

Belenios with cast-as-intended

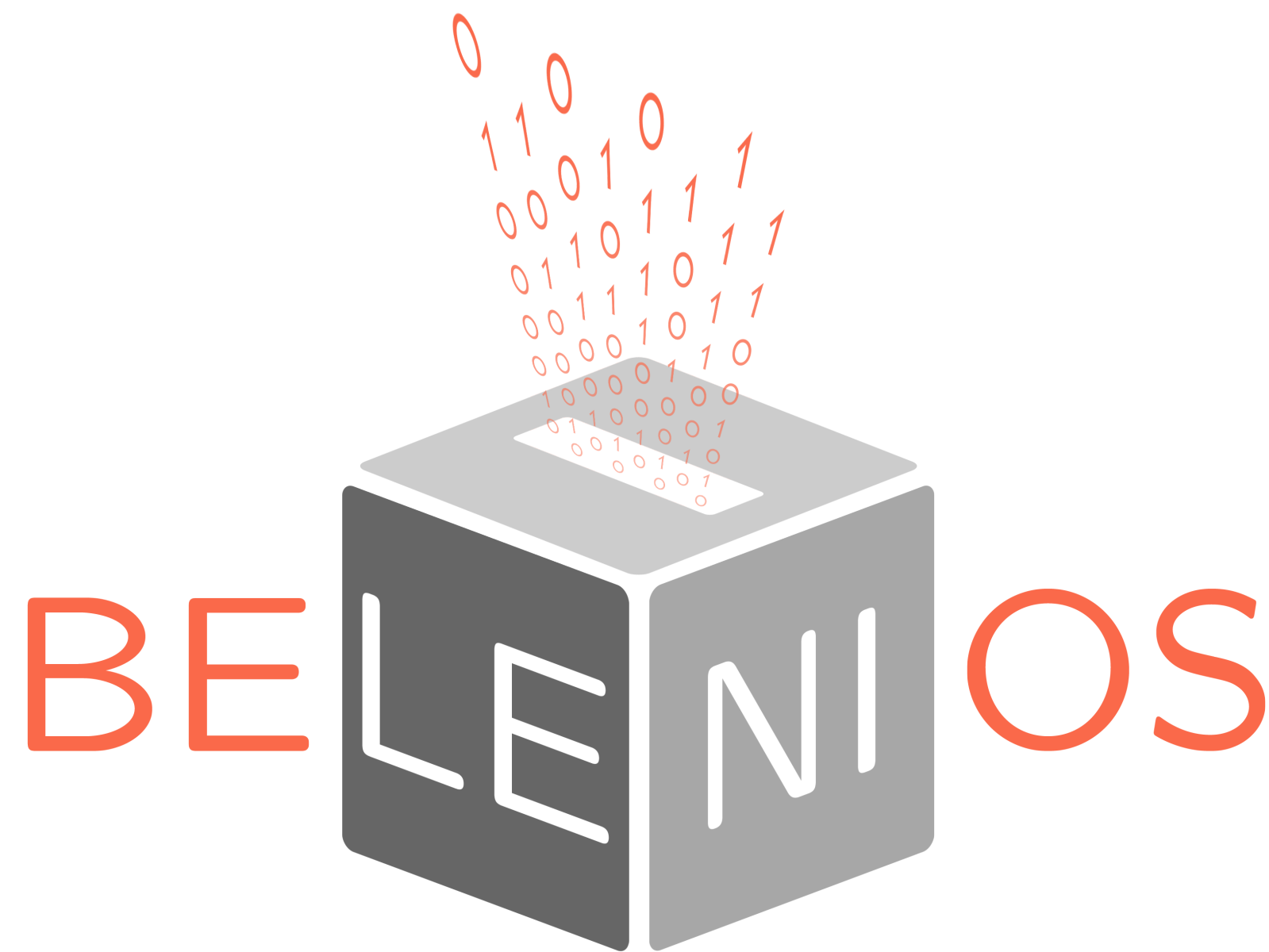
*Véronique Cortier, **Alexandre Debant**, Pierrick Gaudry, Stéphane Glondu*

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**8th Voting workshop
Bol, Brač, Croatia**



Belenios



Belenios is a protocol firstly introduced in 2014

- ▶ it extends Helios [Adida - 2008]
- ▶ it has many extensions: BeleniosVS, BeleniosRF, ...
- ▶ often studied in the literature
















Belenios is a software

- ▶ open source (GNU AGPLv3)
- ▶ developed in OCaml and Javascript

Belenios is a platform - <https://vote.belenios.org/admin>

- ▶ mainly associations and professional elections
- ▶ + 2000 elections
- ▶ + 100 000 voters

Belenios security
















	Voter	Voting client	Voting server	Registrar	Trustees
Verifiability	 	 	 	 	 
Vote secrecy					 (k out of n)

Election Verifiability with ProVerif. Cortier, Debant, and Cheval - CSF 2023

Provably Improving Election Verifiability in Belenios. Baloglu, Bursuc, Mauw, and Pang - E-Vote-ID 2021.

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Belenios security

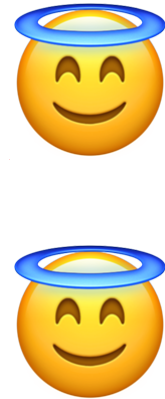

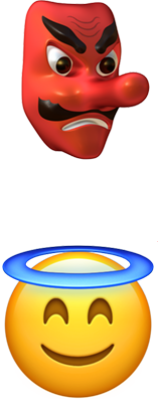







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Can we do better?

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Existing cast-as-intended solutions

- ▶ **Cast-or-audit:** e.g. Benaloh challenge [Benaloh - 2006]
- ▶ **Second device:** e.g. Estonian IVXV protocol
- ▶ **Return codes:** e.g. Swiss Post protocol
- ▶ **Transparent voting:** e.g. sElect and Selene protocols [Ryan *et al* - 2015], [Küsters *et al* - 2016]
- ▶ **Cast-and-audit:** [Cortier *et al* - 2019]

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Cast-and-audit

Idea of cast-and-audit

- ▶ Alice wants to vote for $X \in \{1, \dots, p\}$
- ▶ Alice picks $A \in \{1, \dots, p\}$ at random
- ▶ Alice casts ballot $b = (\{X\}_{pk}^{r_X}, \{A\}_{pk}^{r_A}, \{B\}_{pk}^{r_B}, \text{zkp}(B = X + A \text{ mod } n))$
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If the attacker casts a ballot $b' = (\{X'\}_{pk}^{r'_X}, c_A, c_B, \pi)$ with $X \neq X'$ then either:

- ▶ the proof π is invalid; or
- ▶ c_A does not encrypt A ; or
- ▶ c_B does not encrypt B



