Semantics & Discourse

— Syntax-Semantics Interface —

Philippe de Groote

Outline

Using an object language

Using an object language

natural language

Using an object language

natural language → object language

Using an object language

natural language → object language

Eric is tall and thin

Using an object language

natural language $\rightarrow$ object language

Eric is tall and thin $\quad \rightarrow \quad \mathbf{tall}(\mathbf{e}) \wedge \mathbf{thin}(\mathbf{e})$

Using an object language

natural language $\rightarrow$ object language

Eric is tall and thin $\rightarrow$ $\mathbf{tall}(\mathbf{e}) \wedge \mathbf{thin}(\mathbf{e})$

Rebecca has a husband

Using an object language

natural language → object language

Eric is tall and thin → $\mathbf{tall}(\mathbf{e}) \wedge \mathbf{thin}(\mathbf{e})$

Rebecca has a husband → $\exists x.\, \mathbf{husband}(x, \mathbf{r})$

Using an object language

natural language → object language

Eric is tall and thin → $\mathbf{tall}(\mathbf{e}) \wedge \mathbf{thin}(\mathbf{e})$

Rebecca has a husband → $\exists x.\,\mathbf{husband}(x, \mathbf{r})$

Eric is someone's husband.

Using an object language

natural language → object language

Eric is tall and thin $\rightarrow$ $\mathbf{tall}(\mathbf{e}) \wedge \mathbf{thin}(\mathbf{e})$

Rebecca has a husband $\rightarrow$ $\exists x.\, \mathbf{husband}(x, \mathbf{r})$

Eric is someone's husband. $\rightarrow$ $\exists x.\, \mathbf{husband}(\mathbf{e}, x)$

Using an object language

natural language → object language

Eric is tall and thin → $\mathbf{tall}(\mathbf{e}) \wedge \mathbf{thin}(\mathbf{e})$

Rebecca has a husband → $\exists x. \mathbf{husband}(x, \mathbf{r})$

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Everybody has a father

Using an object language

natural language → object language

Eric is tall and thin → $\mathbf{tall}(\mathbf{e}) \wedge \mathbf{thin}(\mathbf{e})$

Rebecca has a husband → $\exists x.\, \mathbf{husband}(x, \mathbf{r})$

Eric is someone's husband. → $\exists x.\, \mathbf{husband}(\mathbf{e}, x)$

Everybody has a father → $\forall x.\, \exists y.\, \mathbf{Is}(\mathbf{y}, \mathbf{father}(\mathbf{x}))$

Using an object language

Using an object language

natural language

Using an object language

natural language $\rightarrow$ syntactic structure

Using an object language

natural language → syntactic structure → object language

Using an object language

natural language → syntactic structure → object language → model

Using an object language

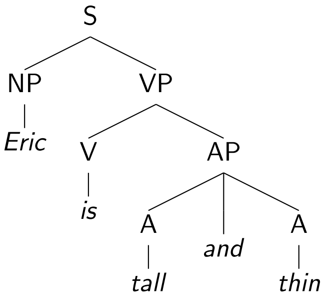
natural language → syntactic structure → object language → model

Eric is tall and thin

Using an object language

natural language → syntactic structure → object language → model

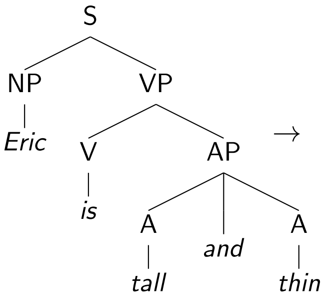
Eric is tall and thin →



Using an object language

natural language → syntactic structure → object language → model

Eric is tall and thin →



→ $\mathbf{tall}(\mathbf{e}) \land \mathbf{thin}(\mathbf{e})$

Compositionality

The principle of compositionality

Also known as Frege's principle:

> *The meaning of a complex expression is determined by its structure and the meanings of its constituents.*

Compositionality

Consider the following non-logical constants:

$$\begin{aligned}\mathbf{e} &: \mathrm{e} \\ \mathbf{tall} &: \mathrm{e} \rightarrow \mathrm{t} \\ \mathbf{thin} &: \mathrm{e} \rightarrow \mathrm{t}\end{aligned}$$

Compositionality

Consider the following non-logical constants:

$$\begin{array}{l} \mathbf{e} : \mathrm{e} \\ \mathbf{tall} : \mathrm{e} \to \mathrm{t} \\ \mathbf{thin} : \mathrm{e} \to \mathrm{t} \end{array}$$

