Structures Informatiques et Logiques pour la Modélisation Linguistique (MPRI 2.27.1 - second part)

Philippe de Groote

Inria

2015-2016

Formal semantics

Formal semantic

A formal point of view

There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of languages within a single natural and mathematically precise theory. On this point I differ from a number of philosophers (...).

R. Montague,
Universal Grammar,
Theoria 36:373–398 (1970)

Semantic translations

- Interpret directly natural language utterances into a model (in the Tarskian tradition).
- Give the semantic interpretation of some logic (intensional logic, in Montague's case). Translate natural language utterances as formulas of this logic.

Montague's legacy

- The notion of fragment.
- Semantics as an homomorphic image of syntax.
- Semantic interpretation through a translation into an intermediate logical form.

Formal semantic

Syntax/semantics interface:

$$\begin{array}{lcl} \text{JOHN} & : & \text{NP} \\ \text{MARY} & : & \text{NP} \\ \text{LOVES} & : & \text{NP} \rightarrow \text{NP} \rightarrow \text{S} \end{array}$$

$$\begin{array}{ll} [\![\text{NP}]\!] & = \mathbf{e} \\ [\![\text{S}]\!] & = \mathbf{t} \end{array}$$

Semantic interpretation:

$$\begin{array}{ll} [\![\text{JOHN}]\!] & = \mathbf{j} \\ [\![\text{MARY}]\!] & = \mathbf{m} \\ [\![\text{LOVES}]\!] & = \lambda y.\, \lambda x.\, \mathbf{love}\, x\, y \end{array}$$

where:

$$\begin{array}{l} \mathbf{j}, \mathbf{m} : \mathbf{e} \\ \mathbf{love} : \mathbf{e} \rightarrow \mathbf{e} \rightarrow \mathbf{t} \end{array}$$

Formal semantic

Syntax/semantics interface:

$$
\begin{array}{lcl}
\text{JOHN} & : & \text{NP} \\
\text{MARY} & : & \text{NP} \\
\text{EVERYBODY} & : & \text{NP} \\
\text{SOMEBODY} & : & \text{NP} \\
\text{LOVES} & : & \text{NP} \rightarrow \text{NP} \rightarrow \text{S}
\end{array}
$$

$$
\begin{array}{ll}
[\![\text{NP}]\!] & = (\mathbf{e} \rightarrow \mathbf{t}) \rightarrow \mathbf{t} \\
[\![\text{S}]\!] & = \mathbf{t}
\end{array}
$$

Semantic interpretation:

$$
\begin{array}{ll}
[\![\text{JOHN}]\!] & = \lambda k.\, k\,\mathbf{j} \\
[\![\text{MARY}]\!] & = \lambda k.\, k\,\mathbf{m} \\
[\![\text{EVERYBODY}]\!] & = \lambda k.\, \forall x.\, k\,x \\
[\![\text{SOMEBODY}]\!] & = \lambda k.\, \exists x.\, k\,x \\
[\![\text{LOVES}]\!] & = \lambda o.\, \lambda s.\, s\,(\lambda x.\, o\,(\lambda y.\, \mathbf{love}\,x\,y))
\end{array}
$$

Formal semantic

Syntax/semantics interface:

$$
\begin{array}{lcl}
\text{JOHN} & : & NP \\
\text{MARY} & : & NP \\
\text{EVERYBODY} & : & NP \\
\text{SOMEBODY} & : & NP \\
\text{MAN} & : & N \\
\text{WOMAN} & : & N \\
\text{EVERY} & : & N \to NP \\
\text{A} & : & N \to NP \\
\text{LOVES} & : & NP \to NP \to S
\end{array}
$$

$$
\begin{array}{ll}
[\![N]\!] & = \mathbf{e} \to \mathbf{t} \\
[\![NP]\!] & = (\mathbf{e} \to \mathbf{t}) \to \mathbf{t} \\
[\![S]\!] & = \mathbf{t}
\end{array}
$$

