# Semantic Representation of Modal Subordination Using Type Theory 

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## Outline

(1) About Modal Subordination
(2) A Montagovian Treatment
(3) Discussion and Alternative Proposals
(4) Conclusion

## Modal Subordination: Some Examples

## Example

(1) A wolf might walk in. It would growl.

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(1) A wolf might walk in. It would growl.
(2) A wolf might walk in. *It will growl.

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## References: DRT and Dynamic Frameworks

- Accommodation of DRSs [Roberts(1989)]
- Modals presuppose their domain [Geurts(1999)]
- Anaphoric context references and graded modality [Frank and Kamp(1997)]
- Compositional DRT extension [Stone and Hardt(1997)]
- Two-dimensionsal approach, accessibility relation and world ordering [van Rooij(2005)]
- DPL and sets of epistemic possibilities [Asher and McCready(2007)]


## DRT Based Account

## Example

A wolf might walk in.

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Note:

- Accessibility conditions


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## Example

A wolf might walk in.lt would growl.

$$
\begin{aligned}
& \diamond \begin{array}{l}
x \\
\begin{array}{l}
\text { wolf }(x) \\
\text { enter }(x)
\end{array} \\
\square \frac{y}{\operatorname{growl}(y)} \\
\hline
\end{array} \\
& \square
\end{aligned}
$$

$$
\begin{array}{|l|}
\hline x \\
\hline \begin{array}{l}
\text { wolf }(x) \\
\text { enter }(x)
\end{array} \\
y \\
\hline \operatorname{growl}(y) \\
\hline
\end{array}
$$

Note:

- Accessibility conditions


## DRT Based Account

## Example

A wolf might walk in.lt would growl.


## Note:

- Accessibility conditions
- Modal base and accommodation


## A Montagovian Treatment

## Our Aim

To consider modal subordination in [de Groote(2006)]'s framework:

- Taking advantages of this framework
- Implementing MS principles in lexical entries
- Without any change to the formal framwork


## The Steps

- Intepretation of (the syntactic type of) the sentences
- Combination rules
- The lexical semantics of MS


## Interpretation of the Sentences

$$
\text { [de Groote(2006)]: } \llbracket s \rrbracket=\gamma \rightarrow(\gamma \rightarrow t) \rightarrow t
$$

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Here:

$$
\llbracket s \rrbracket=\gamma \rightarrow \gamma \rightarrow(\gamma \rightarrow \gamma \rightarrow t) \rightarrow(\gamma \rightarrow \gamma \rightarrow t) \rightarrow(t \rightarrow t \rightarrow t) \rightarrow t
$$

## Interpretation of the Sentences

$$
\begin{array}{ll}
\text { [de Groote(2006)]: } & \llbracket s \rrbracket=\gamma \rightarrow(\gamma \rightarrow t) \rightarrow t \\
\text { Here: } & \llbracket s \rrbracket=\gamma \rightarrow \gamma \rightarrow(\gamma \rightarrow \gamma \rightarrow t) \rightarrow(\gamma \rightarrow \gamma \rightarrow t) \rightarrow(t \rightarrow t \rightarrow t) \rightarrow t
\end{array}
$$

- A modal environment and a factual environment


## Interpretation of the Sentences

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Here:
$\llbracket s \rrbracket=\gamma \rightarrow \gamma \rightarrow(\gamma \rightarrow \gamma \rightarrow t) \rightarrow(\gamma \rightarrow \gamma \rightarrow t) \rightarrow(t \rightarrow t \rightarrow t) \rightarrow t$

- A modal environment and a factual environment
- A modal continuation and a factual continuation (or a modal contribution and a factual contribution of the continuation)


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- a modal part and a factual part


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- A modal environment and a factual environment
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## Note on pairs: $(t, t)$ as $(t \rightarrow t \rightarrow t) \rightarrow t$

- A pair $(a, b)$ is interpreted as $\lambda f$.f $a b$ (selecting two-place functions and applying them to the 1st and the 2nd component)
- An additional parameter:
- How should the modal and the factual part be combined? Typically $\lambda b_{1} b_{2} \cdot b_{1} \wedge b_{2}$
- When should they be combined? Possibility of a Reset operator that close the modal contribution.


