Formal Semantics of Natural Language

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Introduction

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- Entailment

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  → *Eric is the husband of Rebecca.*

  → *Eric is the father of John.*

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  - The meaning of an utterance depends upon its *form*, i.e., its linguistic structure.
  - The tools used to account for the meanings of utterances are *formal* mathematical tools.

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  - The tools used to account for the meanings of utterances are *formal* mathematical tools.
- Truth conditional semantics.
- Model theoretic semantics.

Truth and Entailment

John wears glasses and a cap.

Truth and Entailment

John wears glasses and a cap.

True or false ?

Truth and Entailment

John wears glasses and a cap.

True or false ?

false

Truth and Entailment

John wears glasses and a cap.

True or false ?

false

true

Truth and Entailment

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This idea of a *state of affairs* is captured by the mathematical notion of *model*

Formal languages and Models

A (first-order) language consists of two sets of symbols:

- a set of symbols that denote constants;
- a set of symbols that denote n-ary relations.

Formal languages and Models

Given a (first-order) language, a model

$M = \langle E, I \rangle$ consists of:

- a set E of entities (called the *domain* of the model
- An interpretation function I, such that:
  - $I(c) \in E$, for c a constant symbol;
  - $I(R) \subset E^n$, for R a n-ary relation symbol.

The mathematical notion of truth

- $R(a_1, a_2, \ldots, a_n)$ is true if and only if the entities denoted by $a_1, a_2, \ldots, a_n$ belong to the relation denoted by R:

$$\langle\, I\,(a_1), I\,(a_2), \ldots, I\,(a_n)\,\rangle \in I\,(R)$$

The mathematical notion of truth

- $\alpha \land \beta$ is true iff α is true and β is true;
- $\alpha \lor \beta$ is true iff α is true or β is true (or both);
- $\alpha \rightarrow \beta$ is true iff β is true whenever α is;
- $\neg\ \alpha$ is true iff α is not true;

The mathematical notion of truth

- $\forall x.\alpha[x]$ is true iff for every $a \in \mathrm{E}$, α is true when the variable x is interpreted as the entity a;
- $\exists x.\alpha[x]$ is true iff for some $a \in \mathrm{E}$, α is true when the variable x is interpreted as the entity a;

By analogy…

John wears glasses and a cap is true

By analogy…

John wears glasses and a cap is true

If and only if

John wears glasses and John wears a cap is true

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If and only if

John wears glasses is true **and** *John wears a cap* is true

By analogy…

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If and only if

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If and only if

the entity denoted by *John* belongs to the subset of *E* that denotes the glasses wearers **and** the entity denoted by *John* belongs to the subset of *E* that denotes the cap wearers

