Election Verifiability with ProVerif

(published at CSF'23)

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Laboratoire lorrain de recherche en informatique et ses applications

FRANCE 2030

Security properties

Vote secrecy

“No one should know who I voted for”

Verifiability

“No one can modify the outcome of the election”

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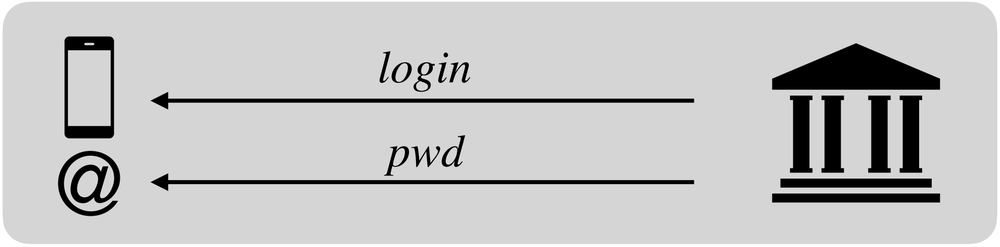
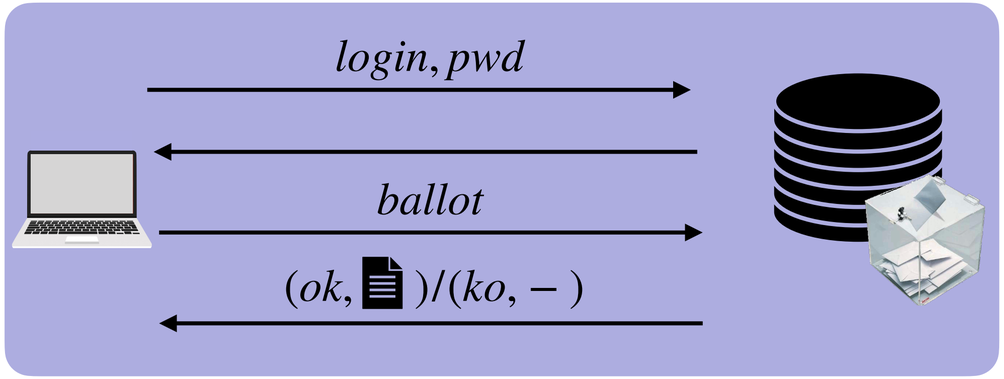
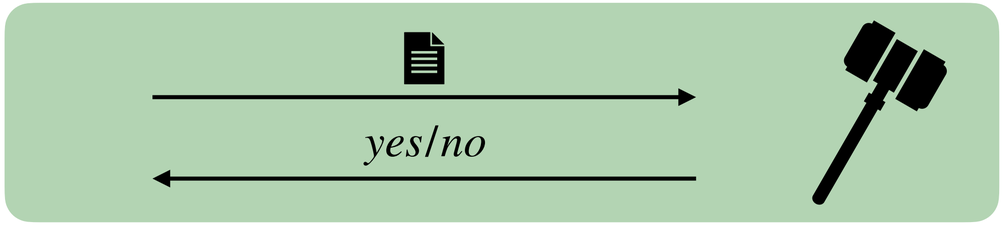
E-voting protocol
- overview -



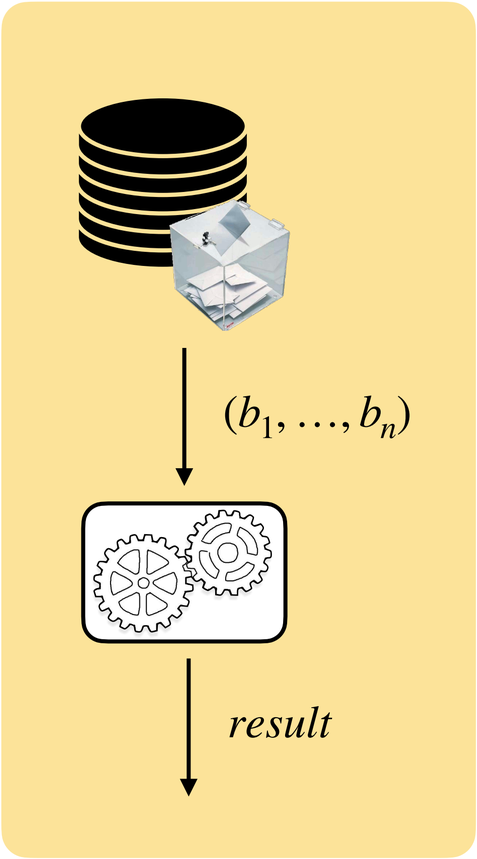
Setup phase



Voting phase



Verification phase



Tally phase

E2E verifiability [Cortier et al - ESORICS'14]

Definition - An evoting protocol satisfies E2E verifiability if for any execution,

$$\text{result} = V_{\mathtt{HV}} \quad V_{\mathtt{HNV}} \quad V_{\mathtt{D}}$$

where:

- $V_{\mathtt{HV}}$ is the multiset of votes of honest voters who verify
- $V_{\mathtt{HNV}}$ is a submultiset of the multiset of votes of honest voters who do not verify
- $V_{\mathtt{D}}$ contains at most one vote per dishonest voter

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Approaches based on sub-properties e.g, [Cortier et al - CSF'19], [Baloglu et al - CSF'21]

- **Eligibility:** each vote has been cast by a legitimate voter
- **Individual verifiability**
  - **Cast-as-intended:** the voter's ballot contains their intended vote
  - **Recorded-as-cast:** the counted ballot corresponds to the cast one
- **Universal verifiability:** the result corresponds to the content of the ballot-box
- **No clash attacks:** two voters cannot agree on the same ballot

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These are only sufficient conditions...

