Formal Semantics of Natural Language

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Yoad Winter

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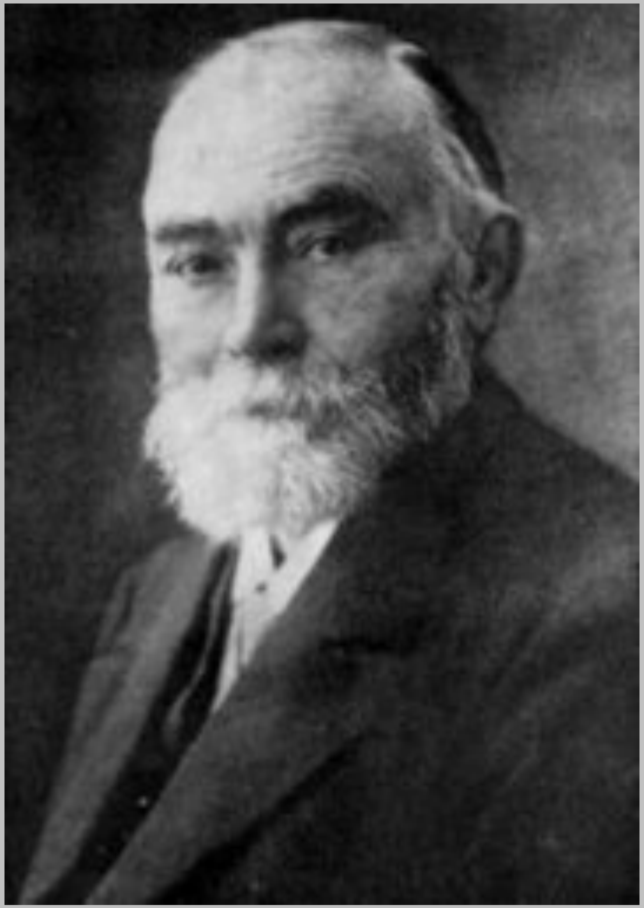
Additional Topics:

Intensionality

Sinn und bedeutung



Gottlob Frege (1848-1925)

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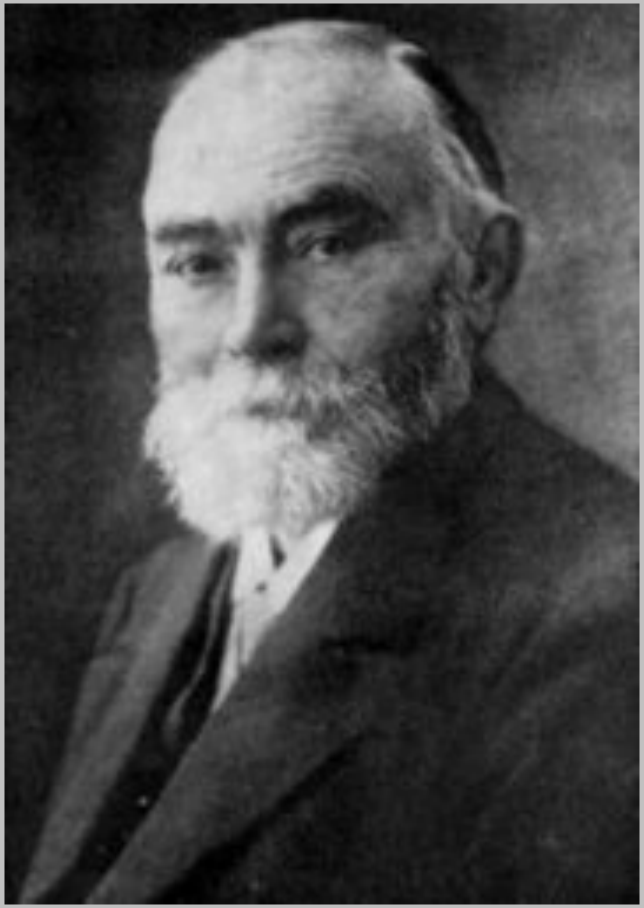