Semantic interpretation:

$$
\begin{aligned}
[\![\text{JOHN}]\!] &= \lambda k.\, k\,\mathbf{j} \\
[\![\text{MARY}]\!] &= \lambda k.\, k\,\mathbf{m} \\
[\![\text{EVERYBODY}]\!] &= \lambda k.\, \forall x.\, k\,x \\
[\![\text{SOMEBODY}]\!] &= \lambda k.\, \exists x.\, k\,x \\
[\![\text{MAN}]\!] &= \lambda x.\, \mathbf{man}\,x \\
[\![\text{WOMAN}]\!] &= \lambda x.\, \mathbf{woman}\,x \\
[\![\text{EVERY}]\!] &= \lambda n.\, \lambda m.\, \forall x.\, n\,x \supset m\,x \\
[\![\text{A}]\!] &= \lambda n.\, \lambda m.\, \exists x.\, n\,x \wedge m\,x \\
[\![\text{LOVES}]\!] &= \lambda o.\, \lambda s.\, s\,(\lambda x.\, o\,(\lambda y.\, \mathbf{love}\,x\,y))
\end{aligned}
$$

where:

$$\mathbf{woman}, \mathbf{man} : \mathbf{e} \rightarrow \mathbf{t}$$

Formal semantic

Syntax/semantics interface:

$$
\begin{array}{lcl}
\mathrm{JOHN} & : & \mathrm{NP} \\
\mathrm{MARY} & : & \mathrm{NP} \\
\mathrm{EVERYBODY} & : & \mathrm{NP} \\
\mathrm{SOMEBODY} & : & \mathrm{NP} \\
\mathrm{MAN} & : & \mathrm{N} \\
\mathrm{WOMAN} & : & \mathrm{N} \\
\mathrm{EVERY} & : & \mathrm{N} \rightarrow \mathrm{NP} \\
\mathrm{A} & : & \mathrm{N} \rightarrow \mathrm{NP} \\
\mathrm{LOVES} & : & \mathrm{NP} \rightarrow \mathrm{NP} \rightarrow \mathrm{S} \\
\mathrm{WHO} & : & (\mathrm{NP} \rightarrow \mathrm{S}) \rightarrow \mathrm{N} \rightarrow \mathrm{N}
\end{array}
$$

$$
\begin{array}{ll}
[\![\mathrm{N}]\!] & = \mathbf{e} \rightarrow \mathbf{t} \\
[\![\mathrm{NP}]\!] & = (\mathbf{e} \rightarrow \mathbf{t}) \rightarrow \mathbf{t} \\
[\![\mathrm{S}]\!] & = \mathbf{t}
\end{array}
$$

Semantic interpretation:

$$
\begin{aligned}
[\![\text{JOHN}]\!] &= \lambda k.\, k\, \mathbf{j} \\
[\![\text{MARY}]\!] &= \lambda k.\, k\, \mathbf{m} \\
[\![\text{EVERYBODY}]\!] &= \lambda k.\, \forall x.\, k\, x \\
[\![\text{SOMEBODY}]\!] &= \lambda k.\, \exists x.\, k\, x \\
[\![\text{MAN}]\!] &= \lambda x.\, \mathbf{man}\, x \\
[\![\text{WOMAN}]\!] &= \lambda x.\, \mathbf{woman}\, x \\
[\![\text{EVERY}]\!] &= \lambda n.\, \lambda m.\, \forall x.\, n\, x \supset m\, x \\
[\![\text{A}]\!] &= \lambda n.\, \lambda m.\, \exists x.\, n\, x \wedge m\, x \\
[\![\text{LOVES}]\!] &= \lambda o.\, \lambda s.\, s\, (\lambda x.\, o\, (\lambda y.\, \mathbf{love}\, x\, y)) \\
[\![\text{WHO}]\!] &= \lambda r.\, \lambda n.\, \lambda x.\, n\, x \wedge r\, (\lambda k.\, k\, x)
\end{aligned}
$$

Formal semantic

Syntax/semantics interface:

$$
\begin{array}{lcl}
\text{JOHN} & : & \text{NP} \\
\text{MARY} & : & \text{NP} \\
\text{EVERYBODY} & : & \text{NP} \\
\text{SOMEBODY} & : & \text{NP} \\
\text{MAN} & : & \text{N} \\
\text{WOMAN} & : & \text{N} \\
\text{EVERY} & : & \text{N} \rightarrow \text{NP} \\
\text{A} & : & \text{N} \rightarrow \text{NP} \\
\text{FRENCH} & : & \text{N} \rightarrow \text{N} \\
\text{LOVES} & : & \text{NP} \rightarrow \text{NP} \rightarrow \text{S} \\
\text{WHO} & : & (\text{NP} \rightarrow \text{S}) \rightarrow \text{N} \rightarrow \text{N}
\end{array}
$$

$$
\begin{array}{ll}
[\![\text{N}]\!] & = \mathbf{e} \rightarrow \mathbf{t} \\
[\![\text{NP}]\!] & = (\mathbf{e} \rightarrow \mathbf{t}) \rightarrow \mathbf{t} \\
[\![\text{S}]\!] & = \mathbf{t}
\end{array}
$$