Contributions

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Theorem - An evoting protocol satisfies E2E verifiability if and only if it satisfies Query 1 and Query 2

Contributions

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2. **A ProVerif framework to analyze evoting protocols**

Applied to several protocols: Helios, Belenios, Swiss Post, CHVote

ProVerif

What is ProVerif?

- is an automatic prover for symbolic analysis
  - messages abstracted with terms
  - Dolev-Yao attacker model (intercept/inject/modify)
- can model an unbounded number of sessions
- handles trace-based properties
- handles equivalence-based properties
- has already been used to analyse voting protocols, e.g., Helios, Belenios, Swiss Post, CHVote, etc

$$
\begin{aligned}
P, Q := \; & 0 \\
& \mid \texttt{new}\ n; P \\
& \mid \texttt{let}\ x = v\ \texttt{in}\ P\ \texttt{else}\ Q; \\
& \mid \texttt{in}(c, x); P \\
& \mid \texttt{out}(c, u); P \\
& \mid (P \mid Q) \\
& \mid !P \\
& \mid \texttt{event}\ e(u_1, \ldots, u_n); P
\end{aligned}
$$

A trace tr is a finite sequence of $\texttt{in}$, $\texttt{out}$, or $\texttt{event}(\texttt{e}(\texttt{u}_1, \ldots, \texttt{u}_n))$.

Queries

Event satisfaction - A trace $tr = tr_1 \ldots tr_n$ executes event $E(u_1, \ldots, u_n)$ at time $\tau \in \{1, \ldots, n\}$, noted $(tr, \tau) \vdash E(u_1, \ldots, u_n)$, if $tr_\tau = \texttt{event}(E(u_1, \ldots, u_n))$

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Query formula - A trace $tr = tr_1 \dots tr_n$ satisfies a query of the form

$$\bigwedge_{k=1}^{p} F_k(v_1, \dots, v_{l_k}) \Rightarrow \bigvee_{i=1}^{m} \bigwedge_{j=1}^{n_i} E_{i,j}(u_1^{i,j}, \dots, u_{l_{i,j}}^{i,j})$$

if for all substitution σ such that for all k, $(tr, \tau_k) \vdash F_k(v_1, \dots, v_{l_k})\sigma$ for some τ_k, there exists σ' and i such that for all j, there exists $\tau_{i,j}$ such that $(tr, \tau_{i,j}) \vdash E_{i,j}(u_1^{i,j}, \dots, u_{l_{i,j}}^{i,j})\sigma'$ and $F_k(v_1, \dots, v_{l_k})\sigma = F_k(v_1, \dots, v_{l_k})\sigma'$

Injective queries

Injective query - A trace $tr = tr_1 \dots tr_n$ satisfies an injective query of the form

$$\texttt{inj} - F_0(v_0, \dots, v_{l_0}) \wedge \bigwedge_{k=1}^{p} F_k(v_1, \dots, v_{l_k}) \Rightarrow \bigvee_{i=1}^{m} \texttt{inj} - E_{i,0}(u_1^{i,0}, \dots, u_{l_{i,0}}^{i,0}) \wedge \bigwedge_{j=1}^{n_i} E_{i,j}(u_1^{i,j}, \dots, u_{l_{i,j}}^{i,j})$$

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Moreover, there exists an injective function $f : \mathscr{F}_0(tr) \to \mathscr{E}_0(tr)$ such that if $(tr, \alpha) \vdash F_0(v_1, \dots, v_{l_0})\sigma$ then $(tr, f(\alpha)) \vdash E_{i,0}(u_1^{i,0}, \dots, u_{l_{i,0}}^{i,0})\sigma'$.

$\mathscr{F}_0(tr), \mathscr{E}_0(tr) \subseteq \{1, \dots, n\}$ are the sets of indices matching respectively $F_0(v_0, \dots, v_{l_0})$ and $E_{i,0}(u_1^{i,0}, \dots, u_{l_{i,0}}^{i,0})$

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Injective query - A trace $tr = tr_1 \ldots tr_n$ satisfies an injective query of the form

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Example: $\rho = \texttt{inj} - F(x) \Rightarrow \texttt{inj} - E_0(x)$

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- $tr_1 = \texttt{event}(E_0(a)) \,.\, \texttt{event}(E_1(a)) \,.\, \texttt{event}(F(a)) \,.\, \texttt{event}(F(a))$

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tr_1 satisfies ρ ✓

tr_2 does not satisfy ρ ✗

E2E verifiability

Events used to model E2E verifiability

Honesty and behavior of voter:

- `hv(id)`, an honest voter who verifies
- `hnv(id)`, an honest voter who does not verify
- `corrupt(id)`, a dishonest voter

Protocol steps

- `voted`(id, v), voter id has cast a vote v
- `verified`(id, v), voter id has cast a vote v and verified
- `counted`(v), a vote for v has been counted during the tally
- `finish`, the tally has been completed