## Interpretation of the Sentences

[de Groote(2006)]: $\llbracket s \rrbracket=\gamma \rightarrow(\gamma \rightarrow t) \rightarrow t$
Here:
$\llbracket s \rrbracket=\gamma \rightarrow \gamma \rightarrow(\gamma \rightarrow \gamma \rightarrow t) \rightarrow(\gamma \rightarrow \gamma \rightarrow t) \rightarrow(t \rightarrow t \rightarrow t) \rightarrow t$

- A modal environment and a factual environment
- A modal continuation and a factual continuation (or a modal contribution and a factual contribution of the continuation)
- a modal part and a factual part
- $\llbracket n p \rrbracket=(e \rightarrow \llbracket s \rrbracket) \rightarrow \llbracket s \rrbracket, \llbracket n \rrbracket=e \rightarrow \llbracket s \rrbracket$, etc.


## Note on pairs: $(t, t)$ as $(t \rightarrow t \rightarrow t) \rightarrow t$

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## $S_{1} . S_{2}$ when $S_{2}$ has a factual mood

$$
\llbracket S_{1} \cdot S_{2} \rrbracket=\lambda i_{1} i_{2} k_{1} k_{2} f . \llbracket S_{1} \rrbracket i_{1} i_{2} k_{1}\left(\lambda i_{1}^{\prime} i_{2}^{\prime} . \llbracket S_{2} \rrbracket i_{1}^{\prime} i_{2}^{\prime} k_{1} k_{2} \Pi_{2}\right) f
$$

(with $\Pi_{2}=\lambda a b . b$ the second projection)

## $S_{1} . S_{2}$ when $S_{2}$ has a factual mood

$$
\llbracket S_{1} \cdot S_{2} \rrbracket=\lambda i_{1} i_{2} k_{1} k_{2} f . \llbracket S_{1} \rrbracket i_{1} i_{2} k_{1}\left(\lambda i_{1}^{\prime} i_{2}^{\prime} . \llbracket S_{2} \rrbracket i_{1}^{\prime} i_{2}^{\prime} k_{1} k_{2} \Pi_{2}\right) f
$$

(with $\Pi_{2}=\lambda a b . b$ the second projection)
$S_{1} . S_{2}$ when $S_{2}$ has a nonfactual mood

$$
\llbracket S_{1} \cdot S_{2} \rrbracket=\lambda i_{1} i_{2} k_{1} k_{2} f \cdot \llbracket S_{1} \rrbracket i_{1} i_{2}\left(\lambda i_{1}^{\prime} i_{2}^{\prime} \cdot \llbracket S_{2} \rrbracket i_{1}^{\prime} i_{2}^{\prime} k_{1} k_{2} \Pi_{1}\right) k_{2} f
$$

(with $\Pi_{1}=\lambda a b . a$ the first projection)

## Example

$$
\llbracket S_{1} . S_{2} \rrbracket=\lambda i_{1} i_{2} k_{1} k_{2} f . \llbracket S_{1} \rrbracket i_{1} i_{2} k_{1}\left(\lambda i_{1}^{\prime} i_{2}^{\prime} . \llbracket S_{2} \rrbracket i_{1}^{\prime} i_{2}^{\prime} k_{1} k_{2} \Pi_{2}\right) f
$$

## Example

$$
\begin{array}{ll}
\llbracket A \text { wolf might walk in } \rrbracket=\lambda i_{1} i_{2} k_{1} k_{2} f . f & \left(\diamond\left(\exists x .(\text { wolf } x) \wedge\left((\text { enter } x) \wedge\left(k_{1}\left(x:: i_{1}\right) i_{2}\right)\right)\right)\right) \\
& \left(k_{2} i_{1} i_{2}\right)
\end{array}
$$

## Example

$$
\llbracket S_{1} \cdot S_{2} \rrbracket=\lambda i_{1} i_{2} k_{1} k_{2} f . \llbracket S_{1} \rrbracket i_{1} i_{2} k_{1}\left(\lambda i_{1}^{\prime} i_{2}^{\prime} \cdot \llbracket S_{2} \rrbracket i_{1}^{\prime} i_{2}^{\prime} k_{1} k_{2} \Pi_{2}\right) f
$$