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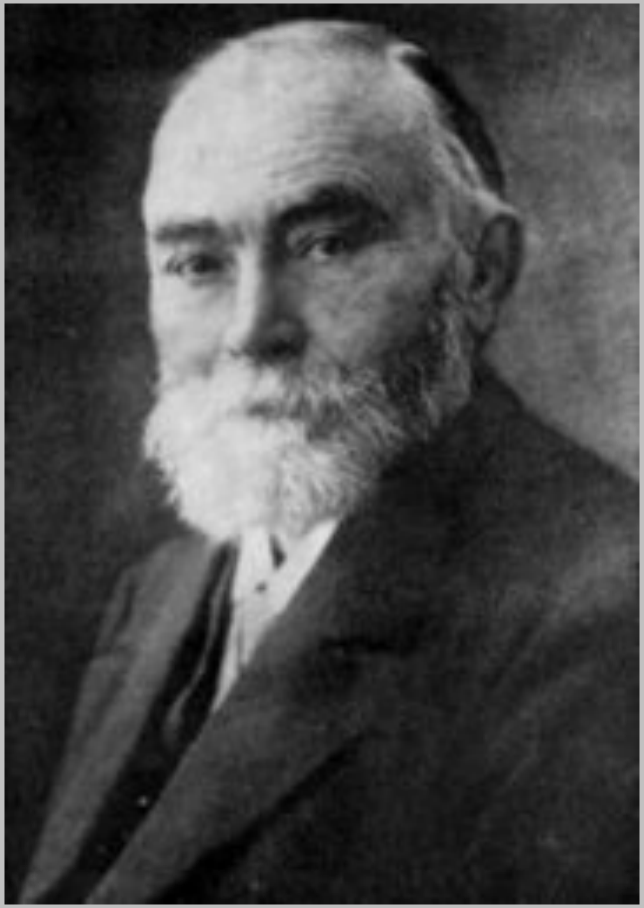

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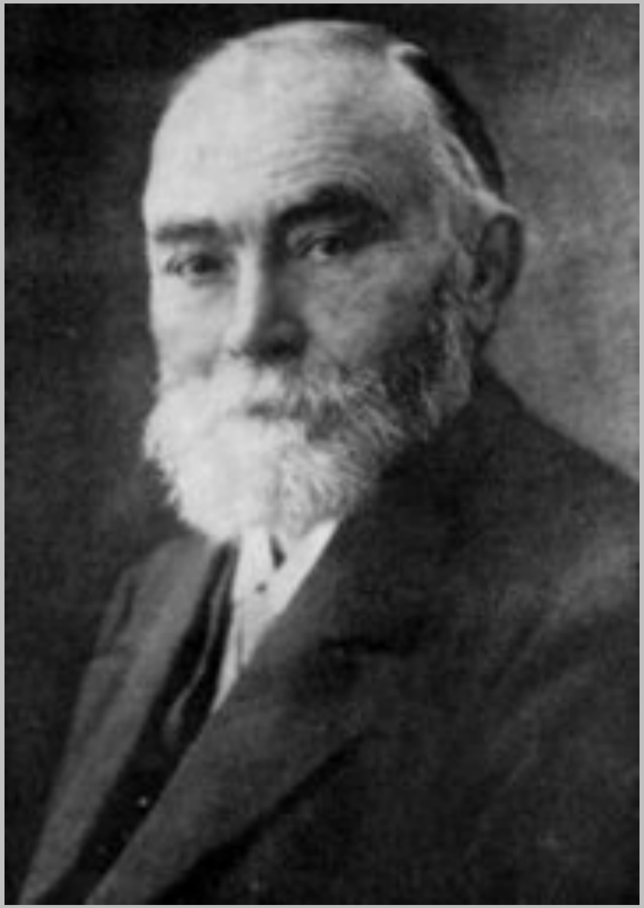

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- According to Frege, the sense of an expression is its “mode of presentation”, while the reference or denotation of an expression is the object it refers to.