Grammar:

$$\begin{array}{lcl} \mathrm{S} & \to & \mathrm{NP\ VP} \\ \mathrm{VP} & \to & \mathrm{V\ AP} \\ \mathrm{AP} & \to & \mathrm{A}\ \mathit{and}\ \mathrm{A} \\ \mathrm{A} & \to & \mathit{tall} \\ \mathrm{A} & \to & \mathit{thin} \\ \mathrm{NP} & \to & \mathit{Eric} \end{array}$$

Compositionality

Consider the following non-logical constants:

$$\mathbf{e} : \mathrm{e}$$
$$\mathbf{tall} : \mathrm{e} \to \mathrm{t}$$
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Grammar:

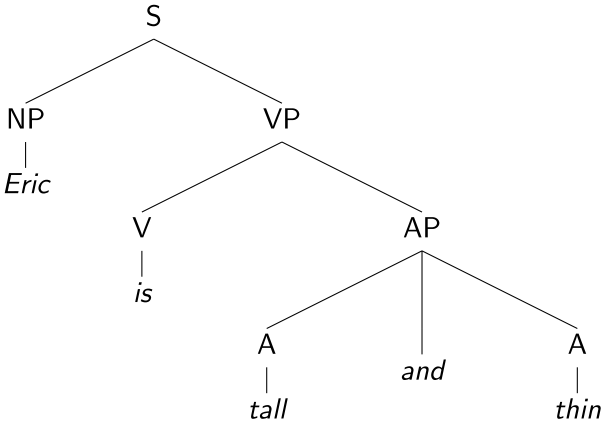
$$\begin{aligned} \mathrm{S} &\to \mathrm{NP}\ \mathrm{VP} \\ \mathrm{VP} &\to \mathrm{V}\ \mathrm{AP} \\ \mathrm{AP} &\to \mathrm{A}\ \mathit{and}\ \mathrm{A} \\ \mathrm{A} &\to \mathit{tall} \\ \mathrm{A} &\to \mathit{thin} \\ \mathrm{NP} &\to \mathit{Eric} \end{aligned}$$

Semantic rules:

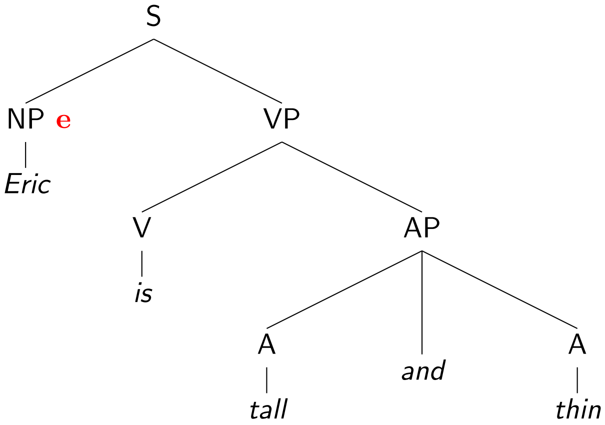
$$\begin{aligned} [\![\mathrm{S}]\!] &= [\![\mathrm{VP}]\!]\,[\![\mathrm{NP}]\!] \\ [\![\mathrm{VP}]\!] &= [\![\mathrm{AP}]\!] \\ [\![\mathrm{AP}]\!] &= \lambda x.\, ([\![\mathrm{A}_1]\!]\, x) \wedge ([\![\mathrm{A}_2]\!]\, x) \\ [\![\mathrm{A}]\!] &= \lambda x.\, \mathbf{tall}\, x \\ [\![\mathrm{A}]\!] &= \lambda x.\, \mathbf{thin}\, x \\ [\![\mathrm{NP}]\!] &= \mathbf{e} \end{aligned}$$