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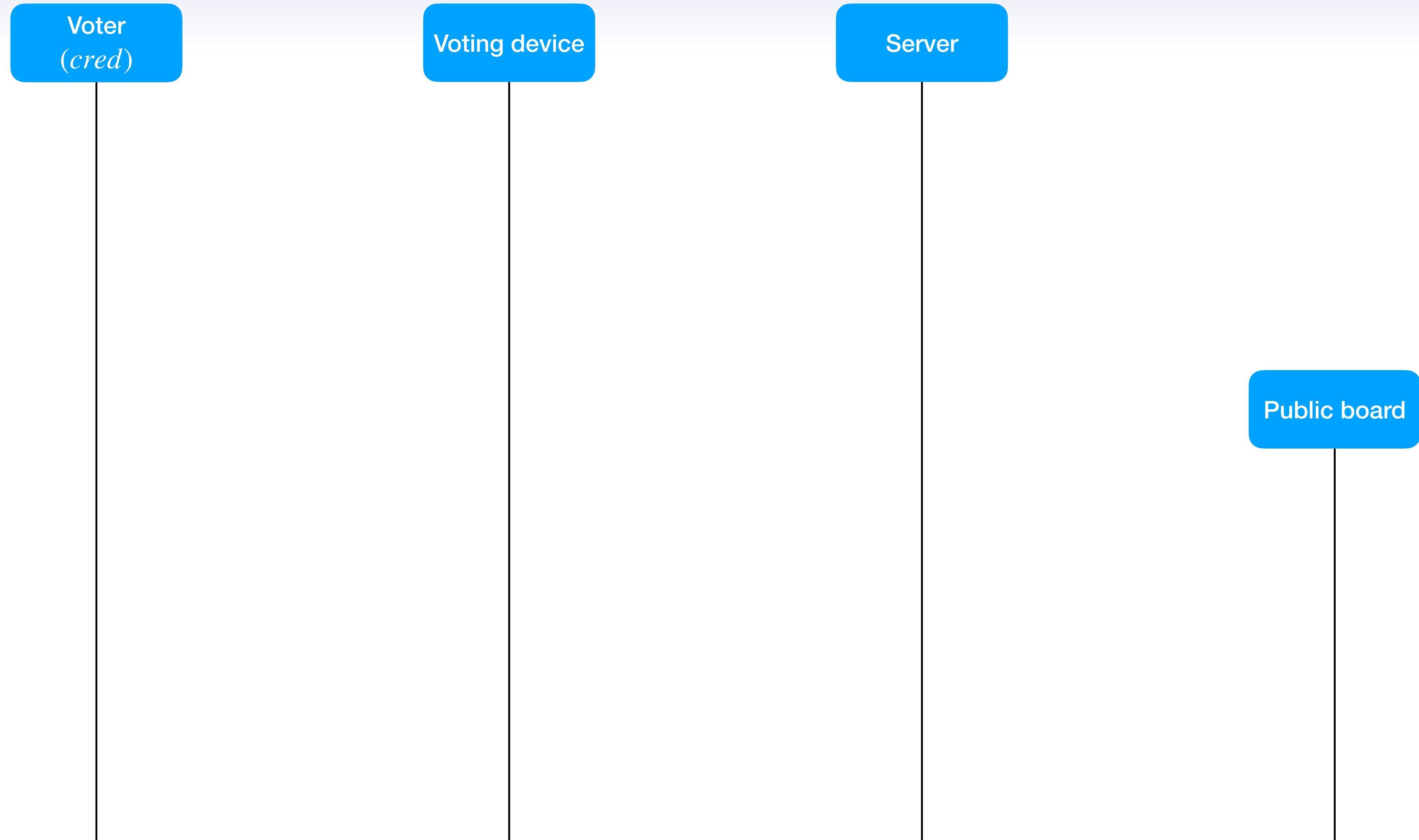
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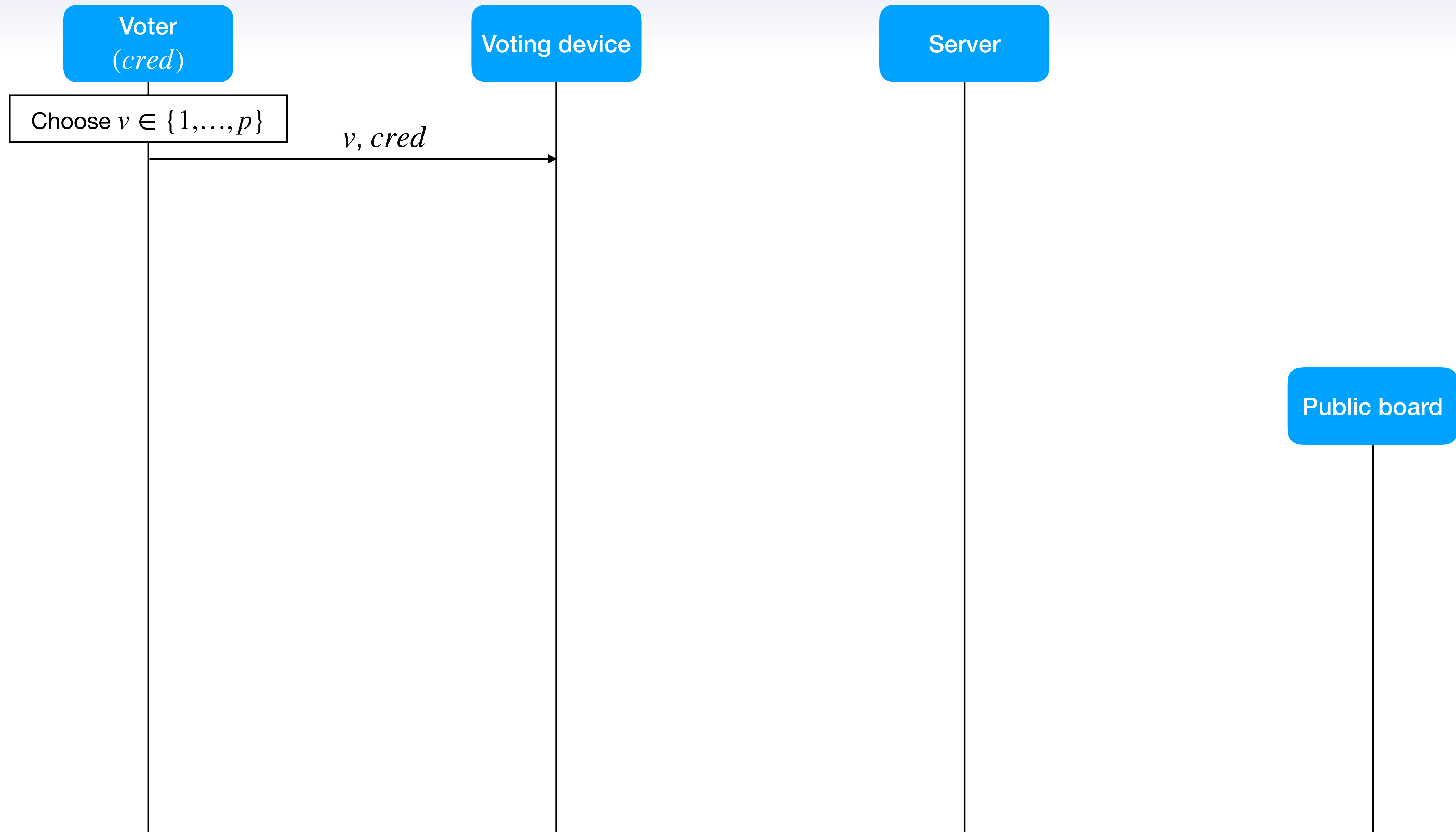