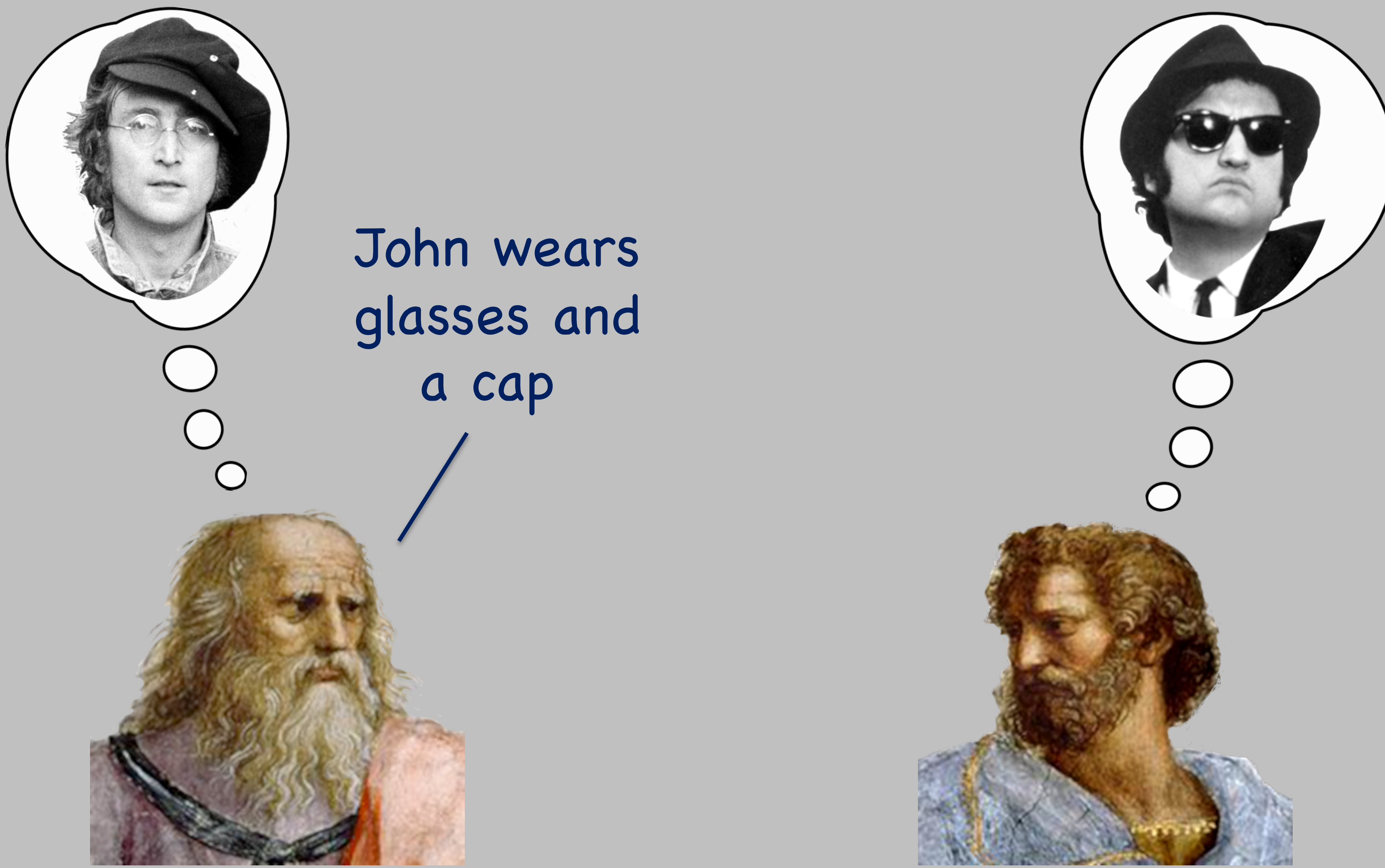
Entailment

Entailment

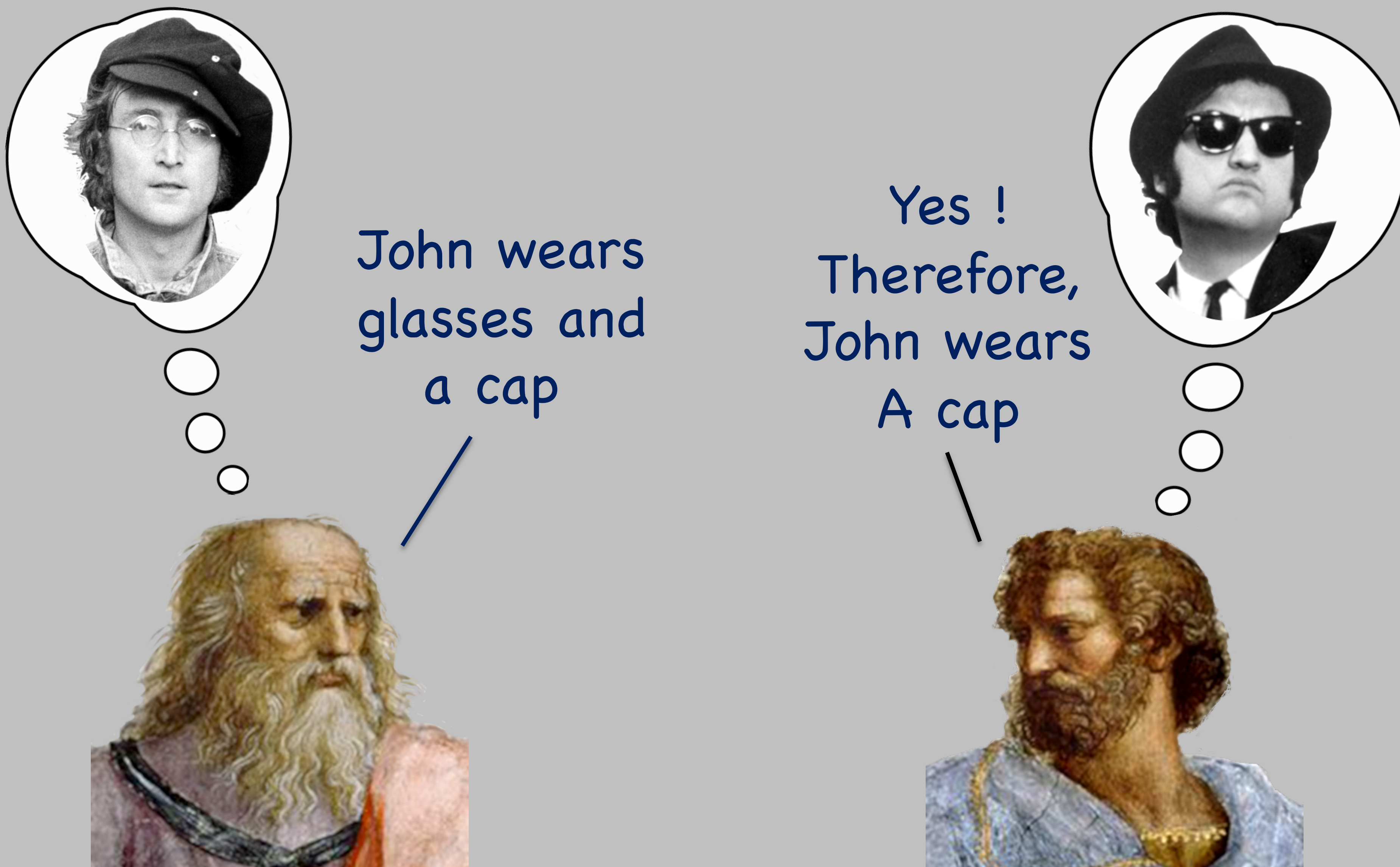
Entailment

John wears glasses and a cap

Entailment



Entailment



Entailment

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Whatever entity is denoted by *John*, wathever relation is denoted by *wears*, whatever subsets of entities are denoted by *glasses* and *cap*…

John wears glasses and a cap

entails

John wears a cap

Entailment and meaning postulates

α : *a dog entered the room*

⇒ ?

β : *an animal entered the room*

Entailment and meaning postulates

α : *a dog entered the room*

⇒ ?