Compositionality

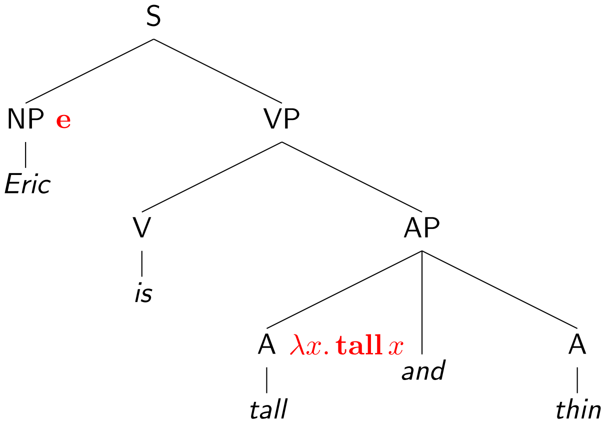
Compositionality



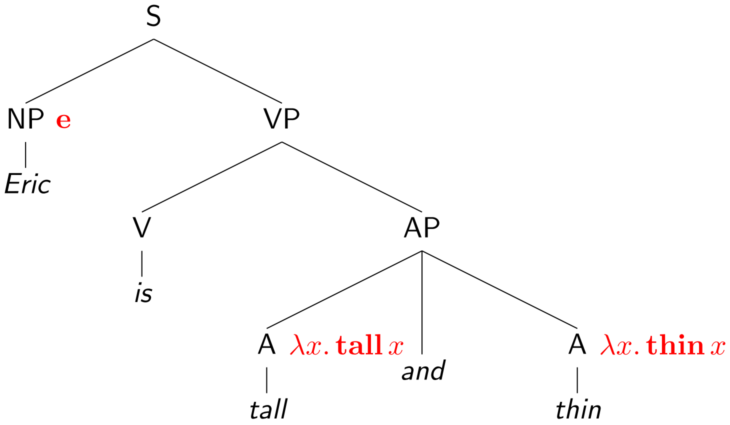
Compositionality



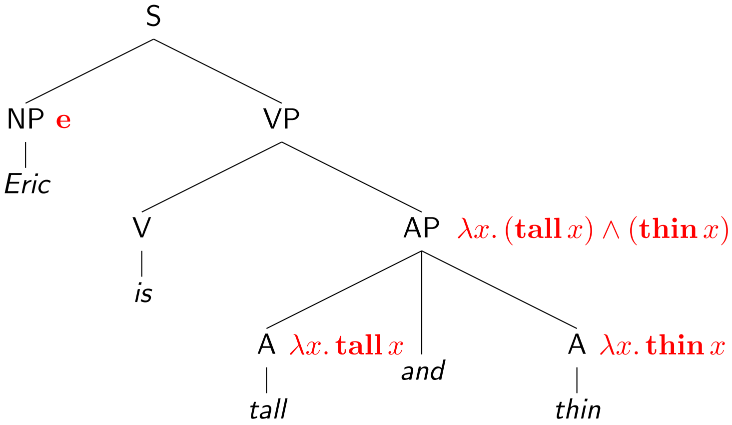
Compositionality



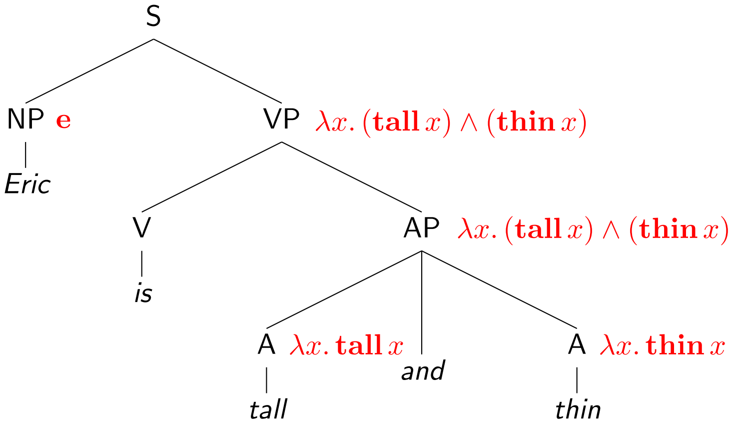
Compositionality



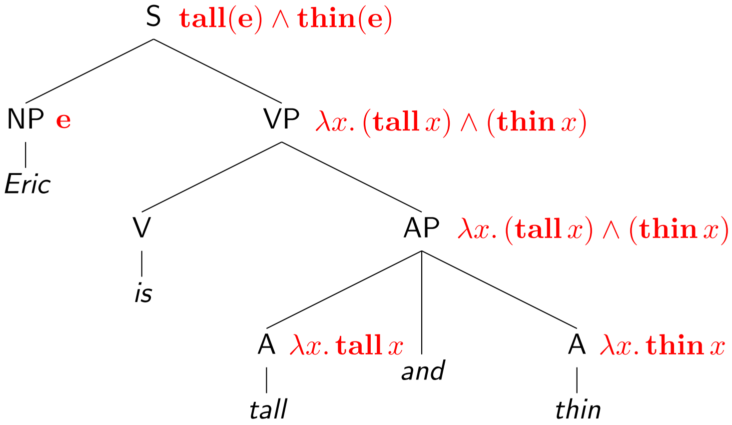
Compositionality



Compositionality



Compositionality



The homomorphism requirement

Richard Montague (1970)
Universal Grammar, *Theoria* 36:373–398.