Alice will detect any modification of X with probability $1/2$

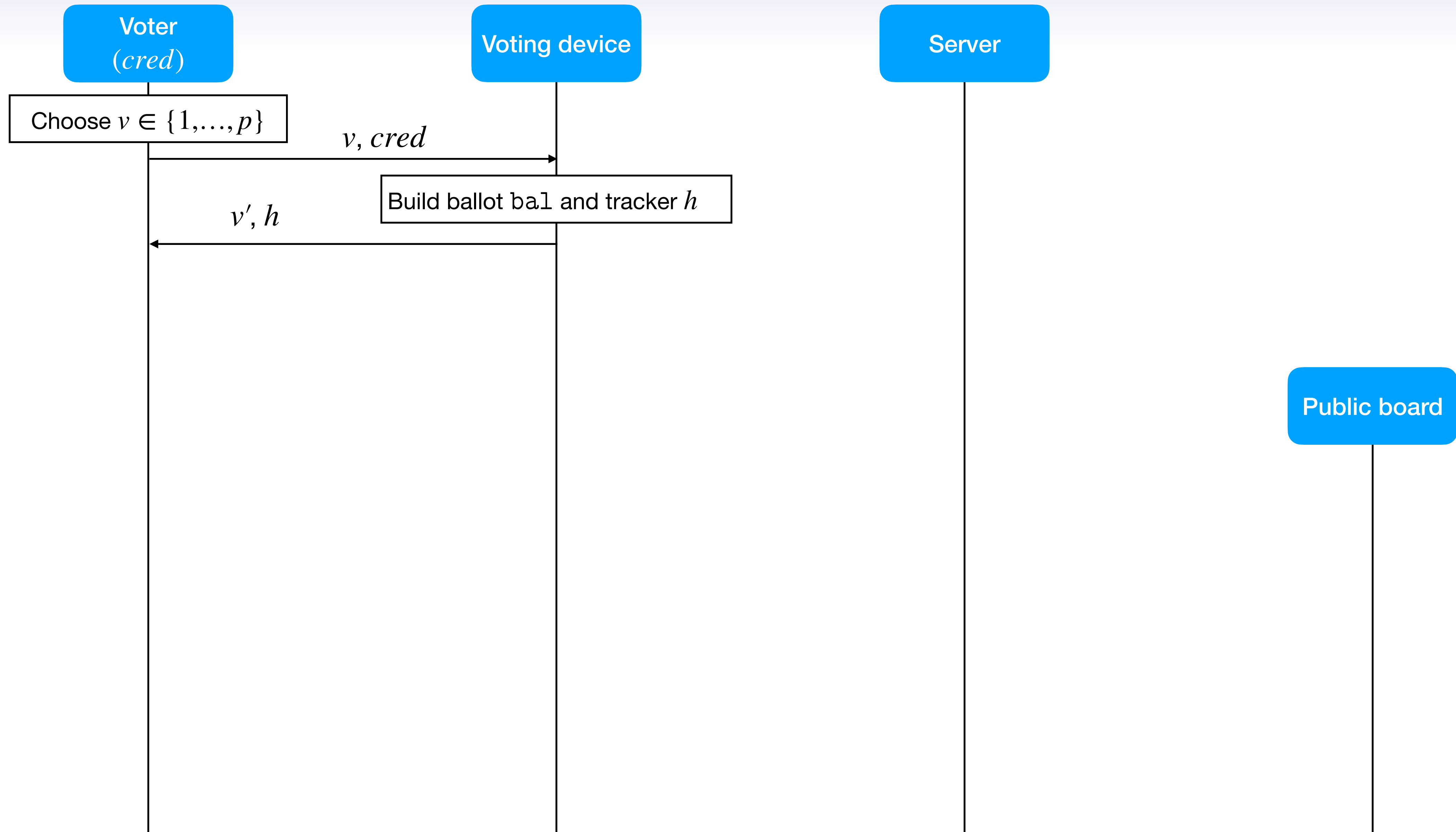
Belenios protocol



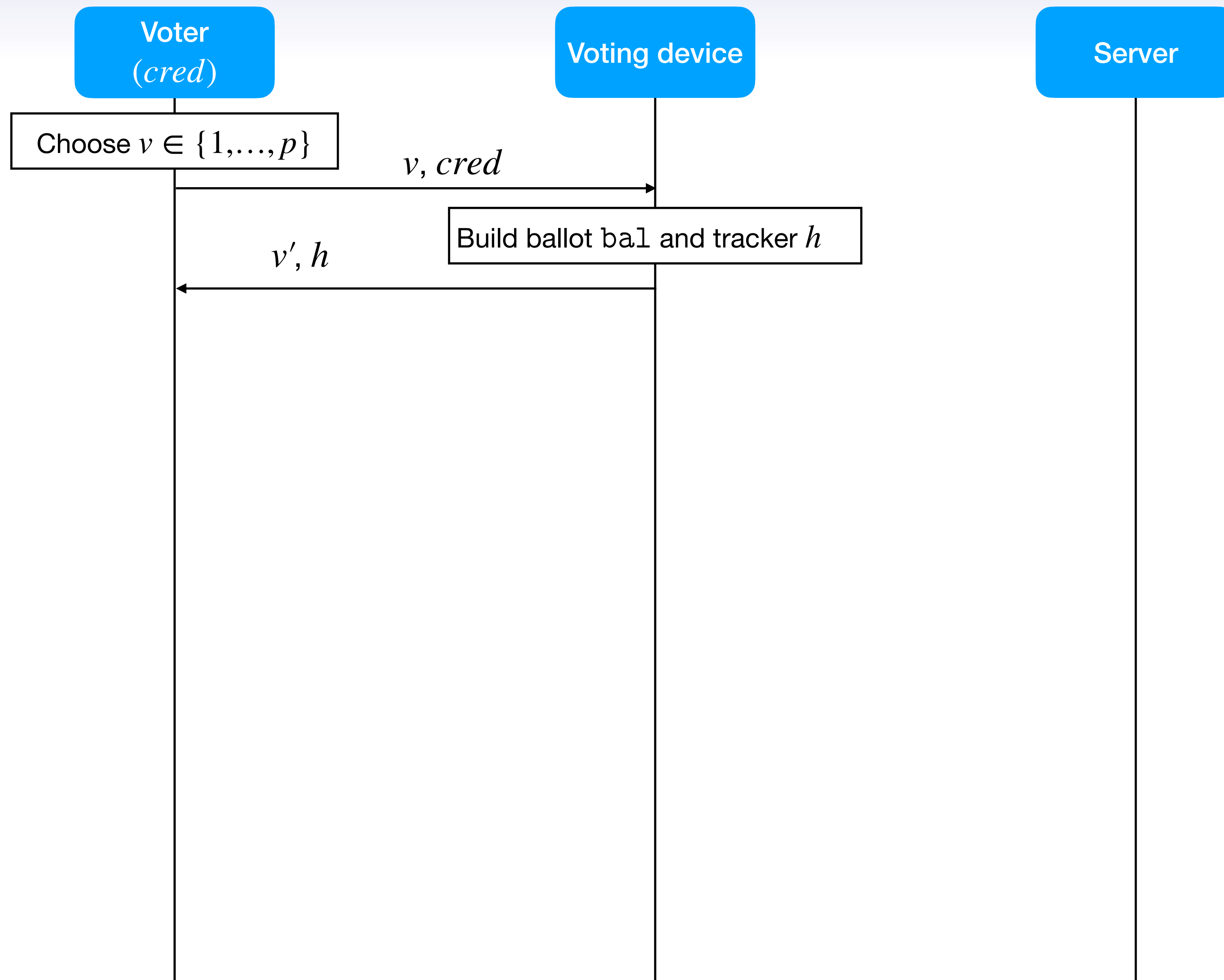
Belenios protocol



Belenios protocol



Belenios protocol



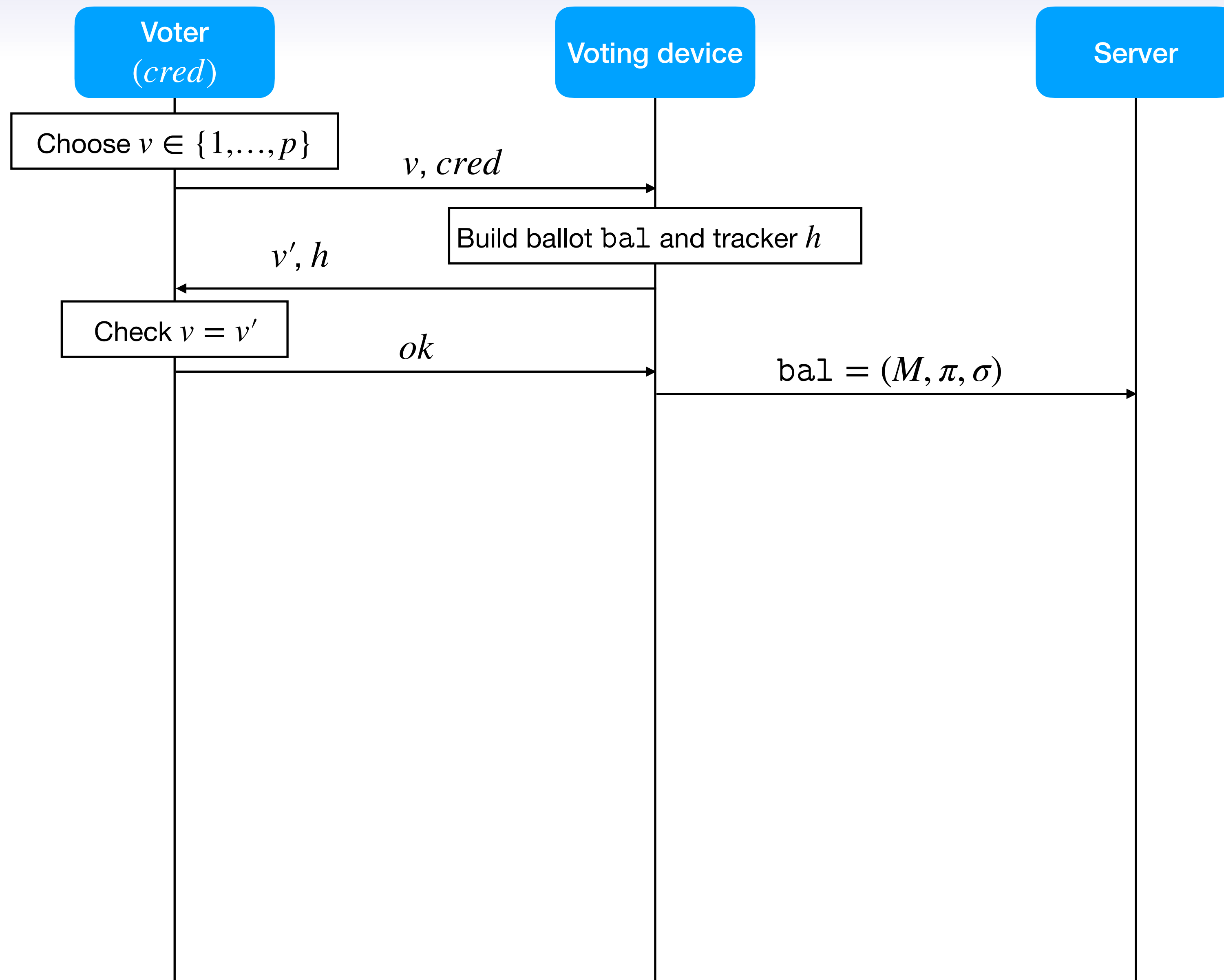
Build ballot ba_1 and tracker h

$$M = \begin{pmatrix} \{X_1\}_{pk}^{r_1} \\ \vdots \\ \{X_k\}_{pk}^{r_k} \end{pmatrix}$$

- $\pi = \text{zkp}(M \text{ is a valid vote})$
- $\sigma = \text{sign}_{cred}(M, \pi)$
- $h = \text{hash}(M, \pi, \sigma)$