β : *an animal entered the room*

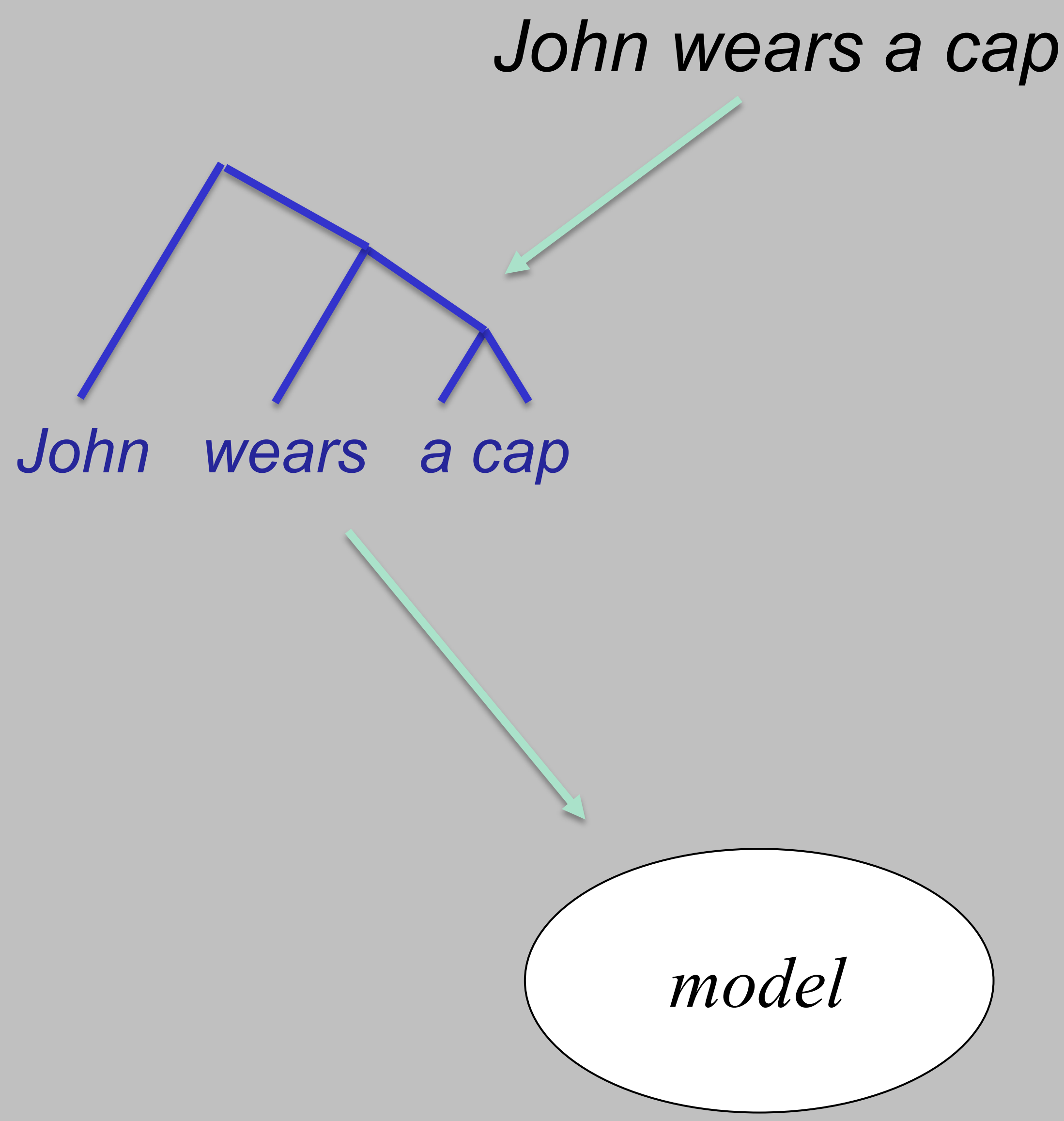
in every model such that the set denoted by *dog* is a subset of the set denoted by *animal*, α entails β.

