpha \in \mathscr{T}}, (V_\alpha)_{\alpha \in \mathscr{T}} \rangle$

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- $D_{\mathbf{s}} = W$
- $V_{\mathbf{s} \to \mathbf{t}}(p)(w) = 1$ iff $w \in V(p)$
- $V_{\mathbf{t} \to \mathbf{t}}(\neg) = \{(0,1), (1,0)\}$
- $V_{\mathbf{t} \to \mathbf{t} \to \mathbf{t}}(\wedge) = \{(0, \{(0,0), (1,0)\}), (1, \{(0,0), (1,1)\})\}$

# Hybrid Logic

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$$\mathfrak{M}, g, w \vDash \varphi \text{ iff } [\![\varphi^{\ddagger}]\!]_g^{\mathfrak{M}'}(w) = 1$$

# Application

$$\textit{Q1}: \downarrow_s. (\varphi \to \psi) \to (\varphi \to \downarrow_s. \psi) \quad s \notin \mathrm{FV}(\varphi)$$
$$K_{@}: @_s (\varphi \to \psi) \to (@_s \varphi \to @_s \psi)$$
$$\textit{Scope}: @_s (@_j \varphi) \leftrightarrow @_j \varphi$$

# Application

$$\mathit{Q1} : {\downarrow_s}.\,(\varphi \to \psi) \to (\varphi \to {\downarrow_s}.\,\psi) \quad s \notin \mathrm{FV}(\varphi)$$
$$K_{@} : @_s\,(\varphi \to \psi) \to (@_s\,\varphi \to @_s\,\psi)$$
$$\mathit{Scope} : @_s\,(@_j\,\varphi) \leftrightarrow @_j\,\varphi$$

Instances of $\alpha \to \alpha$

# Application

$$(\downarrow_s.\,(\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \downarrow_s.\,\psi))^{\ddagger}$$

# Application

$$(\downarrow_s .\, (\varphi \to \psi) \to (\varphi \to \downarrow_s .\, \psi))^{\ddagger}$$
$$= \lambda i.\, ((\downarrow_s .\, (\varphi \to \psi))^{\ddagger}\, i) \to ((\varphi \to \downarrow_s .\, \psi)^{\ddagger}\, i)$$

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$$= \lambda i.\,((\lambda s.\,(\varphi \to \psi)^{\ddagger}\, s)\, i) \to ((\varphi \to \downarrow_s.\,\psi)^{\ddagger}\, i)$$

# Application

$$
\begin{aligned}
&(\downarrow_s.\,(\varphi \to \psi) \to (\varphi \to \downarrow_s.\,\psi))^{\ddagger} \\
&\quad = \lambda i.\,((\downarrow_s.\,(\varphi \to \psi))^{\ddagger}\, i) \to ((\varphi \to \downarrow_s.\,\psi)^{\ddagger}\, i) \\
&\quad = \lambda i.\,((\lambda s.\,(\varphi \to \psi)^{\ddagger}\, s)\, i) \to ((\varphi \to \downarrow_s.\,\psi)^{\ddagger}\, i) \\
&\quad = \lambda i.\,((\lambda s.\,(\lambda i.\,(\varphi^{\ddagger}\, i) \to (\psi^{\ddagger}\, i))\, s)\, i) \to ((\varphi \to \downarrow_s.\,\psi)^{\ddagger}\, i)
\end{aligned}
$$

# Application

$$
\begin{aligned}
&(\downarrow_s . (\varphi \to \psi) \to (\varphi \to \downarrow_s . \psi))^\ddagger \\
&\quad = \lambda i. ((\downarrow_s . (\varphi \to \psi))^\ddagger\, i) \to ((\varphi \to \downarrow_s . \psi)^\ddagger\, i) \\
&\quad = \lambda i. ((\lambda s. (\varphi \to \psi)^\ddagger\, s)\, i) \to ((\varphi \to \downarrow_s . \psi)^\ddagger\, i) \\
&\quad = \lambda i. ((\lambda s. (\lambda i. (\varphi^\ddagger\, i) \to (\psi^\ddagger\, i))\, s)\, i) \to ((\varphi \to \downarrow_s . \psi)^\ddagger\, i) \\
&\quad \to_\beta \lambda i. ((\lambda s. (\varphi^\ddagger\, s) \to (\psi^\ddagger\, s))\, i) \to ((\varphi \to \downarrow_s . \psi)^\ddagger\, i)
\end{aligned}
$$