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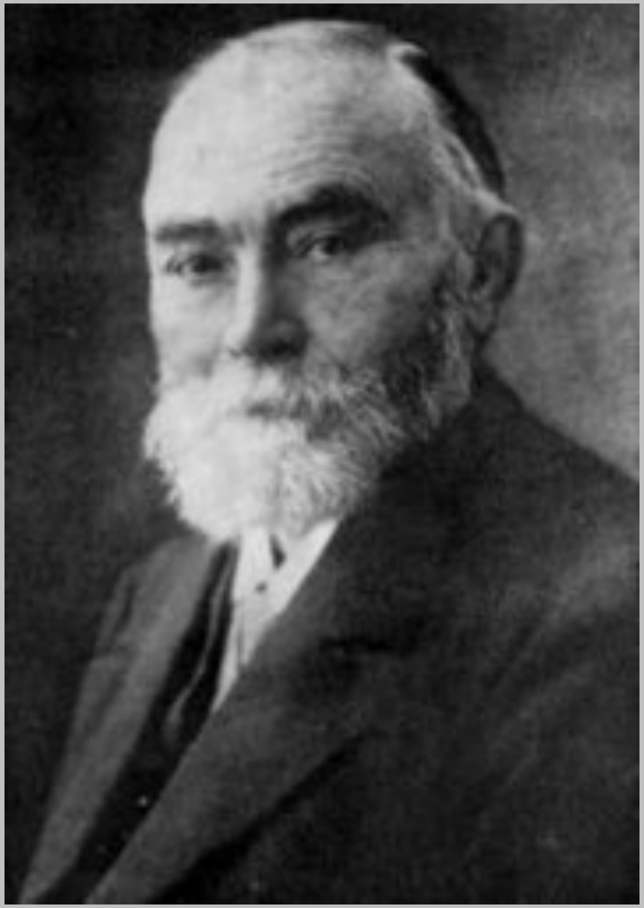

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- According to Frege, the sense of an expression is its “mode of presentation”, while the reference or denotation of an expression is the object it refers to.
- For instance, both expressions *“1 + 1”* and *“2”* have the same denotation but not the same sense.

Intensional propositions

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- Frege gives the example of *“the morning star”* and *“the evening star”* which both refer to the planet Venus.
- Compare *“the morning star is the evening star”* with *“the ancients did not know that the morning star is the evening star”*.

Possible world semantics

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Pangloss enseignait la métaphysico-théologo-cosmolo-nigologie.
Il prouvait admirablement qu'il n'y a point d'effet sans cause, et que,
dans ce meilleur des mondes possibles, le château de monseigneur le baron
était le plus beau des châteaux et madame la meilleure des baronnes possibles.

Voltaire (Candide)

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  - Epistemic logic: *Bob knows that... Bob ignores that...*
  - Temporal logic: *It will always be the case that... It will eventually be the case that...*

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$$F \;::=\; a \mid \neg F \mid F \vee F \mid \Box F$$

Define the other connectives in the usual way. Define $\Diamond A$ as $\neg\Box\neg A$.

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Validity:

let $\mathcal{M} = \langle W, P\rangle$, where W is a set of "possible worlds", and P is a function that asigns to each atomic proposition a subset of W.

- $\mathcal{M}, s \models a$ iff $s \in P(a)$.
- $\mathcal{M}, s \models \neg A$ iff not $\mathcal{M}, s \models A$.
- $\mathcal{M}, s \models A \vee B$ iff either $\mathcal{M}, s \models A$ or $\mathcal{M}, s \models B$, or both.
- $\mathcal{M}, s \models \Box A$ iff for every $t \in W$, $\mathcal{M}, t \models A$.

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$$\mathbf{necessarily} := \boldsymbol{\lambda}\, A\, w.\ \forall v.\ (A\, v)$$

Intension and extension

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This skillful surgeon is Dr Johnson

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$$(\forall x.\,(\textsf{\textbf{surgeon}}\,x) \leftrightarrow (\textsf{\textbf{driver}}\,x))$$

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Solution:

$$\textbf{surgeon} : \mathrm{e}\,(\mathrm{s}\,\mathrm{t})$$
$$\textbf{driver} : \mathrm{e}\,(\mathrm{s}\,\mathrm{t})$$
$$\textbf{skillful} : (\mathrm{e}\,(\mathrm{s}\,\mathrm{t}))\,\mathrm{e}\,(\mathrm{s}\,\mathrm{t})$$