E-voting protocol
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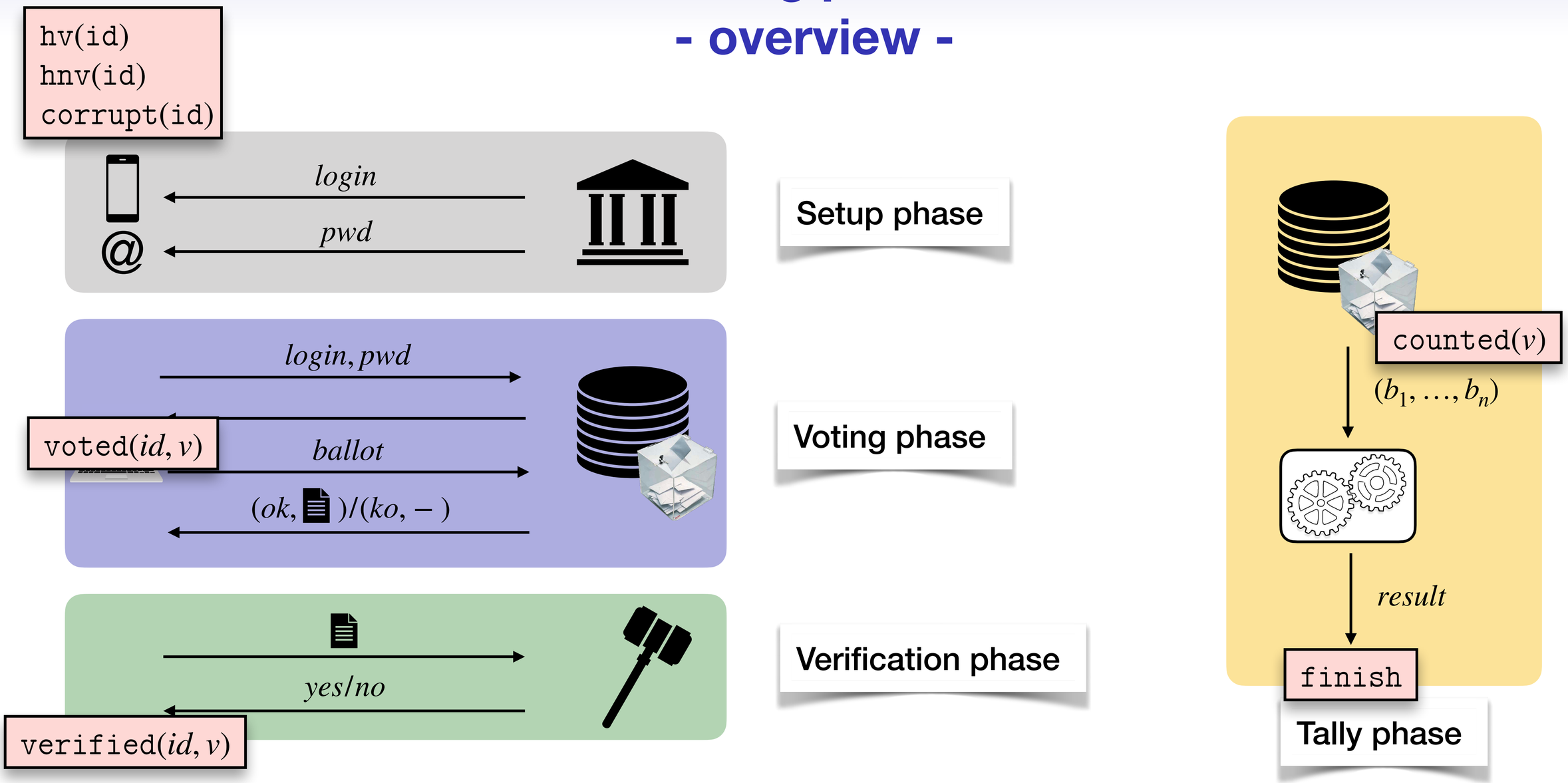

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- $V_{\texttt{HNV}} \subseteq_m \{\!\{v \mid (tr, \tau) \vdash \texttt{voted}(id, v) \text{ and } (tr, \tau') \vdash \texttt{hnv}(id)\}\!\}$
- $|V_{\texttt{D}}| \leq |\texttt{D}|$ where $D = \{id \mid (tr, \tau) \vdash \texttt{corrupt}(id)\}$

Exact characterization of E2E verifiability

Theorem - An evoting protocol satisfies E2E verifiability if and only if it all its traces tr satisfy:

- (Query 1) $\texttt{finish} \wedge \texttt{inj} - \texttt{counted}(x) \Rightarrow \texttt{inj} - \texttt{hv}(\texttt{z}) \wedge \texttt{verified}(z, x)$
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$\boxed{\Leftarrow}$ “more difficult”…

$\Leftarrow$ idea of the proof

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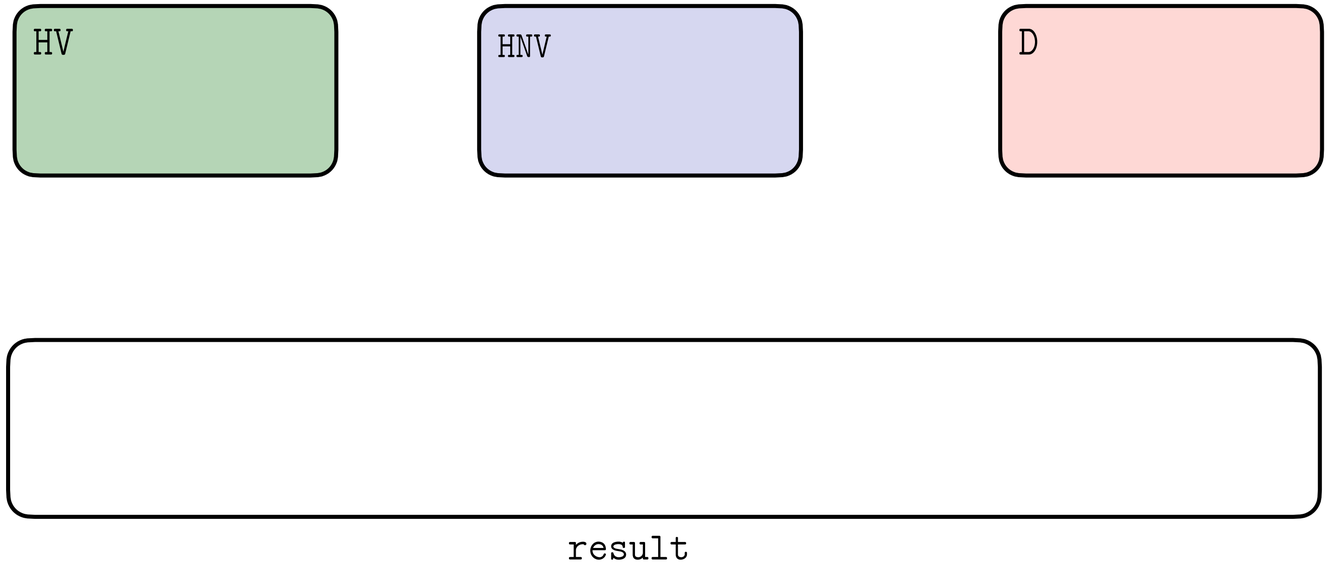