Public board

Belenios protocol

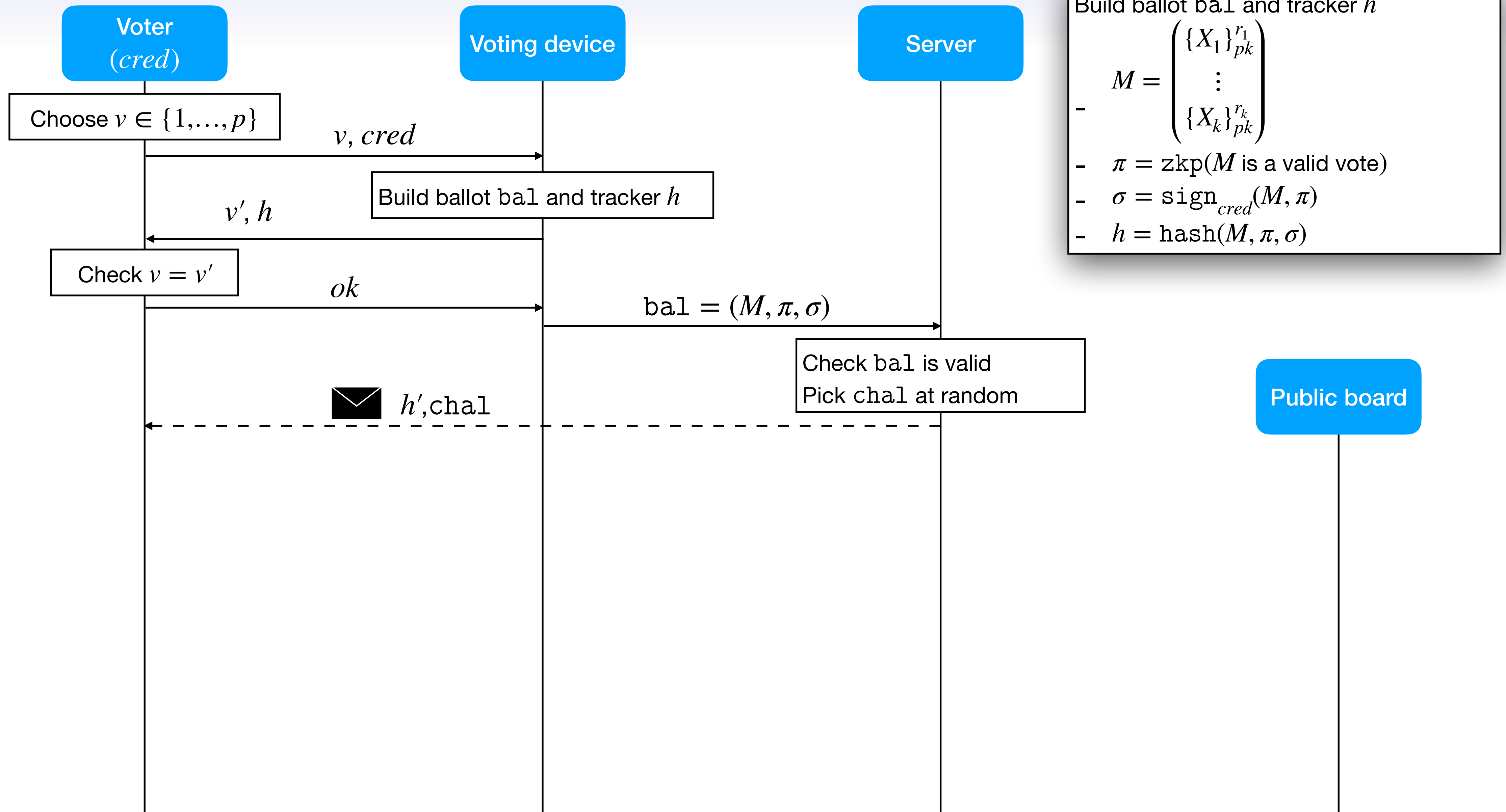


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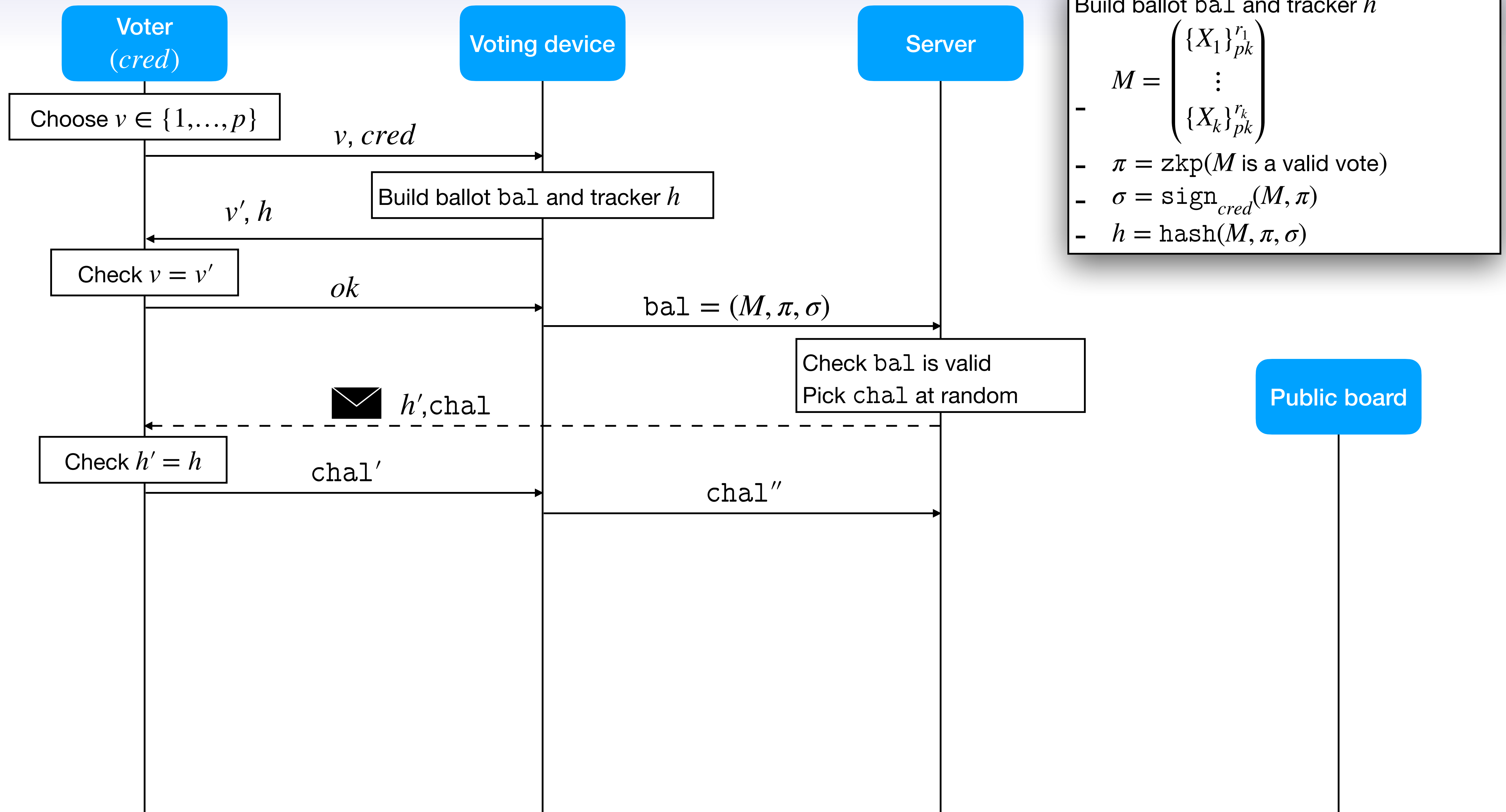
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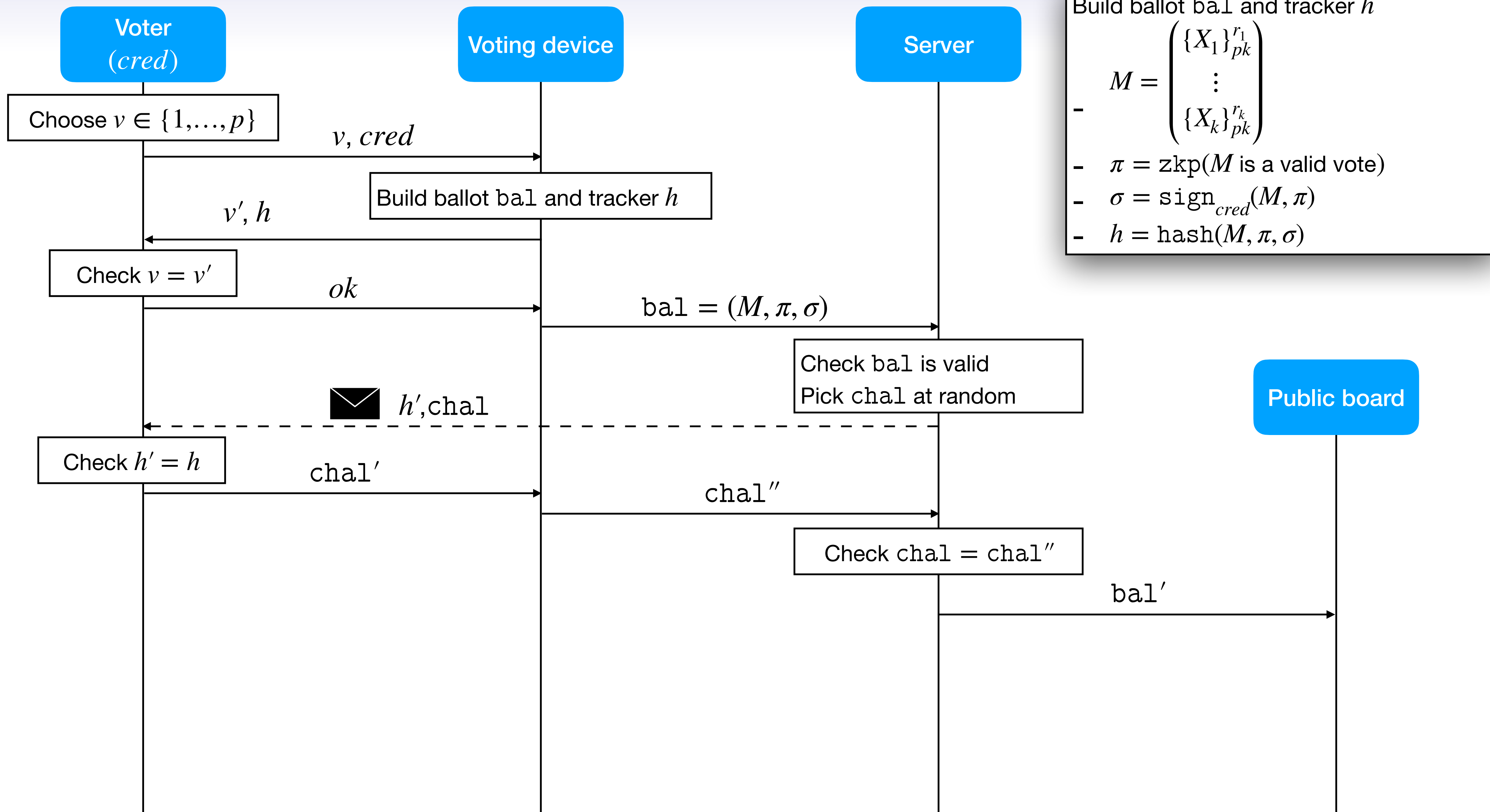