Semantic interpretation:

$$
\begin{aligned}
[\![\text{JOHN}]\!] &= \lambda k.\, k\,\mathbf{j} \\
[\![\text{MARY}]\!] &= \lambda k.\, k\,\mathbf{m} \\
[\![\text{EVERYBODY}]\!] &= \lambda k.\, \forall x.\, k\,x \\
[\![\text{SOMEBODY}]\!] &= \lambda k.\, \exists x.\, k\,x \\
[\![\text{MAN}]\!] &= \lambda x.\, \mathbf{man}\,x \\
[\![\text{WOMAN}]\!] &= \lambda x.\, \mathbf{woman}\,x \\
[\![\text{EVERY}]\!] &= \lambda n.\, \lambda m.\, \forall x.\, n\,x \supset m\,x \\
[\![\text{A}]\!] &= \lambda n.\, \lambda m.\, \exists x.\, n\,x \wedge m\,x \\
[\![\text{FRENCH}]\!] &= \lambda n.\, \lambda x.\, n\,x \wedge \mathbf{french}\,x \\
[\![\text{LOVES}]\!] &= \lambda o.\, \lambda s.\, s\,(\lambda x.\, o\,(\lambda y.\, \mathbf{love}\,x\,y)) \\
[\![\text{WHO}]\!] &= \lambda r.\, \lambda n.\, \lambda x.\, n\,x \wedge r\,(\lambda k.\, k\,x)
\end{aligned}
$$

where:

$$\mathbf{french} : \mathbf{e} \rightarrow \mathbf{t}$$

Adjective classification:

- **Intersective**:

  French, sick, carnivorous, red, ...

- **Subsective** but non intersective:

  typical, recent, skillful, ...

- **Privative**:

  fake, former, spurious, ...

- **Plain nonsubsective**:

  alleged, arguable, putative, ...

Meaning postulates:

$$\mathrm{INT}(A) = \exists P. \forall Q\, x.\, A\, Q\, x \equiv (P\, x \wedge Q\, x)$$

$$\mathrm{SUB}(A) = \forall Q\, x.\, A\, Q\, x \supset Q\, x$$

$$\mathrm{PRIV}(A) = \forall Q\, x.\, A\, Q\, x \supset \neg(Q\, x)$$

Beware!!!! Some intensionality involved!

Formal semantic

Scope ambiguities

Every man loves a woman

$$\forall x.\mathbf{man}\, x \supset (\exists y.\mathbf{woman}\, y \wedge \mathbf{love}\, x\, y)$$
$$\exists y.\mathbf{woman}\, y \wedge (\forall x.\mathbf{man}\, x \wedge \mathbf{love}\, x\, y)$$

Subject wide scope:

$$[\![\mathrm{LOVES}]\!] = \lambda o.\, \lambda s.\, s\, (\lambda x.\, o\, (\lambda y.\, \mathbf{love}\, x\, y))$$

Object wide scope:

$$[\![\mathrm{LOVES}_{\mathrm{ows}}]\!] = \lambda o.\, \lambda s.\, o\, (\lambda y.\, s\, (\lambda x.\, \mathbf{love}\, x\, y))$$

Scope ambiguities

Another solution:

$$\mathrm{QR} \ : \ \mathrm{NP} \to (\mathrm{NP} \to \mathrm{S}) \to \mathrm{S}$$

$$[\![\mathrm{QR}]\!] \ = \ \lambda n.\, \lambda p.\, n\, (\lambda x.\, p\, (\lambda k.\, k\, x))$$

Wide scope reading:

$$\mathrm{QR}\,(\mathrm{A\ WOMAN})\,(\lambda o.\, \mathrm{LOVE}\, o\, (\mathrm{EVERY\ MAN}))$$

Scope ambiguities

Yet another solution:

$$\begin{array}{rcl} \text{every} & : & \mathrm{N} \to (\mathrm{NP} \to \mathrm{S}) \to \mathrm{S} \\ \text{a} & : & \mathrm{N} \to (\mathrm{NP} \to \mathrm{S}) \to \mathrm{S} \end{array}$$

with

$$\begin{array}{rcl} [\![\mathrm{S}]\!] & = & \mathbf{t} \\ [\![\mathrm{NP}]\!] & = & \mathbf{e} \end{array}$$

$$\begin{array}{ll} [\![\textsc{every}]\!] & = \lambda n.\, \lambda m.\, \forall x.\, n\, x \supset m\, x \\ [\![\textsc{a}]\!] & = \lambda n.\, \lambda m.\, \exists x.\, n\, x \wedge m\, x \\ [\![\textsc{loves}]\!] & = \lambda o.\, \lambda s.\, \mathbf{love}\, s\, o \end{array}$$