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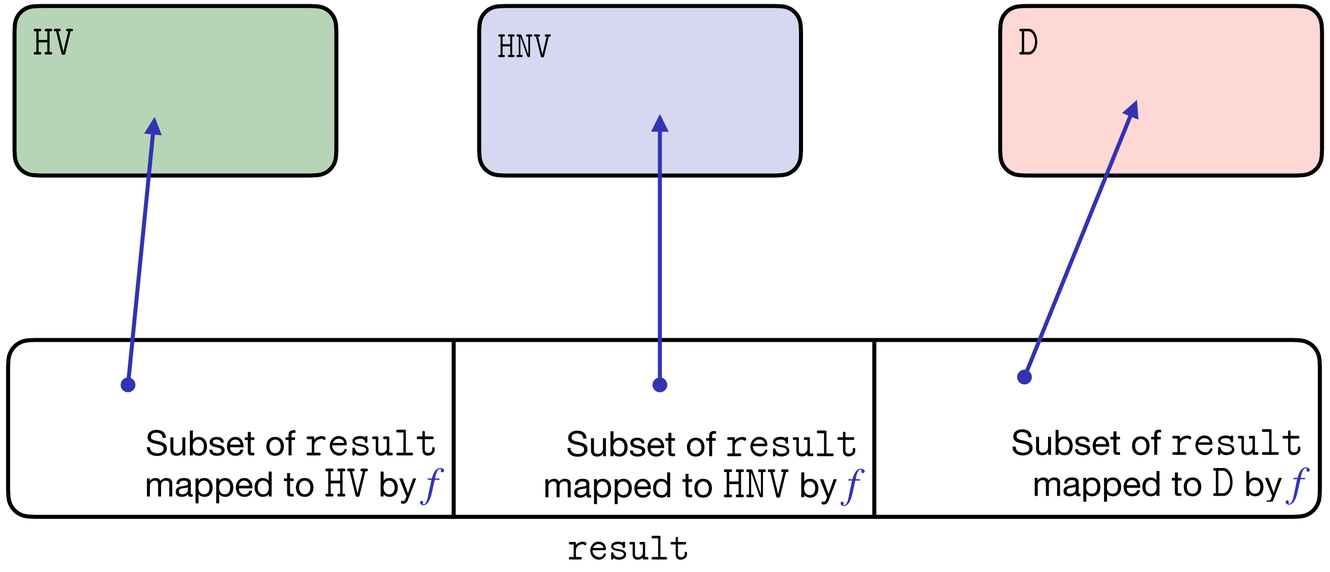

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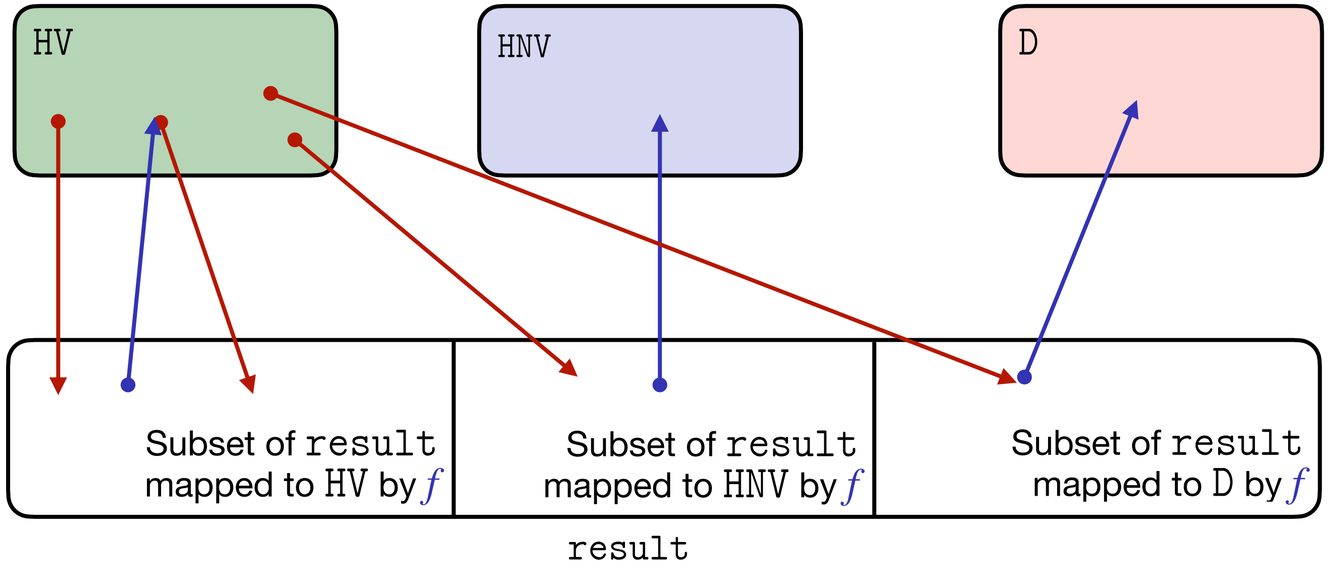