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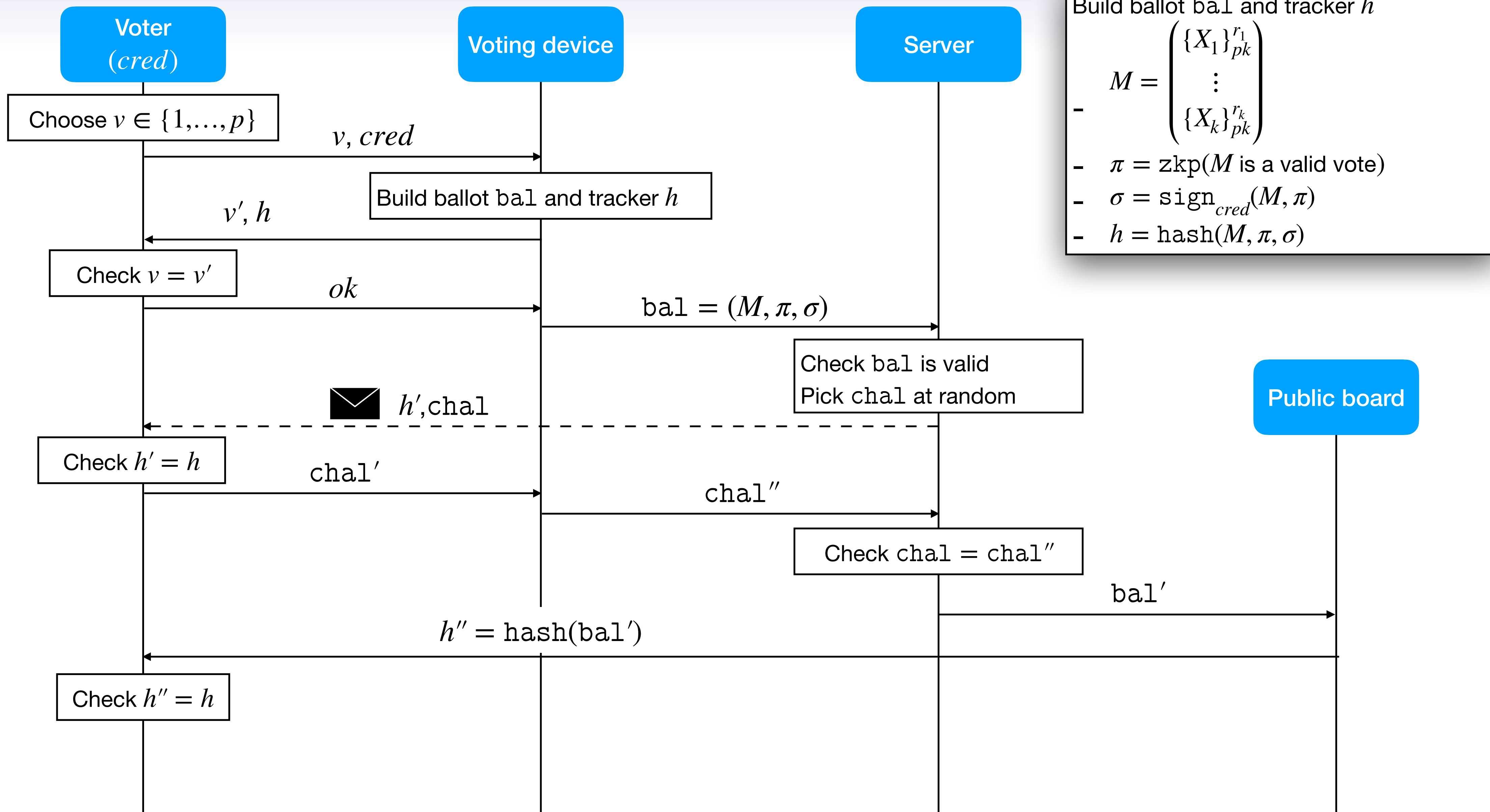
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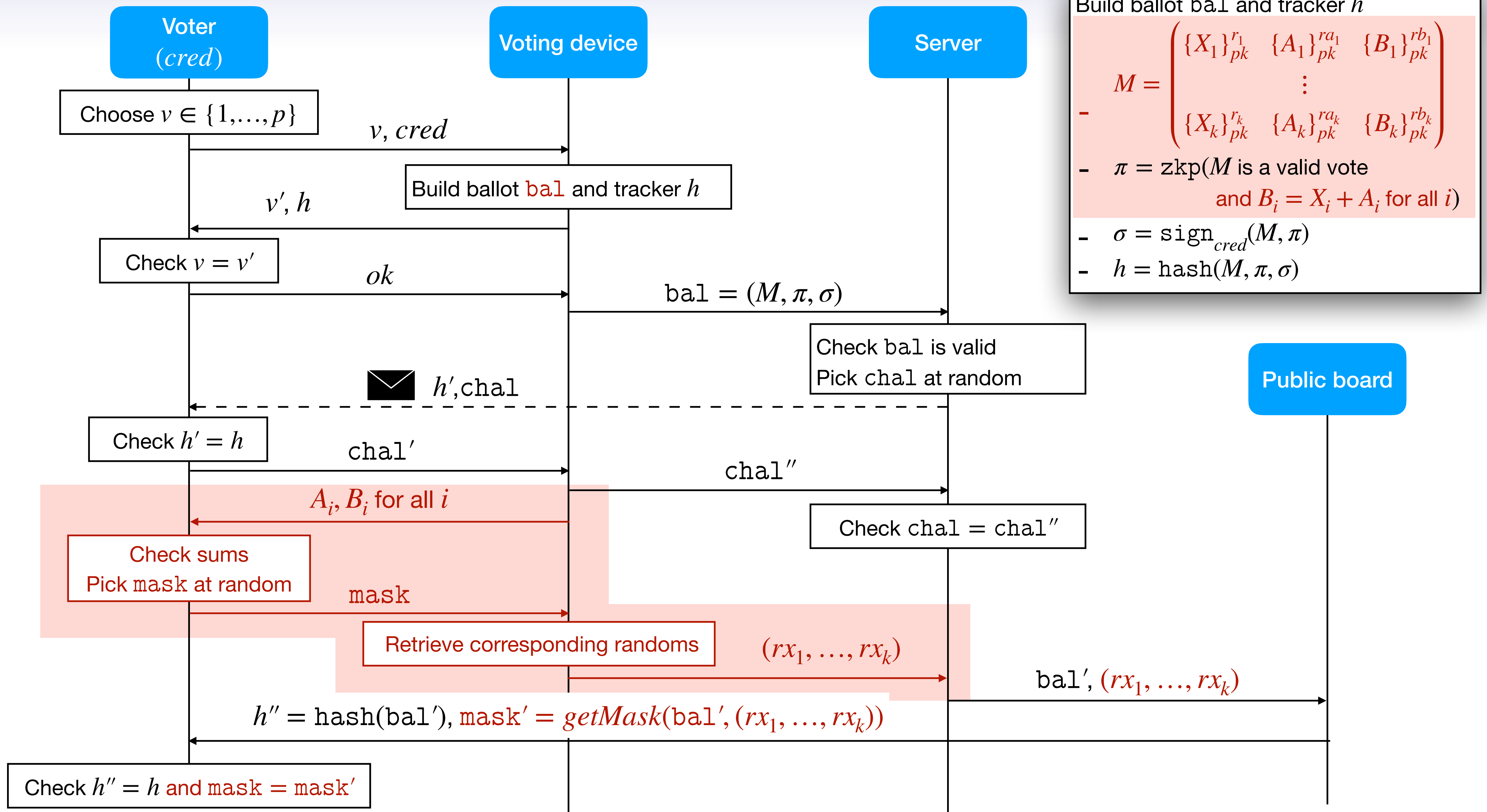
Build ballot bal and tracker h