- Semantics must be obtained as a homorphic image of syntax.

Abstract parse tree

Grammar:

$$
\begin{array}{lrcl}
P_0 : & \mathrm{S} & \rightarrow & \mathrm{NP}\ \mathrm{VP} \\
P_1 : & \mathrm{VP} & \rightarrow & \mathrm{V}\ \mathrm{AP} \\
P_2 : & \mathrm{AP} & \rightarrow & \mathrm{A}\ \mathit{and}\ \mathrm{A} \\
P_3 : & \mathrm{A} & \rightarrow & \mathit{tall} \\
P_4 : & \mathrm{A} & \rightarrow & \mathit{thin} \\
P_5 : & \mathrm{NP} & \rightarrow & \mathit{Eric} \\
P_6 : & \mathrm{V} & \rightarrow & \mathit{is}
\end{array}
$$

Abstract parse tree

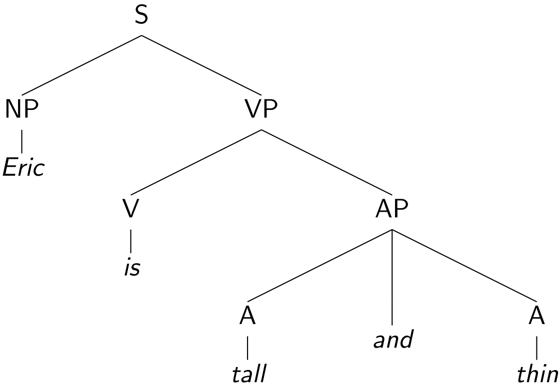
Grammar:

$$P_0 : \quad \mathrm{S} \;\rightarrow\; \mathrm{NP}\ \mathrm{VP}$$
$$P_1 : \quad \mathrm{VP} \;\rightarrow\; \mathrm{V}\ \mathrm{AP}$$
$$P_2 : \quad \mathrm{AP} \;\rightarrow\; \mathrm{A}\ \mathit{and}\ \mathrm{A}$$
$$P_3 : \quad \mathrm{A} \;\rightarrow\; \mathit{tall}$$
$$P_4 : \quad \mathrm{A} \;\rightarrow\; \mathit{thin}$$
$$P_5 : \quad \mathrm{NP} \;\rightarrow\; \mathit{Eric}$$
$$P_6 : \quad \mathrm{V} \;\rightarrow\; \mathit{is}$$

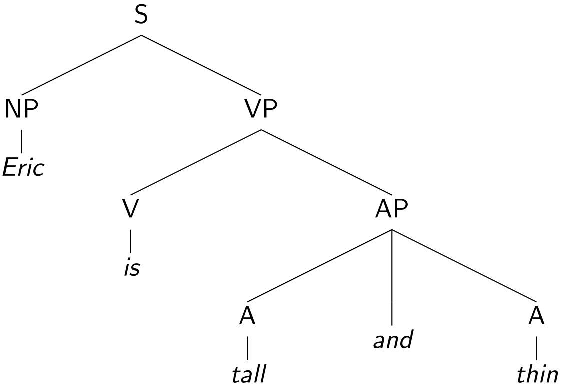
First-order multisorted signature:

$$\mathrm{P}_0 : \quad \mathrm{NP} \times \mathrm{VP} \rightarrow \mathrm{S}$$
$$\mathrm{P}_1 : \quad \mathrm{V} \times \mathrm{AP} \rightarrow \mathrm{VP}$$
$$\mathrm{P}_2 : \quad \mathrm{A} \times \mathrm{A} \rightarrow \mathrm{AP}$$
$$\mathrm{P}_3 : \quad \mathrm{A}$$
$$\mathrm{P}_4 : \quad \mathrm{A}$$
$$\mathrm{P}_5 : \quad \mathrm{NP}$$
$$\mathrm{P}_6 : \quad \mathrm{V}$$