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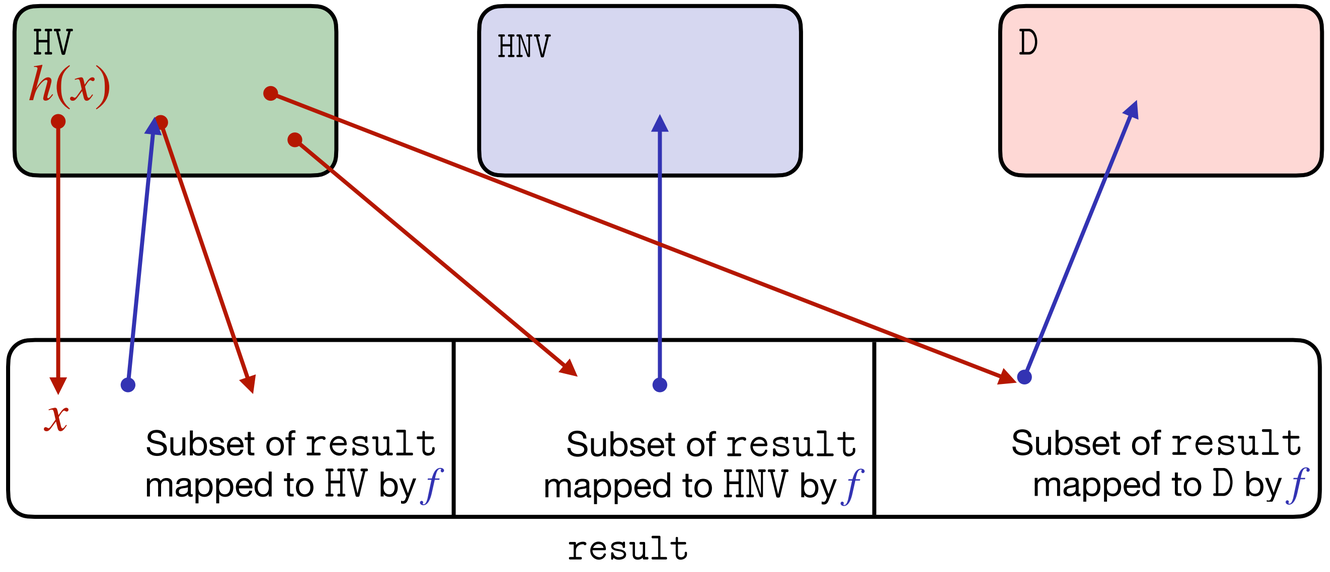
h(x) =

$$\begin{cases} g^{-1}(x) & \text{if } x \in g(\texttt{HV}) \\ & \end{cases}$$



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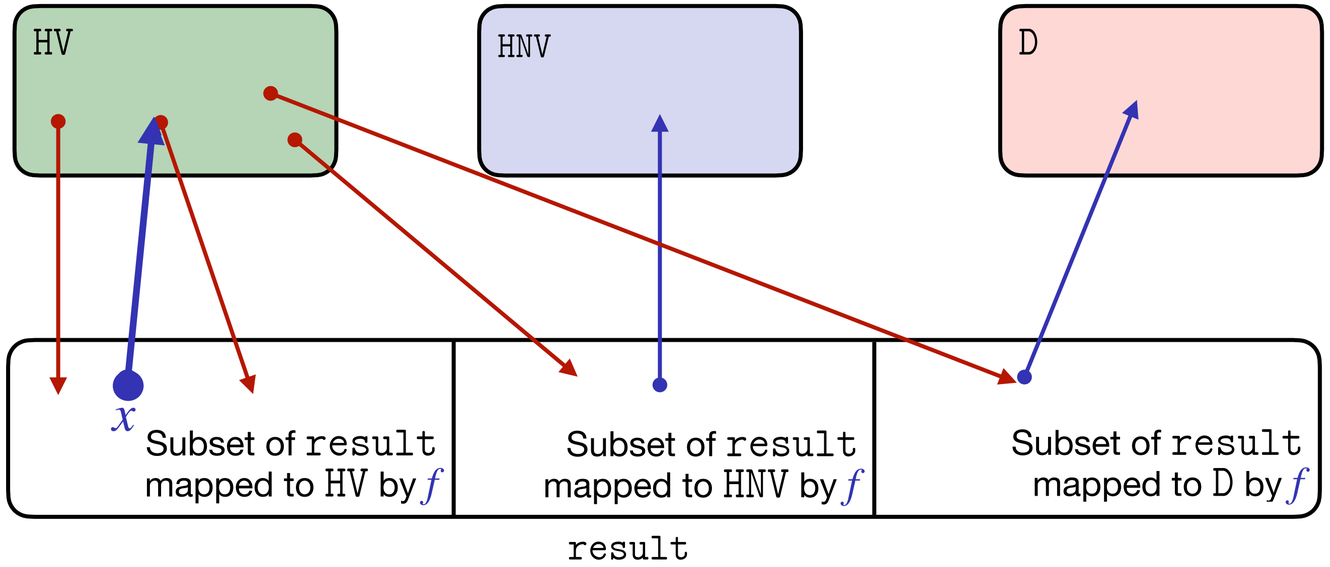