Compositionality

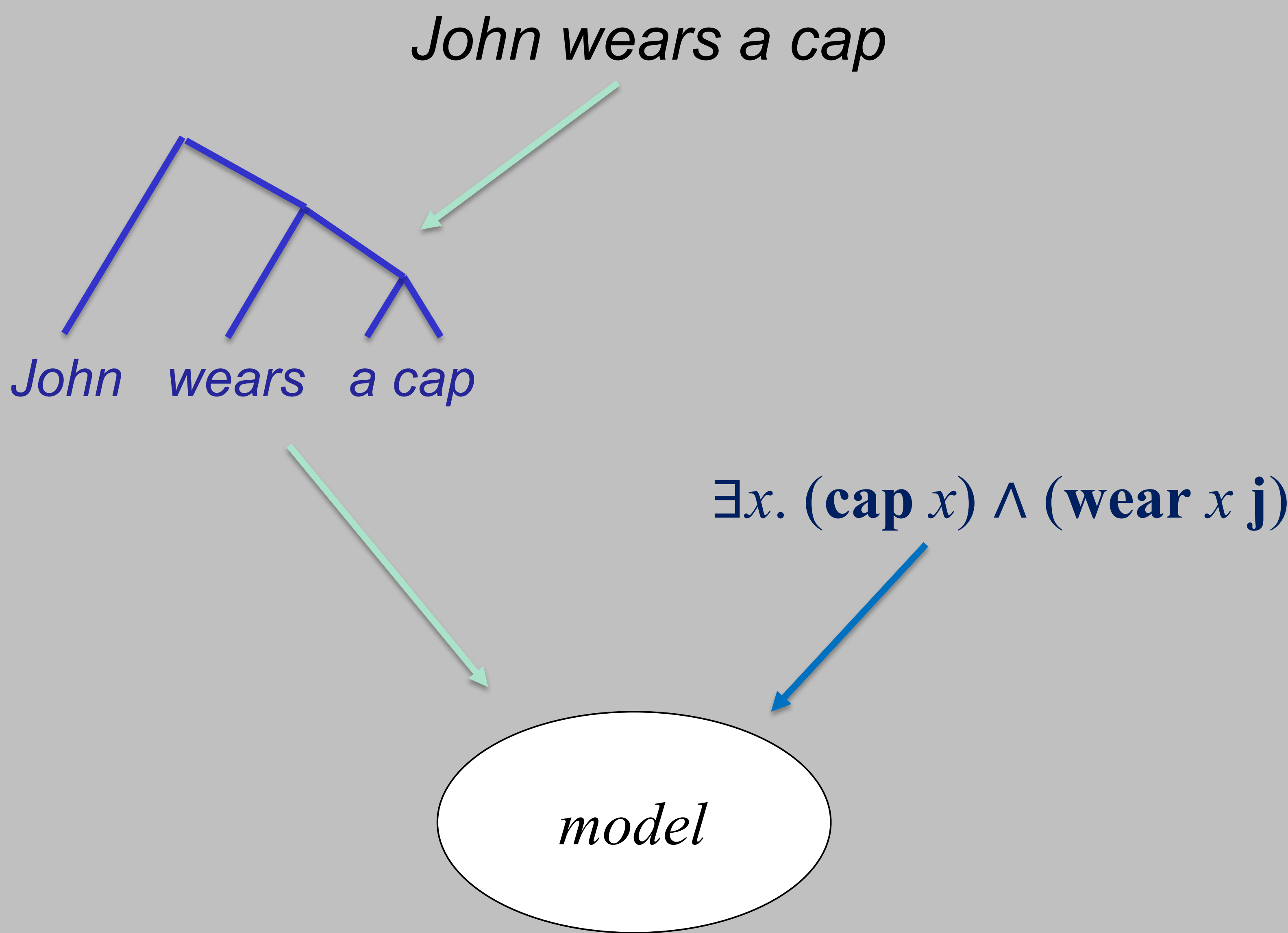
Compositionality principle

The denotation of a complex expression is determined by the denotations of its immediate parts and the ways they combined with each other