Abstract parse tree



Abstract parse tree



$$P_0(P_5, P_1(P_6, P_2(P_3, P_4)))$$

Lexicalized case

Grammar:

$$
\begin{aligned}
P_0 : &\quad \mathrm{S} \rightarrow \mathrm{NP}\ \textit{is}\ \mathrm{AP} \\
P_1 : &\quad \mathrm{AP} \rightarrow \mathrm{AP}\ \textit{and}\ \mathrm{AP} \\
P_2 : &\quad \mathrm{AP} \rightarrow \textit{tall} \\
P_3 : &\quad \mathrm{AP} \rightarrow \textit{thin} \\
P_4 : &\quad \mathrm{NP} \rightarrow \textit{Eric}
\end{aligned}
$$

Lexicalized case

Grammar:

$$P_0 : \quad \mathrm{S} \;\rightarrow\; \mathrm{NP}\ \mathit{is}\ \mathrm{AP}$$
$$P_1 : \quad \mathrm{AP} \;\rightarrow\; \mathrm{AP}\ \mathit{and}\ \mathrm{AP}$$
$$P_2 : \quad \mathrm{AP} \;\rightarrow\; \mathit{tall}$$
$$P_3 : \quad \mathrm{AP} \;\rightarrow\; \mathit{thin}$$
$$P_4 : \quad \mathrm{NP} \;\rightarrow\; \mathit{Eric}$$

First-order multisorted signature:

$$\textsc{is} : \quad \mathrm{AP} \times \mathrm{NP} \rightarrow \mathrm{S}$$
$$\textsc{and} : \quad \mathrm{AP} \times \mathrm{AP} \rightarrow \mathrm{AP}$$
$$\textsc{tall} : \quad \mathrm{AP}$$
$$\textsc{thin} : \quad \mathrm{AP}$$
$$\textsc{eric} : \quad \mathrm{NP}$$

Lexicalized case

Grammar:

P_0 : S → NP *is* AP

P_1 : AP → AP *and* AP

P_2 : AP → *tall*

P_3 : AP → *thin*

P_4 : NP → *Eric*

First-order multisorted signature:

IS : AP × NP → S

AND : AP × AP → AP

TALL : AP

THIN : AP

ERIC : NP

IS(AND(TALL, THIN), ERIC)

Syntactic strucutre as λ-terms

First-order multisorted signature:

$$\begin{array}{ll} \text{IS}: & \text{AP} \times \text{NP} \rightarrow \text{S} \\ \text{AND}: & \text{AP} \times \text{AP} \rightarrow \text{AP} \\ \text{TALL}: & \text{AP} \\ \text{THIN}: & \text{AP} \\ \text{ERIC}: & \text{NP} \end{array}$$

Syntactic strucutre as λ-terms

First-order multisorted signature:

$$\begin{array}{ll} \text{IS} : & AP \times NP \to S \\ \text{AND} : & AP \times AP \to AP \\ \text{TALL} : & AP \\ \text{THIN} : & AP \\ \text{ERIC} : & NP \end{array}$$

Higher-order signature:

$$\begin{array}{ll} \text{IS} : & AP \to (NP \to S) \\ \text{AND} : & AP \to (AP \to AP) \\ \text{TALL} : & AP \\ \text{THIN} : & AP \\ \text{ERIC} : & NP \end{array}$$

Syntactic strucutre as λ-terms

First-order multisorted signature:

$$\begin{array}{ll} \textsc{is}: & AP \times NP \rightarrow S \\ \textsc{and}: & AP \times AP \rightarrow AP \\ \textsc{tall}: & AP \\ \textsc{thin}: & AP \\ \textsc{eric}: & NP \end{array}$$

Higher-order signature:

$$\begin{array}{ll} \textsc{is}: & AP \rightarrow (NP \rightarrow S) \\ \textsc{and}: & AP \rightarrow (AP \rightarrow AP) \\ \textsc{tall}: & AP \\ \textsc{thin}: & AP \\ \textsc{eric}: & NP \end{array}$$

$$\textsc{is}\,(\textsc{and}\ \textsc{tall}\ \textsc{thin})\ \textsc{eric}$$

Compositionality as a homomorphism

Compositionality as a homomorphism

Abstract language:

$$\mathrm{AP}, \mathrm{NP}, \mathrm{S} : \text{type}$$

$$\begin{aligned}
\textsc{is} &: \mathrm{AP} \rightarrow (\mathrm{NP} \rightarrow \mathrm{S}) \\
\textsc{and} &: \mathrm{AP} \rightarrow (\mathrm{AP} \rightarrow \mathrm{AP}) \\
\textsc{tall} &: \mathrm{AP} \\
\textsc{thin} &: \mathrm{AP} \\
\textsc{eric} &: \mathrm{NP}
\end{aligned}$$