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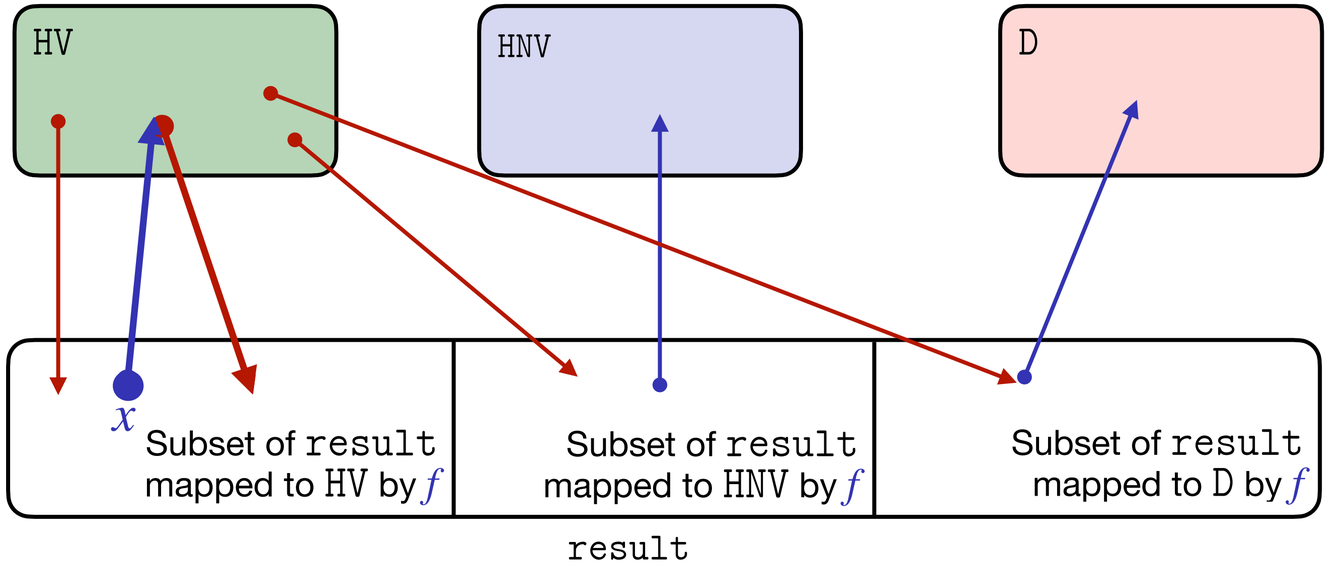

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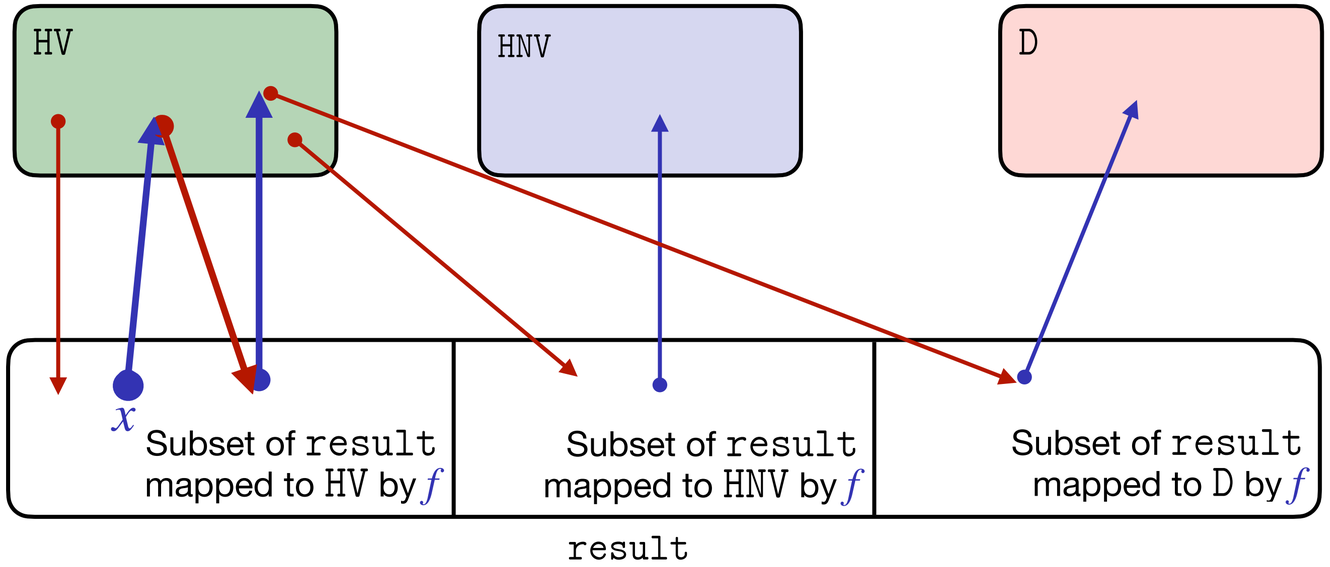