## Example

$$
\begin{array}{lll}
\llbracket A \text { wolf might walk in } & =\lambda i_{1} i_{2} k_{1} k_{2} f . f & \left(\diamond\left(\exists x .(\operatorname{wolf} x) \wedge\left((\text { enter } x) \wedge\left(k_{1}\left(x:: i_{1}\right) i_{2}\right)\right)\right)\right) \\
\llbracket I t \text { would grow } \rrbracket & =\lambda i_{1} i_{2} k_{1} k_{2} f . f & \left(\square \left(\left(k_{2} i_{1} i_{2}\right)\right.\right. \\
\left.\left(\square\left(\operatorname{growl}\left(\operatorname{sel}\left(i_{1} \cup i_{2}\right)\right)\right) \wedge\left(k_{1} i_{1} i_{2}\right)\right)\right) & \left(k_{2} i_{1} i_{2}\right)
\end{array}
$$

## Example

$$
\llbracket S_{1} . S_{2} \rrbracket=\lambda i_{1} i_{2} k_{1} k_{2} f . \llbracket S_{1} \rrbracket i_{1} i_{2} k_{1}\left(\lambda i_{1}^{\prime} i_{2}^{\prime} . \llbracket S_{2} \rrbracket i_{1}^{\prime} i_{2}^{\prime} k_{1} k_{2} \Pi_{2}\right) f
$$

## Example



```
(k2 it iz)
|t would grow\rrbracket\ = \i1 i i2 k
(\square((growl (sel (i, 隹)))})^(\mp@subsup{k}{1}{}\mp@subsup{i}{1}{}\mp@subsup{i}{2}{})))\quad(\mp@subsup{k}{2}{}\mp@subsup{i}{1}{}\mp@subsup{i}{2}{}
```



## Example

$$
\llbracket S_{1} \cdot S_{2} \rrbracket=\lambda i_{1} i_{2} k_{1} k_{2} f \cdot \llbracket S_{1} \rrbracket i_{1} i_{2} k_{1}\left(\lambda i_{1}^{\prime} i_{2}^{\prime} \cdot \llbracket S_{2} \rrbracket i_{1}^{\prime} i_{2}^{\prime} k_{1} k_{2} \Pi_{2}\right) f
$$

## Example

```
|A wolf might walk in\rrbracket=\lambdai
(k2 i}\mp@subsup{i}{1}{}\mp@subsup{i}{2}{}
|lt would grow|| llll
```

Let:

- Nil be the empty environment (sel Nil always fails)
- $\mathbb{T}$ be the trivial continuation $\left(\lambda i_{1} i_{2} . \top\right)$
- Conj be the conjunction $\left(\lambda b_{1} b_{2} . b_{1} \wedge b_{2}\right)$


## Example

$$
\llbracket S_{1} \cdot S_{2} \rrbracket=\lambda i_{1} i_{2} k_{1} k_{2} f \cdot \llbracket S_{1} \rrbracket i_{1} i_{2} k_{1}\left(\lambda i_{1}^{\prime} i_{2}^{\prime} \cdot \llbracket S_{2} \rrbracket i_{1}^{\prime} i_{2}^{\prime} k_{1} k_{2} \Pi_{2}\right) f
$$

## Example

```
|A wolf might walk in\rrbracket=\lambdai
(k2 i i i i2)
```



```
|lt will grow|\rrbracket = = i\mp@subsup{i}{1}{}\mp@subsup{i}{2}{}\mp@subsup{k}{1}{}\mp@subsup{k}{2}{}f.f (k1\mp@subsup{i}{1}{}\mp@subsup{i}{2}{})
```

Let:

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- Conj be the conjunction $\left(\lambda b_{1} b_{2} . b_{1} \wedge b_{2}\right)$

We can then evaluate ( $\mathrm{NilNilT} \mathbb{T}$ Conj parameters are omitted):

## Example (A wolf might walk in. It would growl)

$$
\llbracket S \rrbracket=(\diamond(\exists x .(\text { wolf } x) \wedge((\text { enter } x) \wedge(\square((\text { growl }(\operatorname{sel}(x:: \operatorname{Nil}) \cup \operatorname{Nil})) \wedge \top))))) \wedge \top
$$