Two readings:

$$\begin{array}{l} \textsc{every man}\, (\lambda s.\, \textsc{a woman}\, (\lambda o.\, \textsc{love}\, o\, s)) \\ \textsc{a woman}\, (\lambda o.\, \textsc{every man}\, (\lambda s.\, \textsc{love}\, o\, s)) \end{array}$$

Formal semantic

De re and de dicto readings as scope ambiguities

John seeks a unicorn

$$\exists x.\mathbf{unicorn}\, x \wedge \mathbf{try}\, \mathbf{j}\, (\lambda z.\, \mathbf{find}\, z\, x)$$
$$\mathbf{try}\, \mathbf{j}\, (\lambda z.\, \exists x.\mathbf{unicorn}\, x \wedge \mathbf{find}\, z\, x)$$

De re reading:

$$[\![\mathrm{SEEK}_{\mathrm{re}}]\!] \;=\; \lambda o.\, \lambda s.\, s\, (\lambda x.\, o\, (\lambda y.\, \mathbf{try}\, x\, (\lambda z.\, \mathbf{find}\, z\, y)))$$

De dicto reading:

$$[\![\mathrm{SEEK}_{\mathrm{dicto}}]\!] \;=\; \lambda o.\, \lambda s.\, s\, (\lambda x.\, \mathbf{try}\, x\, (\lambda z.\, o\, (\lambda y.\, \mathbf{find}\, z\, y)))$$

Beware!!!! Some intensionality involved!

Formal semantic

Intensionality

This red car is a Ferrari

This skillful surgeon is Dr Johnson

$$(\forall x.\,(\textbf{surgeon}\,x) \leftrightarrow (\textbf{driver}\,x)) \\ \rightarrow (\forall x.\,((\textbf{skillful}\,\textbf{surgeon})\,x) \rightarrow ((\textbf{skillful}\,\textbf{driver})\,x))$$

Solution:

$$\begin{aligned} \textbf{surgeon} &: \mathrm{e} \rightarrow (\mathrm{s} \rightarrow \mathrm{t}) \\ \textbf{driver} &: \mathrm{e} \rightarrow (\mathrm{s} \rightarrow \mathrm{t}) \\ \textbf{skillful} &: (\mathrm{e} \rightarrow (\mathrm{s} \rightarrow \mathrm{t})) \rightarrow \mathrm{e} \rightarrow (\mathrm{s} \rightarrow \mathrm{t}) \end{aligned}$$

Formal semantic

Intensionalization

Type transformation:

$$\overline{\mathbf{e}} = \mathbf{s} \to \mathbf{e}$$
$$\overline{\mathbf{t}} = \mathbf{s} \to \mathbf{t}$$
$$\overline{\alpha \to \beta} = \overline{\alpha} \to \overline{\beta}$$

Term transformation:

$$\overline{x} = x$$
$$\overline{\lambda x.\, t} = \lambda x.\, \overline{t}$$
$$\overline{t\, u} = \overline{t}\, \overline{u}$$
$$\overline{c} = \mathbb{I}_{\alpha}\, c$$

Intensionalization

$$\mathbb{I}_\alpha : (\mathbf{s} \to \alpha) \to \overline{\alpha} \qquad \mathbb{E}_\alpha : \overline{\alpha} \to \mathbf{s} \to \alpha$$

$$\begin{aligned}
\mathbb{I}_{\mathbf{e}}\, a &= a \\
\mathbb{I}_{\mathbf{t}}\, P &= P \\
\mathbb{I}_{\alpha\to\beta}\, f &= \lambda a.\, \mathbb{I}_\beta\, (\lambda i.\, f\, i\, (\mathbb{E}_\alpha\, a\, i))
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}_{\mathbf{e}}\, a\, i &= a\, i \\
\mathbb{E}_{\mathbf{t}}\, P\, i &= P\, i \\
\mathbb{E}_{\alpha\to\beta}\, f\, i &= \lambda a.\, \mathbb{E}_\beta\, (f\, (\mathbb{I}_\alpha\, (\lambda k.\, a)))\, i
\end{aligned}$$