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Belenios protocol



BeleniosCal protocol



Build ballot $ba1$ and tracker h

- $M = \begin{pmatrix} \{X_1\}_{pk}^{r_1} & \{A_1\}_{pk}^{ra_1} & \{B_1\}_{pk}^{rb_1} \\ \vdots \\ \{X_k\}_{pk}^{r_k} & \{A_k\}_{pk}^{ra_k} & \{B_k\}_{pk}^{rb_k} \end{pmatrix}$
- $\pi = \text{zkp}(M \text{ is a valid vote and } B_i = X_i + A_i \text{ for all } i)$
- $\sigma = \text{sign}_{cred}(M, \pi)$
- $h = \text{hash}(M, \pi, \sigma)$

User experience

Candidate 1

Candidate 2

Candidate 3

Select your favorite candidate

Confirm

Back

User experience

Candidate 1

Candidate 2

Candidate 3

Select your favorite candidate

Confirm

Back

Selected?

Candidate 1 1

Candidate 2 0

Candidate 3 0

Your ballot tracker is:

bwWypF5ysEazLzRADO7ttm2K...

Confirm and
send challenge

Back

User experience

Candidate 1

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Select your favorite candidate

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Confirm and send challenge Back

Your ballot tracker is:
bwWypF5ysEazLzRAdo7ttm2K...

Check your ballot tracker and enter the challenge:

Confirm Back

User experience

Candidate 1

Candidate 2

Candidate 3

Select your favorite candidate

Confirm Back

Selected?

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Confirm Back

	Selected?		Audit codes
Candidate 1	1	+	9 = 10
Candidate 2	0	+	3 = 3
Candidate 3	0	+	6 = 6

Check that the additions are correct and select one audit code per line

Cast ballot Back

User experience

Candidate 1

Candidate 2

Candidate 3

Select your favorite candidate

Confirm Back

Selected?

Candidate 1 1

Candidate 2 0

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Cast ballot Back

User experience

Candidate 1

Candidate 2

Candidate 3

Select your favorite candidate

Confirm Back

Selected?

Candidate 1

Candidate 2

Candidate 3

Your ballot track
bwWypF5ysEazL...

Confirm and send challenge

Bulletin board

Candidate 1 - + 9 = -

Candidate 2 - + - = 3

Candidate 3 - + - = 6

Ballot tracker:
bwWypF5ysEazLzRADO7ttm2K...

Selected? Audit codes

Candidate 1 1 + 9 = 10

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Computation cost

Belenios ballot

Builds ballot ba_1 and tracker h

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BeleniosCal ballot

Builds ballot ba_1 and tracker h

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and $B_i = X_i + A_i$ for all $i)$
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Ciphertexts: 1 ElGamal encryption per candidate

ZKP:

- $X_i \in \{0,1\} \Rightarrow 5$ exponentiations per candidate
- the voter chooses a valid combination \Rightarrow it depends

Signature: 1 signature

BeleniosCal ballot

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Ciphertexts: 1+2 ElGamal encryptions per candidate

ZKP:

- $X_i \in \{0,1\} \Rightarrow 5$ exponentiations per candidate
- the voter chooses a valid combination \Rightarrow it depends

- $B_i = X_i + A_i \pmod n \Rightarrow 5$ exponentiations per candidate

Signature: 1 signature

Computation cost

Belenios ballot

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A BeleniosCal ballot is (at most) 3 times more expensive

BeleniosCal ballot

Builds ballot ba_1 and tracker h

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Security analysis

ProVerif

- ▶ An automatic prover for **symbolic analysis**
- ▶ Handle **trace-based properties** for verifiability
- ▶ Handle **equivalence-based** properties for vote secrecy

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2 main challenges

- ▶ **Probabilities:** ProVerif does not support probabilistic choices
The voter receives a commitment on their ballot before doing their choice
 - ➔ It is enough to model that the voter could audit both codes

Security analysis

ProVerif

- ▶ An automatic prover for **symbolic analysis**
- ▶ Handle **trace-based properties** for verifiability
- ▶ Handle **equivalence-based** properties for vote secrecy

2 main challenges

- ▶ **Probabilities:** ProVerif does not support probabilistic choices
The voter receives a commitment on their ballot before doing their choice
➔ It is enough to model that the voter could audit both codes
- ▶ **Additions:** ProVerif does not support arithmetics in \mathbb{Z}_n
 - ➔ reachability: over-approximate the “+” operator
 - ➔ equivalence: prove a relation preservation

[Cortier et al - 2022]

Modeling arithmetics in \mathbb{Z}_n

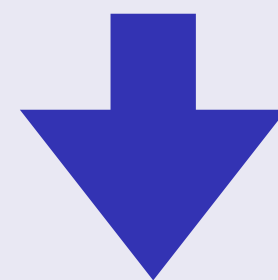
- Modeling:**
- ▶ integers are modeled by **abstract atomic values**, x, y, a, b, c, \dots
 - ▶ whenever someone checks $b \stackrel{?}{=} x + a$, we **execute the event** $isSum(x, a, b)$

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Reachability properties:

« For all $x, a \in \mathbb{Z}_n$, there exists a unique
 $b \in \mathbb{Z}_n$ such that $b = x + a$ »



Restrictions such that

$$isSum(x, a, b) \wedge isSum(x, a, b') \Rightarrow b = b'$$

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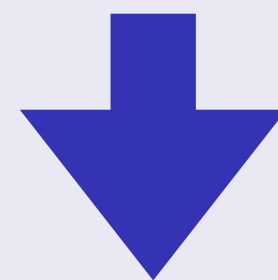
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...

Equivalence properties: relation preservation

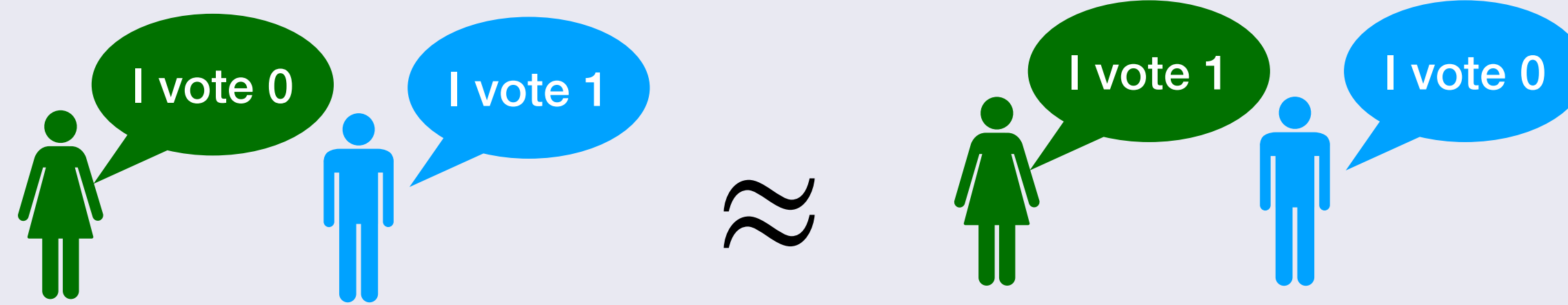
Lemma (intuition): given two processes P and Q , for all traces $tr_P \in Traces(P)$ and $tr_Q \in Traces(Q)$ such that $tr_P \approx tr_Q$ we have:

$$isSum(x, a, b) \in tr_P \Leftrightarrow isSum(x, a, b) \in tr_Q$$

(related to the notion of bi-process and diff-equivalence)

Security properties

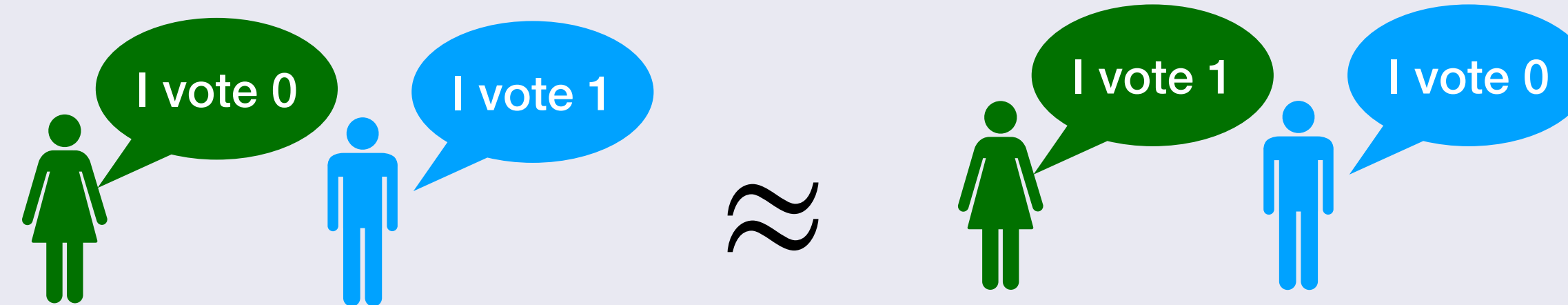
Vote secrecy
[Kremer *et al* - 2009]



