

Structures Informatiques et Logiques pour la Modélisation Linguistique (MPRI 2.27.1 - second part)

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Inria

2021-2022

- 1 Montague semantics
- 2 A direct naive interpretation
- 3 Quantified noun phrases
- 4 Noun and determiners
- 5 Relative clauses
- 6 Adjectives
- 7 Scope ambiguities
- 8 De re and de dicto readings
- 9 Intensionality
- 10 Intensionalization

Formal semantic

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A formal point of view

There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of languages within a single natural and mathematically precise theory. On this point I differ from a number of philosophers (...).

R. Montague,
Universal Grammar,
Theoria 36:373–398 (1970)

Semantic translations

- Interpret directly natural language utterances into a model (in the Tarskian tradition).
- Give the semantic interpretation of some logic (intensional logic, in Montague's case). Translate natural language utterances as formulas of this logic.

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Montague's legacy

- The notion of fragment.
- Semantics as an homomorphic image of syntax.
- Semantic interpretation through a translation into an intermediate logical form.

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Syntax/semantics interface:

JOHN : NP

MARY : NP

LOVES : NP \rightarrow NP \rightarrow S

[[NP]] = e

[[S]] = t

Semantic interpretation:

[[JOHN]] = j

[[MARY]] = m

[[LOVES]] = $\lambda y. \lambda x. \text{love } x y$

where:

j, m : e

love : e \rightarrow e \rightarrow t

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Syntax/semantics interface:

JOHN : NP

MARY : NP

EVERYBODY : NP

SOMEBODY : NP

LOVES : NP \rightarrow NP \rightarrow S $[[\text{NP}]] = (\text{e} \rightarrow \text{t}) \rightarrow \text{t}$ $[[\text{S}]] = \text{t}$

Semantic interpretation:

 $[[\text{JOHN}]] = \lambda k. k \text{ j}$ $[[\text{MARY}]] = \lambda k. k \text{ m}$ $[[\text{EVERYBODY}]] = \lambda k. \forall x. (\text{human } x) \supset (k x)$ $[[\text{SOMEBODY}]] = \lambda k. \exists x. (\text{human } x) \wedge (k x)$ $[[\text{LOVES}]] = \lambda o. \lambda s. s (\lambda x. o (\lambda y. \text{love } x y))$

Syntax/semantics interface:

JOHN : NP
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EVERY	:	$N \rightarrow NP$
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where:

$$\text{woman, man} : e \rightarrow t$$

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french : $e \rightarrow t$

Adjective classification:

- **Intersective:**

French, sick, carnivorous, red, ...

- **Subjective** but non intersective:

typical, recent, skillful, ...

- **Privative:**

fake, former, spurious, ...

- **Plain nonsubjective:**

alleged, arguable, putative, ...

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Meaning postulates:

$$\text{INT}(A) = \exists P. \forall Q x. A Q x \equiv (P x \wedge Q x)$$

$$\text{SUB}(A) = \forall Q x. A Q x \supset Q x$$

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Beware!!!! Some intensionality involved!

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Scope ambiguities

Every man loves a woman

$$\begin{aligned} \forall x. \text{man } x \supset (\exists y. \text{woman } y \wedge \text{love } x y) \\ \exists y. \text{woman } y \wedge (\forall x. \text{man } x \supset \text{love } x y) \end{aligned}$$

Subject wide scope:

$$\llbracket \text{LOVES} \rrbracket = \lambda o. \lambda s. s (\lambda x. o (\lambda y. \text{love } x y))$$

Object wide scope:

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Scope ambiguities

Another solution:

$$\text{QR} : \text{NP} \rightarrow (\text{NP} \rightarrow \text{S}) \rightarrow \text{S}$$

$$[[\text{QR}]] = \lambda n. \lambda p. n (\lambda x. p (\lambda k. k x))$$

Wide scope reading:

$$\text{QR} (\text{A WOMAN}) (\lambda o. \text{LOVE } o (\text{EVERY MAN}))$$

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Yet another solution:

$$\begin{aligned} \text{every} & : N \rightarrow (NP \rightarrow S) \rightarrow S \\ \text{a} & : N \rightarrow (NP \rightarrow S) \rightarrow S \end{aligned}$$

with

$$\begin{aligned} \llbracket S \rrbracket & = t \\ \llbracket NP \rrbracket & = e \end{aligned}$$

$$\begin{aligned} \llbracket \text{EVERY} \rrbracket & = \lambda n. \lambda m. \forall x. n x \supset m x \\ \llbracket \text{A} \rrbracket & = \lambda n. \lambda m. \exists x. n x \wedge m x \\ \llbracket \text{LOVES} \rrbracket & = \lambda o. \lambda s. \text{love } s o \end{aligned}$$

Two readings:

$$\begin{aligned} \text{EVERY MAN } (\lambda s. \text{A WOMAN } (\lambda o. \text{LOVE } o s)) \\ \text{A WOMAN } (\lambda o. \text{EVERY MAN } (\lambda s. \text{LOVE } o s)) \end{aligned}$$

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Formal semantic

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De re and de dicto readings as scope ambiguities

John seeks a unicorn

$$\exists x.\text{unicorn } x \wedge \text{try } j (\lambda z.\text{find } z x)$$

$$\text{try } j (\lambda z.\exists x.\text{unicorn } x \wedge \text{find } z x)$$

De re reading:

$$[[\text{SEEK}_{\text{re}}]] = \lambda o.\lambda s.s (\lambda x.o (\lambda y.\text{try } x (\lambda z.\text{find } z y)))$$

De dicto reading:

$$[[\text{SEEK}_{\text{dicto}}]] = \lambda o.\lambda s.s (\lambda x.\text{try } x (\lambda z.o (\lambda y.\text{find } z y)))$$

Beware!!!! Some intensionality involved!

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Intensionality

This red car is a Ferrari

This skillful surgeon is Dr Johnson

$$\begin{aligned}
 &(\forall x. (\mathbf{surgeon} \ x) \leftrightarrow (\mathbf{driver} \ x)) \\
 &\quad \rightarrow (\forall x. ((\mathbf{skillful} \ \mathbf{surgeon}) \ x) \rightarrow ((\mathbf{skillful} \ \mathbf{driver}) \ x))
 \end{aligned}$$

Solution:

surgeon : $e \rightarrow (s \rightarrow t)$

driver : $e \rightarrow (s \rightarrow t)$

skillful : $(e \rightarrow (s \rightarrow t)) \rightarrow e \rightarrow (s \rightarrow t)$

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Intensionalization

Type transformation:

$$\bar{e} = s \rightarrow e$$

$$\bar{t} = s \rightarrow t$$

$$\overline{\alpha \rightarrow \beta} = \bar{\alpha} \rightarrow \bar{\beta}$$

Term transformation:

$$\bar{x} = x$$

$$\overline{\lambda x. t} = \lambda x. \bar{t}$$

$$\overline{t u} = \bar{t} \bar{u}$$

$$\bar{c} = \mathbb{I}_\alpha c$$

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Intensionalization

$$\mathbb{I}_\alpha : (\mathbf{s} \rightarrow \alpha) \rightarrow \bar{\alpha} \quad \mathbb{E}_\alpha : \bar{\alpha} \rightarrow \mathbf{s} \rightarrow \alpha$$

$$\mathbb{I}_e a = a$$

$$\mathbb{I}_t P = P$$

$$\mathbb{I}_{\alpha \rightarrow \beta} f = \lambda a. \mathbb{I}_\beta (\lambda i. f i (\mathbb{E}_\alpha a i))$$

$$\mathbb{E}_e a i = a i$$

$$\mathbb{E}_t P i = P i$$

$$\mathbb{E}_{\alpha \rightarrow \beta} f i = \lambda a. \mathbb{E}_\beta (f (\mathbb{I}_\alpha (\lambda k. a))) i$$

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