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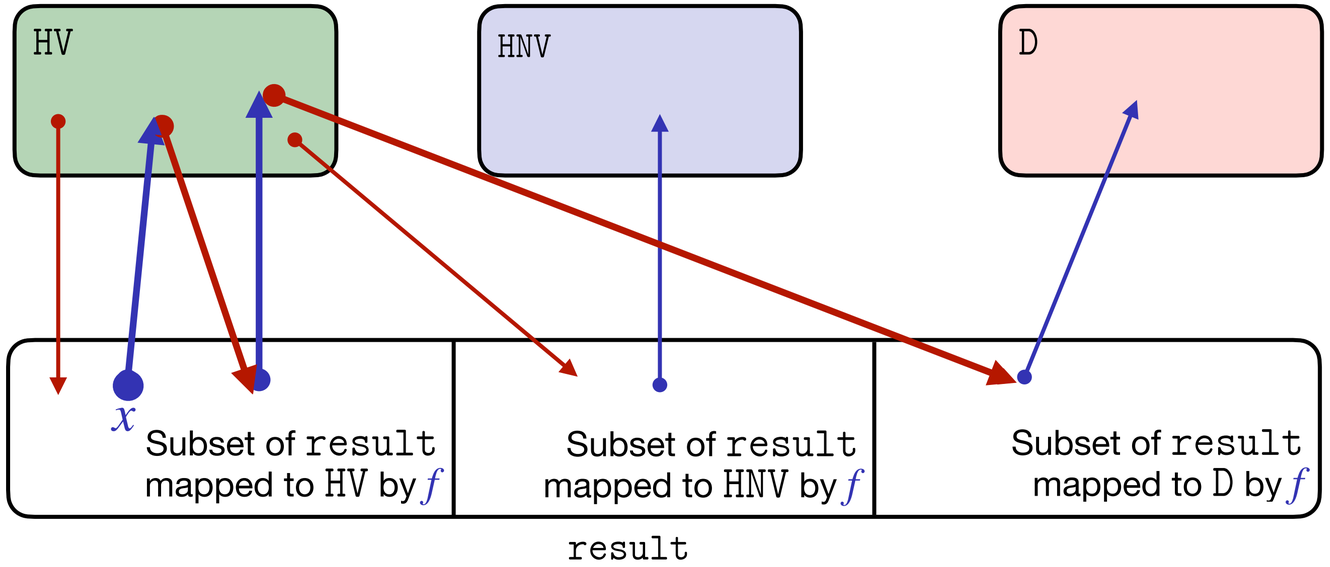

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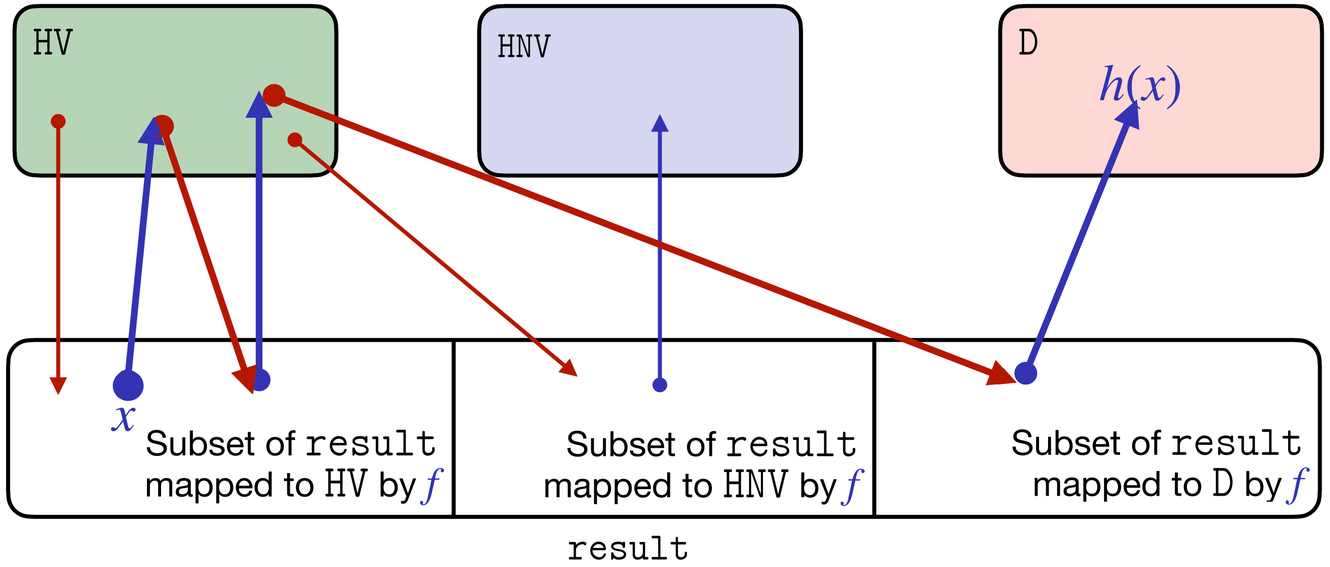