Security properties

Vote secrecy

[Kremer *et al* - 2009]



Verifiability

► Cast-as-intended:

$\text{Verified}(\text{LR}, id, h, v) \wedge \text{Honest}(id, vk) \Rightarrow \text{onBoard}(vk', b, h, r, X) \wedge (b \text{ encrypts candidate } v)$

► No clash attack:

$\text{Verified}(\text{L}, id, h, v) \wedge \text{Verified}(\text{R}, id', h, v') \Rightarrow \text{false}$

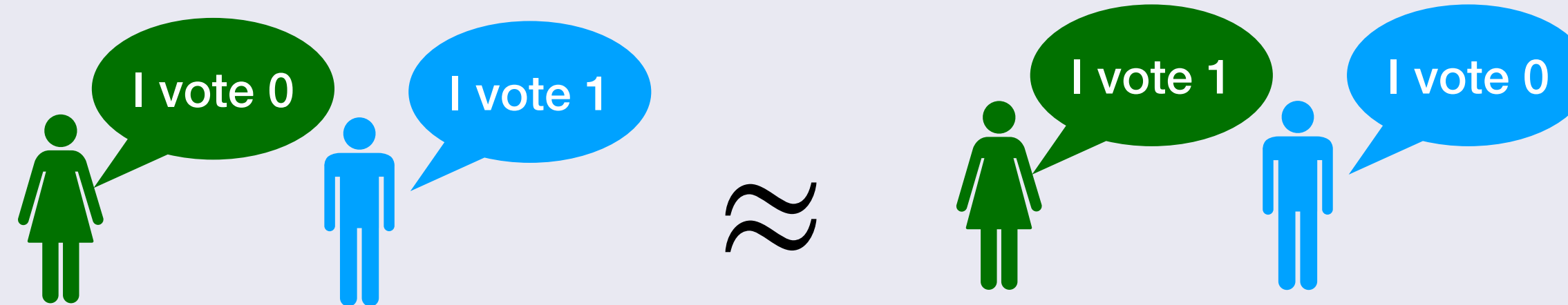
► Recorded-as-cast:

(strong) $\text{onBoard}(vk, b, h, r, X) \wedge \text{Honest}(id, vk) \Rightarrow \text{Voted}(id', vk, h)$

(standard) $\text{onBoard}(vk, b, h, r, X) \wedge \text{Honest}(id, vk) \Rightarrow \text{HasInitiatedVote}(id', vk)$

Security properties

Vote secrecy [Kremer *et al* - 2009]



Verifiability

► Cast-as-intended:

$\text{Verified}(\text{LR}, id, h, v) \wedge \text{Honest}(id, vk) \Rightarrow \text{onBoard}(vk', b, h, r, X) \wedge (b \text{ encrypts candidate } v)$

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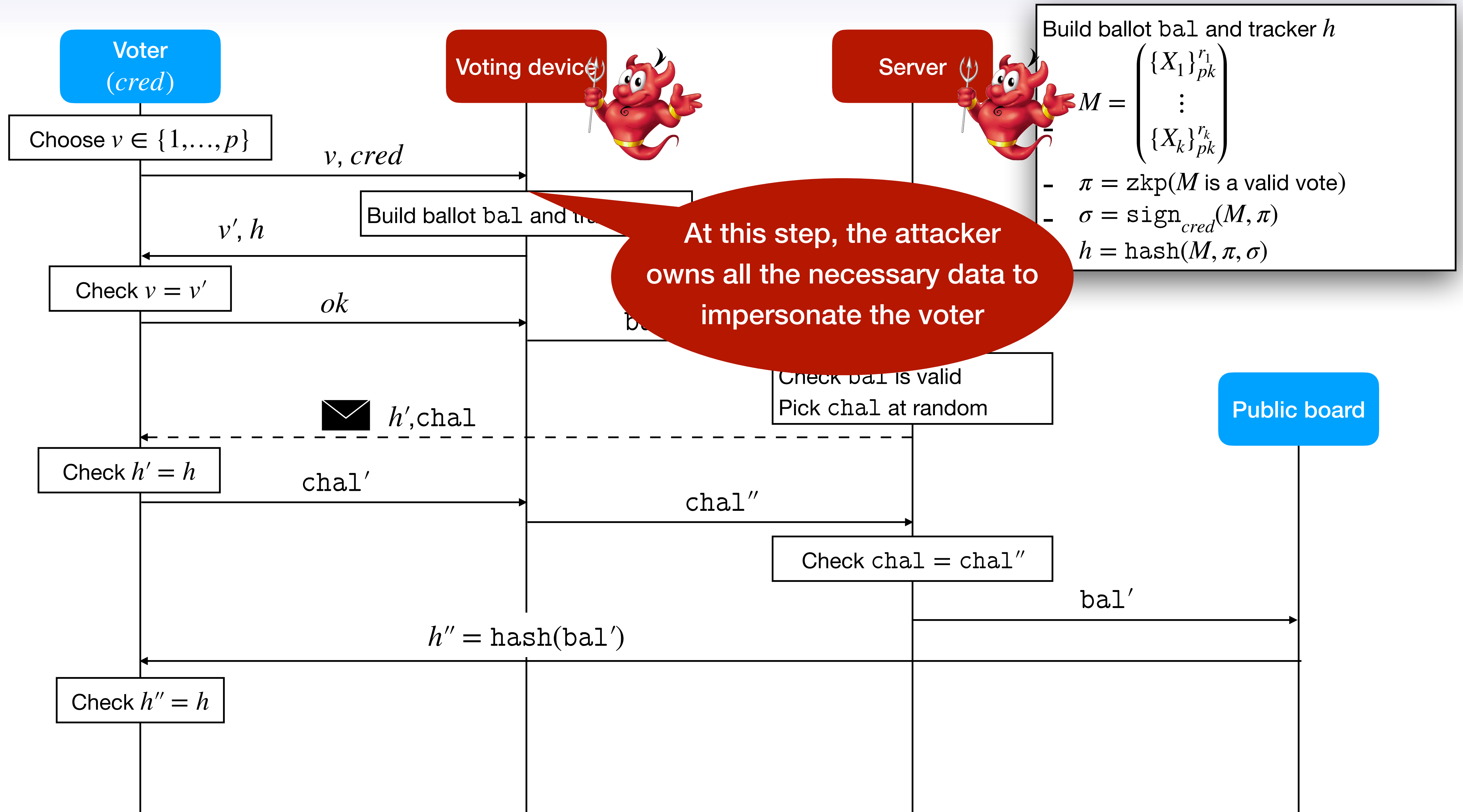
A clash is possible when auditing on the same side...
detected with probability 1/2

► Recorded-as-cast:




































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Recorded-as-cast: strong vs standard






































Results

	Voter	Voting client	Voting server	Registrar	Trustees
Cast-as-intended					
No clash					
Recorded-as-cast (strong)					
					
Recorded-as-cast (standard)					
					
Vote secrecy					 (k out of n)
















Results

	Voter	Voting client	Voting server	Registrar	Trustees
Cast-as-intended					
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Results

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Recorded-as-cast (strong)					
					
Recorded-as-cast (standard)					
					
Vote secrecy					 (k out of n)

Summary

		Voter	Voting client	Voting server	Registrar	Trustees
Verifiability	(standard only)					
	(standard and strong)					
Vote secrecy						 (k out of n)

BeleniosCal

- ▶ extends Belenios **with cast-as-intended**
- ▶ preserves Belenios vote secrecy guarantees
- ▶ is **formally proven** secure
- ▶ still **does not** require expensive computations

Future work

Implement the protocol in Belenios framework
propose it as a new feature



Evaluate its usability in practice

understandability

UX (representation of additions in \mathbb{Z}_n , choice of n , etc)



Evaluate its acceptability

Do people understand and accept “probabilistic security”?