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&(\downarrow_s.\,(\varphi \to \psi) \to (\varphi \to \downarrow_s.\,\psi))^{\ddagger} \\
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&\quad \to_{\beta} \lambda i.\,((\lambda s.\,(\varphi^{\ddagger}\, s) \to (\psi^{\ddagger}\, s))\, i) \to ((\varphi \to \downarrow_s.\,\psi)^{\ddagger}\, i) \\
&\quad \to_{\beta} \lambda i.\,((\varphi^{\ddagger}\, i) \to (\psi^{\ddagger}[s := i]\, i)) \to ((\varphi \to \downarrow_s.\,\psi)^{\ddagger}\, i)
\end{aligned}
$$

# Application

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&(\downarrow_s.\, (\varphi \to \psi) \to (\varphi \to \downarrow_s.\, \psi))^{\ddagger} \\
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&\quad \to_\beta \lambda i.\, ((\lambda s.\, (\varphi^{\ddagger}\, s) \to (\psi^{\ddagger}\, s))\, i) \to ((\varphi \to \downarrow_s.\, \psi)^{\ddagger}\, i) \\
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\end{aligned}
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$$
\begin{aligned}
&(\downarrow_s . (\varphi \to \psi) \to (\varphi \to \downarrow_s . \psi))^{\ddagger} \\
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&\quad = \lambda i.\,((\varphi^{\ddagger}\, i) \to (\psi^{\ddagger}[s := i]\, i)) \to ((\lambda i.\,(\varphi^{\ddagger}\, i) \to ((\downarrow_s.\,\psi)^{\ddagger}\, i))\, i)\\
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\end{aligned}
$$

# Application

$$\textit{Self Dual}_{\downarrow} : {\downarrow_s}.\,\varphi \leftrightarrow \neg{\downarrow_s}.\,\neg\varphi$$
$$\textit{Self Dual}_{@} : @_s\,\varphi \leftrightarrow \neg(@_s\,\neg\varphi)$$

# Application

*Self Dual*$_\downarrow$ : $\downarrow_s . \varphi \leftrightarrow \neg\downarrow_s . \neg\varphi$
*Self Dual*$_@$ : $@_s\, \varphi \leftrightarrow \neg(@_s\, \neg\varphi)$

Instances of $\alpha \leftrightarrow \neg\neg\alpha$

# Application

$$\begin{aligned}
\mathit{Q2}&: \downarrow_s .\, \varphi \to (t \to \varphi[s := t])\\
\mathit{Q3}&: \downarrow_s .\, (s \to \varphi) \to \downarrow_s .\, \varphi\\
\mathit{Introduction}&: (s \wedge \varphi) \to (@_s\, \varphi)\\
\mathit{Label}&: @_s\, s\\
\mathit{Nom}&: (@_s\, t) \to ((@_t\, \varphi) \to (@_s\, \varphi))\\
\mathit{Swap}&: (@_s\, t) \to (@_t\, s)
\end{aligned}$$

# Application

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\textit{Q2}: &\ \downarrow_s.\,\varphi \to (t \to \varphi[s := t]) \\
\textit{Q3}: &\ \downarrow_s.\,(s \to \varphi) \to \downarrow_s.\,\varphi \\
\textit{Introduction}: &\ (s \wedge \varphi) \to (@_s\, \varphi) \\
\textit{Label}: &\ @_s\, s \\
\textit{Nom}: &\ (@_s\, t) \to ((@_t\, \varphi) \to (@_s\, \varphi)) \\
\textit{Swap}: &\ (@_s\, t) \to (@_t\, s)
\end{aligned}$$

Instances of elementary equality reasoning.

# Conclusions

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3. It may be easily extended to cover the higher-order case, resulting in a rebuilding of the Hybrid Type Theory of Areces et al. (2014).

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1. The translation $(\cdot)^{\ddagger}$ provides a way of easily incorporating hybrid logic constructs in a Montague grammar.
2. It allows to establish some results on hybrid formulas simply by using $\beta$-reduction.
3. It may be easily extended to cover the higher-order case, resulting in a rebuilding of the Hybrid Type Theory of Areces et al. (2014).
4. It may be mixed with translation $(\cdot)^{\dagger}$, resulting in translations of (higher-order) Intensional Hybrid Logics .