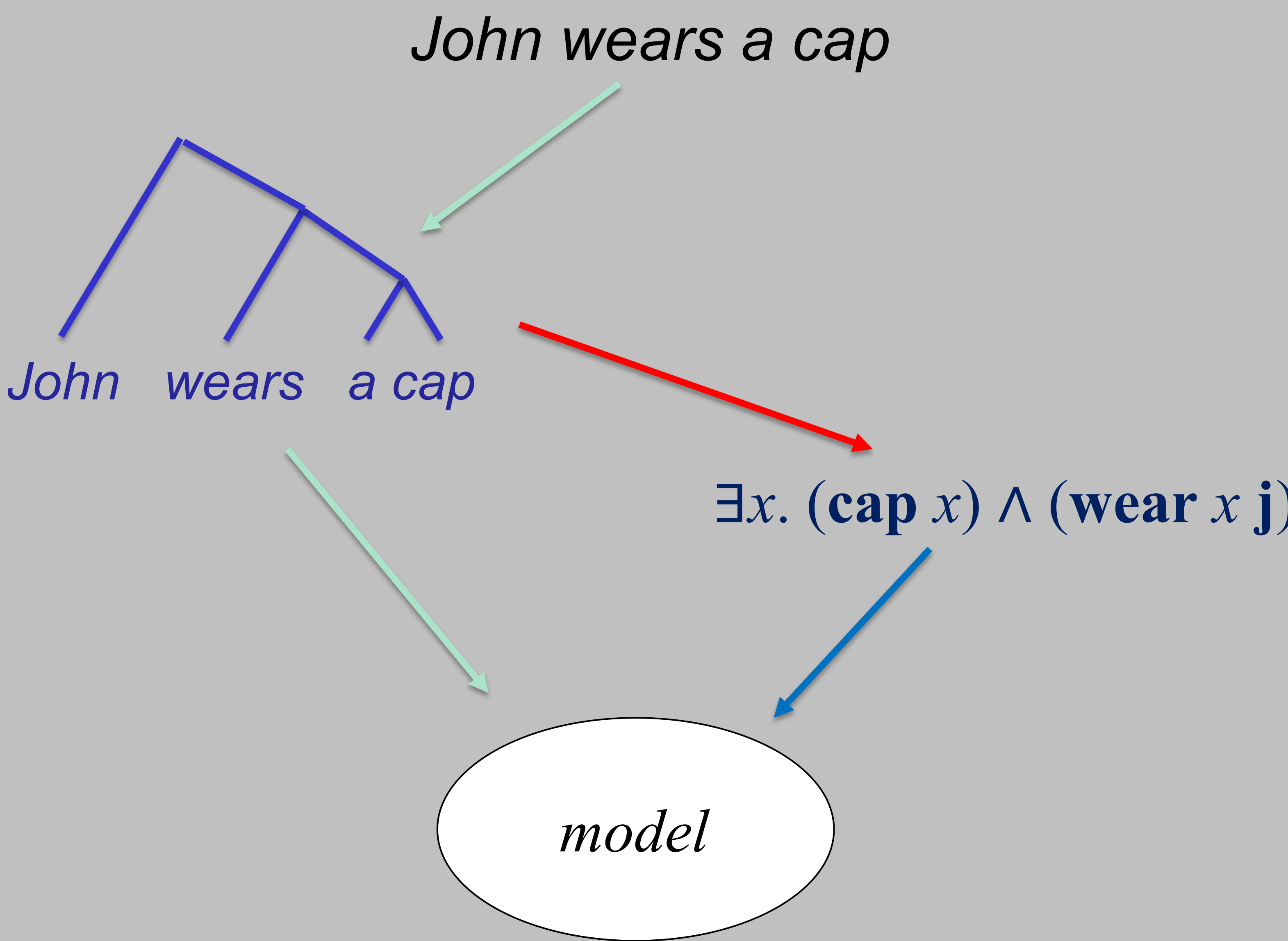
Compositionality



Compositionality

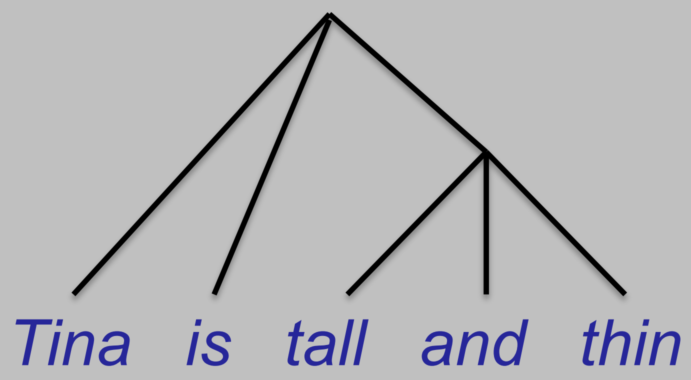


Compositionality



Compositionality

[*Tina is* [*tall and thin*]]



- *Tina is tall and thin* denotes a truth value.
- *Tina* denotes an entity.
- *tall and thin* denotes a subset of entities.
- *is* denotes a relation between entities and sets.
- *tall* denotes a subset of entities.
- *thin* denotes a subset of entities.
- *and* denotes a binary operator on sets.