Compositionality as a homomorphism

Abstract language:

$$\mathrm{AP}, \mathrm{NP}, \mathrm{S} : \text{type}$$

$$\textsc{is} : \mathrm{AP} \to (\mathrm{NP} \to \mathrm{S})$$

$$\textsc{and} : \mathrm{AP} \to (\mathrm{AP} \to \mathrm{AP})$$

$$\textsc{tall} : \mathrm{AP}$$

$$\textsc{thin} : \mathrm{AP}$$

$$\textsc{eric} : \mathrm{NP}$$

Object language:

$$\mathrm{e}, \mathrm{t} : \text{type}$$

$$\mathbf{e} : \mathrm{e}$$

$$\mathbf{tall} : \mathrm{e} \to \mathrm{t}$$

$$\mathbf{thin} : \mathrm{e} \to \mathrm{t}$$

$+$ *the logical constants*

Compositionality as a homomorphism

Abstract language:

$$\mathrm{AP}, \mathrm{NP}, \mathrm{S} : \text{type}$$

$$\textsc{is} : \mathrm{AP} \to (\mathrm{NP} \to \mathrm{S})$$

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$$\textsc{tall} : \mathrm{AP}$$

$$\textsc{thin} : \mathrm{AP}$$

$$\textsc{eric} : \mathrm{NP}$$

Object language:

$$\mathrm{e}, \mathrm{t} : \text{type}$$

$$\mathbf{e} : \mathrm{e}$$

$$\mathbf{tall} : \mathrm{e} \to \mathrm{t}$$

$$\mathbf{thin} : \mathrm{e} \to \mathrm{t}$$

$+$ *the logical constants*

Interpretation of types:

$$\mathrm{AP} := \mathrm{e} \to \mathrm{t}$$

$$\mathrm{NP} := \mathrm{e}$$

$$\mathrm{S} := \mathrm{t}$$

Compositionality as a homomorphism

Abstract language:

$$\mathrm{AP}, \mathrm{NP}, \mathrm{S} : \text{type}$$

$$\textsc{is} : \mathrm{AP} \to (\mathrm{NP} \to \mathrm{S})$$

$$\textsc{and} : \mathrm{AP} \to (\mathrm{AP} \to \mathrm{AP})$$

$$\textsc{tall} : \mathrm{AP}$$

$$\textsc{thin} : \mathrm{AP}$$

$$\textsc{eric} : \mathrm{NP}$$

Interpretation of types:

$$\mathrm{AP} := \mathrm{e} \to \mathrm{t}$$

$$\mathrm{NP} := \mathrm{e}$$

$$\mathrm{S} := \mathrm{t}$$

Object language:

$$\mathrm{e}, \mathrm{t} : \text{type}$$

$$\mathbf{e} : \mathrm{e}$$

$$\mathbf{tall} : \mathrm{e} \to \mathrm{t}$$

$$\mathbf{thin} : \mathrm{e} \to \mathrm{t}$$

$+$ *the logical constants*

Interpretation of terms:

$$\textsc{is} := \lambda xy.\, x\, y$$

$$\textsc{and} := \lambda xyz.\, (x\, z) \wedge (y\, z)$$

$$\textsc{tall} := \lambda x.\, \mathbf{tall}\, x$$

$$\textsc{thin} := \lambda x.\, \mathbf{thin}\, x$$

$$\textsc{eric} := \mathbf{e}$$

Compositionality as a homomorphism

$\text{IS}\ (\text{AND TALL THIN})\ \text{ERIC}$

Compositionality as a homomorphism

$$\text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} \;=\; (\lambda xy.\,x\,y)\,(\text{AND TALL THIN})\,\text{ERIC}$$

Compositionality as a homomorphism

$$\begin{aligned} \text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} &= (\lambda xy.\, x\, y)\,(\text{AND TALL THIN})\,\text{ERIC} \\ &\to_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\,\text{ERIC} \end{aligned}$$