Example (A wolf might walk in. It will growl)

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\llbracket S \rrbracket=(\diamond(\exists x .(\text { wolf } x) \wedge((\text { enter } x) \wedge \top))) \wedge(\text { growl }(\text { sel } \mathrm{Nil}))
$$

## Example (cont'd)

## Example (A wolf might walk in. It will growl)

$$
\llbracket S \rrbracket=(\diamond(\exists x .(\text { wolf } x) \wedge((\text { enter } x) \wedge T))) \wedge(\text { growl }(\text { sel Nil }))
$$

Example (A wolf walks in. It might growl)

$$
\llbracket S \rrbracket=\exists x \cdot(\diamond((\text { howl }(\operatorname{sel}(\operatorname{Nil} \cup(x:: \operatorname{Nil})))) \wedge T)) \wedge((\text { wolf } x) \wedge((\text { enter } x) \wedge T))
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$$

## Lexical Semantics

$$
\left.\llbracket m i g h t \rrbracket=\lambda v s . \lambda i_{1} i_{2} k_{1} k_{2} f . f\left(\diamond\left(v s i_{1} i_{2} k_{1} k_{2} \Pi_{1}\right)\right)\left(k_{2} i_{1} i_{2}\right)\right)
$$

## Example (cont'd)

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\llbracket S \rrbracket=(\diamond(\exists x .(\text { wolf } x) \wedge((\text { enter } x) \wedge T))) \wedge(\text { growl }(\text { sel Nil }))
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$$
\begin{aligned}
\llbracket m i g h t \rrbracket & \left.=\lambda v s . \lambda i_{1} i_{2} k_{1} k_{2} f . f\left(\diamond\left(v s i_{1} i_{2} k_{1} k_{2} \Pi_{1}\right)\right)\left(k_{2} i_{1} i_{2}\right)\right) \\
\llbracket a_{n f \rrbracket} & =\lambda P Q . \lambda i_{1} i_{2} k_{1} k_{2} f . \exists x . P \times\left(x:: i_{1}\right) i_{2}\left(\lambda i j . Q \times i j k_{1} k_{2} \Pi_{1}\right) k_{2} f
\end{aligned}
$$

## Example (cont'd)

## Example (A wolf might walk in. It will growl)

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\llbracket S \rrbracket=(\diamond(\exists x .(\text { wolf } x) \wedge((\text { enter } x) \wedge T))) \wedge(\text { growl }(\text { sel Nil }))
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\llbracket a_{n f \rrbracket} & =\lambda P Q \cdot \lambda i_{1} i_{2} k_{1} k_{2} f . \exists x . P \times\left(x:: i_{1}\right) i_{2}\left(\lambda i j \cdot Q \times i j k_{1} k_{2} \Pi_{1}\right) k_{2} f \\
\llbracket a_{f} \rrbracket & =\lambda P Q \cdot \lambda i_{1} i_{2} k_{1} k_{2} f . \exists x . P \times i_{1}\left(x:: i_{2}\right) k_{1}\left(\lambda i j . Q \times i j k_{1} k_{2} \Pi_{2}\right) f
\end{array}
$$

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\begin{array}{ll}
\llbracket m i g h t \rrbracket & \left.=\lambda v s . \lambda i_{1} i_{2} k_{1} k_{2} f . f\left(\diamond\left(v s i_{1} i_{2} k_{1} k_{2} \Pi_{1}\right)\right)\left(k_{2} i_{1} i_{2}\right)\right) \\
\llbracket a_{n f} \rrbracket & =\lambda P Q \cdot \lambda i_{1} i_{2} k_{1} k_{2} f . \exists x . P \times\left(x:: i_{1}\right) i_{2}\left(\lambda i j \cdot Q \times i j k_{1} k_{2} \Pi_{1}\right) k_{2} f \\
\llbracket a_{f} \rrbracket & =\lambda P Q \cdot \lambda i_{1} i_{2} k_{1} k_{2} f . \exists x . P \times i_{1}\left(x:: i_{2}\right) k_{1}\left(\lambda i j \cdot Q \times i j k_{1} k_{2} \Pi_{2}\right) f \\
\text { Reset } & \triangleq \lambda S . \lambda e_{1} e_{2} k_{1} k_{2} f . f\left(k_{1} e_{1} e_{2}\right) \quad\left(S e_{1} e_{2} \mathbb{T} k_{2} \operatorname{Conj}\right)
\end{array}
$$