Compositionality as a homomorphism

$$\begin{aligned} \text{IS}\,\big(\text{AND TALL THIN}\big)\,\text{ERIC} &= (\lambda xy.\, x\, y)\,\big(\text{AND TALL THIN}\big)\,\text{ERIC} \\ &\to_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\, \text{ERIC} \\ &\to_\beta \text{AND TALL THIN ERIC} \end{aligned}$$

Compositionality as a homomorphism

$$
\begin{aligned}
\text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} &= (\lambda xy.\, x\, y)\,(\text{AND TALL THIN})\,\text{ERIC} \\
&\rightarrow_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\,\text{ERIC} \\
&\rightarrow_\beta \text{AND TALL THIN ERIC} \\
&= (\lambda xyz.\, (x\, z) \wedge (y\, z))\,\text{TALL THIN ERIC}
\end{aligned}
$$

Compositionality as a homomorphism

$$\begin{aligned}
\textsf{IS}\,(\textsf{AND TALL THIN})\,\textsf{ERIC} &= (\lambda xy.\, x\, y)\,(\textsf{AND TALL THIN})\,\textsf{ERIC} \\
&\to_\beta (\lambda y.\, \textsf{AND TALL THIN}\, y)\,\textsf{ERIC} \\
&\to_\beta \textsf{AND TALL THIN ERIC} \\
&= (\lambda xyz.\, (x\, z) \wedge (y\, z))\,\textsf{TALL THIN ERIC} \\
&\to_\beta (\lambda yz.\, (\textsf{TALL}\, z) \wedge (y\, z))\,\textsf{THIN ERIC}
\end{aligned}$$

Compositionality as a homomorphism

$$\begin{aligned}
\text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} &= (\lambda xy.\, x\, y)\,(\text{AND TALL THIN})\,\text{ERIC} \\
&\to_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\,\text{ERIC} \\
&\to_\beta \text{AND TALL THIN ERIC} \\
&= (\lambda xyz.\, (x\, z) \wedge (y\, z))\,\text{TALL THIN ERIC} \\
&\to_\beta (\lambda yz.\, (\text{TALL}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&= (\lambda yz.\, ((\lambda x.\, \mathbf{tall}\, x)\, z) \wedge (y\, z))\,\text{THIN ERIC}
\end{aligned}$$

Compositionality as a homomorphism

$$
\begin{aligned}
\text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} &= (\lambda xy.\, x\, y)\,(\text{AND TALL THIN})\,\text{ERIC} \\
&\rightarrow_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\,\text{ERIC} \\
&\rightarrow_\beta \text{AND TALL THIN ERIC} \\
&= (\lambda xyz.\, (x\, z) \wedge (y\, z))\,\text{TALL THIN ERIC} \\
&\rightarrow_\beta (\lambda yz.\, (\text{TALL}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&= (\lambda yz.\, ((\lambda x.\, \mathbf{tall}\, x)\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\rightarrow_\beta (\lambda yz.\, (\mathbf{tall}\, z) \wedge (y\, z))\,\text{THIN ERIC}
\end{aligned}
$$

Compositionality as a homomorphism

$$
\begin{aligned}
\text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} &= (\lambda xy.\, x\, y)\,(\text{AND TALL THIN})\,\text{ERIC} \\
&\to_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\,\text{ERIC} \\
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&= (\lambda xyz.\, (x\, z) \wedge (y\, z))\,\text{TALL THIN ERIC} \\
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&= (\lambda yz.\, ((\lambda x.\, \mathbf{tall}\, x)\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\to_\beta (\lambda yz.\, (\mathbf{tall}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\to_\beta (\lambda z.\, (\mathbf{tall}\, z) \wedge (\text{THIN}\, z))\,\text{ERIC}
\end{aligned}
$$

Compositionality as a homomorphism

$$
\begin{aligned}
\text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} &= (\lambda xy.\, x\, y)\,(\text{AND TALL THIN})\,\text{ERIC} \\
&\rightarrow_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\,\text{ERIC} \\
&\rightarrow_\beta \text{AND TALL THIN ERIC} \\
&= (\lambda xyz.\, (x\, z) \wedge (y\, z))\,\text{TALL THIN ERIC} \\
&\rightarrow_\beta (\lambda yz.\, (\text{TALL}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&= (\lambda yz.\, ((\lambda x.\, \mathbf{tall}\, x)\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\rightarrow_\beta (\lambda yz.\, (\mathbf{tall}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\rightarrow_\beta (\lambda z.\, (\mathbf{tall}\, z) \wedge (\text{THIN}\, z))\,\text{ERIC} \\
&= (\lambda z.\, (\mathbf{tall}\, z) \wedge ((\lambda x.\, \mathbf{thin}\, x)\, z))\,\text{ERIC}
\end{aligned}
$$

Compositionality as a homomorphism

$$
\begin{aligned}
\text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} &= (\lambda xy.\, x\, y)\,(\text{AND TALL THIN})\,\text{ERIC} \\
&\to_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\,\text{ERIC} \\
&\to_\beta \text{AND TALL THIN ERIC} \\
&= (\lambda xyz.\, (x\, z) \wedge (y\, z))\,\text{TALL THIN ERIC} \\
&\to_\beta (\lambda yz.\, (\text{TALL}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&= (\lambda yz.\, ((\lambda x.\, \mathbf{tall}\, x)\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\to_\beta (\lambda yz.\, (\mathbf{tall}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\to_\beta (\lambda z.\, (\mathbf{tall}\, z) \wedge (\text{THIN}\, z))\,\text{ERIC} \\
&= (\lambda z.\, (\mathbf{tall}\, z) \wedge ((\lambda x.\, \mathbf{thin}\, x)\, z))\,\text{ERIC} \\
&\to_\beta (\lambda z.\, (\mathbf{tall}\, z) \wedge (\mathbf{thin}\, z))\,\text{ERIC}
\end{aligned}
$$

Compositionality as a homomorphism

$$
\begin{aligned}
\text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} &= (\lambda xy.\, x\, y)\,(\text{AND TALL THIN})\,\text{ERIC} \\
&\to_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\,\text{ERIC} \\
&\to_\beta \text{AND TALL THIN ERIC} \\
&= (\lambda xyz.\, (x\, z) \wedge (y\, z))\,\text{TALL THIN ERIC} \\
&\to_\beta (\lambda yz.\, (\text{TALL}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&= (\lambda yz.\, ((\lambda x.\, \mathbf{tall}\, x)\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\to_\beta (\lambda yz.\, (\mathbf{tall}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\to_\beta (\lambda z.\, (\mathbf{tall}\, z) \wedge (\text{THIN}\, z))\,\text{ERIC} \\
&= (\lambda z.\, (\mathbf{tall}\, z) \wedge ((\lambda x.\, \mathbf{thin}\, x)\, z))\,\text{ERIC} \\
&\to_\beta (\lambda z.\, (\mathbf{tall}\, z) \wedge (\mathbf{thin}\, z))\,\text{ERIC} \\
&\to_\beta (\mathbf{tall}\,\text{ERIC}) \wedge (\mathbf{thin}\,\text{ERIC})
\end{aligned}
$$

Compositionality as a homomorphism

$$
\begin{aligned}
\text{IS}\,(\text{AND TALL THIN})\,\text{ERIC} &= (\lambda xy.\, x\, y)\,(\text{AND TALL THIN})\,\text{ERIC} \\
&\to_\beta (\lambda y.\, \text{AND TALL THIN}\, y)\,\text{ERIC} \\
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&= (\lambda xyz.\, (x\, z) \wedge (y\, z))\,\text{TALL THIN ERIC} \\
&\to_\beta (\lambda yz.\, (\text{TALL}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&= (\lambda yz.\, ((\lambda x.\, \mathbf{tall}\, x)\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\to_\beta (\lambda yz.\, (\mathbf{tall}\, z) \wedge (y\, z))\,\text{THIN ERIC} \\
&\to_\beta (\lambda z.\, (\mathbf{tall}\, z) \wedge (\text{THIN}\, z))\,\text{ERIC} \\
&= (\lambda z.\, (\mathbf{tall}\, z) \wedge ((\lambda x.\, \mathbf{thin}\, x)\, z))\,\text{ERIC} \\
&\to_\beta (\lambda z.\, (\mathbf{tall}\, z) \wedge (\mathbf{thin}\, z))\,\text{ERIC} \\
&\to_\beta (\mathbf{tall}\,\text{ERIC}) \wedge (\mathbf{thin}\,\text{ERIC}) \\
&= (\mathbf{tall}\,\mathbf{e}) \wedge (\mathbf{thin}\,\mathbf{e})
\end{aligned}
$$