## Example (A wolf might walk in. It would growl)

$$
\llbracket S \rrbracket=(\diamond(\exists x .(\operatorname{wolf} x) \wedge((\text { enter } x) \wedge(\square((\operatorname{growl}(\operatorname{sel}(x:: \operatorname{Nil}) \cup \operatorname{Nil})) \wedge \top))))) \wedge \top
$$

- $\square$ under the scope of $\diamond$
- But what if in the accessed worlds, wolf $x$ is false?


## Discussion

## Example (A wolf might walk in. It would growl)

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$$under the scope of $\diamond$

- But what if in the accessed worlds, wolf $x$ is false?
$\Rightarrow$ Modal base and local accommodation: we would like to have

$$
\begin{aligned}
\llbracket S \rrbracket= & (\diamond(\exists x .(\text { wolf } x) \wedge((\text { enter } x) \wedge \\
& (\square(((\text { wolf } x) \wedge(\text { enter } x)) \Rightarrow(\operatorname{growl}(\operatorname{sel}(x:: \text { Nil }) \cup \operatorname{Nil})) \wedge T))))) \wedge T
\end{aligned}
$$

## Discussion

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\end{aligned}
$$

## Alternative Proposal

$\llbracket s \rrbracket=\gamma \rightarrow \gamma \rightarrow(\gamma \rightarrow \gamma \rightarrow t \rightarrow \kappa \rightarrow t) \rightarrow(\gamma \rightarrow \gamma \rightarrow t \rightarrow \kappa \rightarrow t) \rightarrow(t \rightarrow t \rightarrow t \rightarrow t) \rightarrow t$ with $\kappa \triangleq t \rightarrow t \rightarrow t\left(\right.$ typically $\left.\lambda b_{1} b_{2} \cdot b_{1} \wedge \diamond\left(b_{1} \Rightarrow b_{2}\right)\right)$

## Accommodation: Example

Example (A wolf might enter. It would growl. It would eat you first)
$\diamond \exists x .(($ wolf $x) \wedge($ enter $x) \wedge$
$\square((($ wolf $x) \wedge($ enter $x)) \Rightarrow((\operatorname{growl}(\operatorname{sel}((x::$ Nil) $)+$ Nil) $)) \wedge$
$\square((($ wolf $x) \wedge($ enter $x)) \Rightarrow(($ eat you $(\operatorname{sel}((x::$ Nil $)+\operatorname{Nil}))))))))$

## $\gamma$ as a Macro Definition

- We used $\gamma$ as a list of entities
- But we could introduce $s$ the type of worlds and move to TY2
- Sel function on worlds and explicit reference to worlds (context referents)


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## Example (a wolf might walk in)

$\lambda e_{1} e_{2} k w . \exists w^{\prime} .\left(R w w^{\prime}\right) \wedge\left(\exists x .\left(\right.\right.$ wolf $\left.x w^{\prime}\right) \wedge\left(\left(\right.\right.$ enter $\left.\left.\left.x w^{\prime}\right) \wedge\left(k\left(\left(w^{\prime}, x\right)+e_{1}\right)\left(w^{\prime}:: e_{2}\right) w\right)\right)\right)$

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- Flexibility on factual and nonfactual world interaction


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## Example

John might buy a house $\mathrm{x}_{\mathrm{x}}$. He earns enough to get a mortage. He could rent it $x_{x}$ out for the festival.

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John might buy a house $x_{x}$. He earns enough to get a mortage. He could rent it $\mathrm{t}_{x}$ out for the festival.

## Example

 and Peace.

## Wrapping Up

- Modal subordination in [de Groote(2006)]'s framework
- Flexibility of the approach
- Role of the lexical semantics
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## Conclusion

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## Future Work

- Dynamic modal logic?
- Negation and counterfactuals
- [Veltman(1996)]'s testing and filtering
- Interaction with discourse structure (factual explanations of nonfactual possibilities)
- Hob and Nob sentences

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