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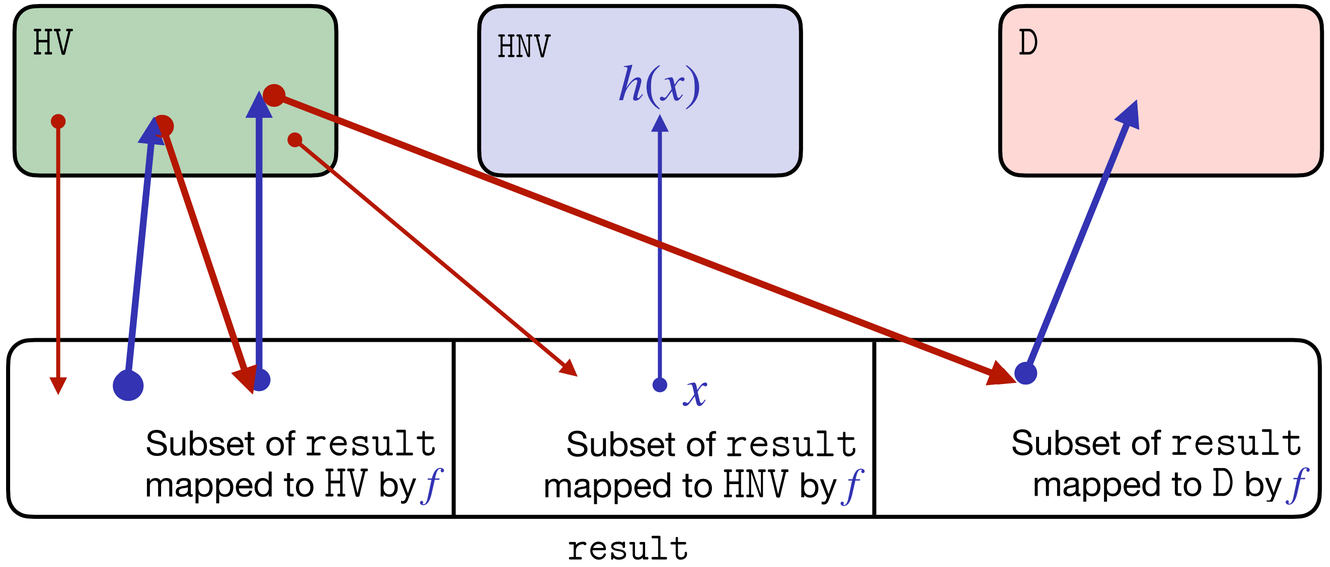


Contributions

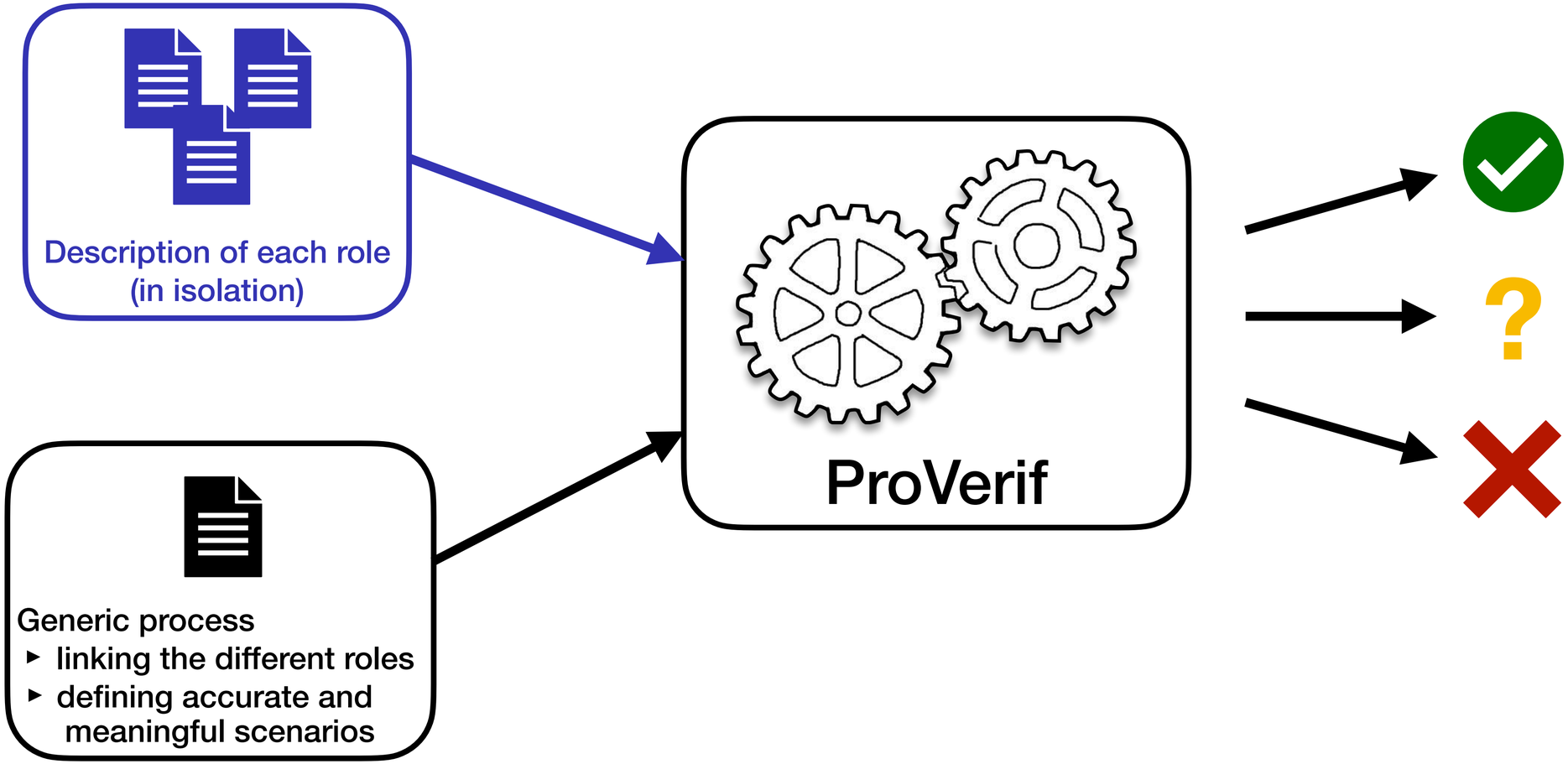
1. **Exact characterization of E2E verifiability**

Theorem - An evoting protocol satisfies E2E verifiability if and only if it satisfies Query 1 and Query 2

2. **A ProVerif framework to analyze evoting protocols**

Applied to several protocols: Helios, Belenios, Swiss Post, CHVote

Our framework



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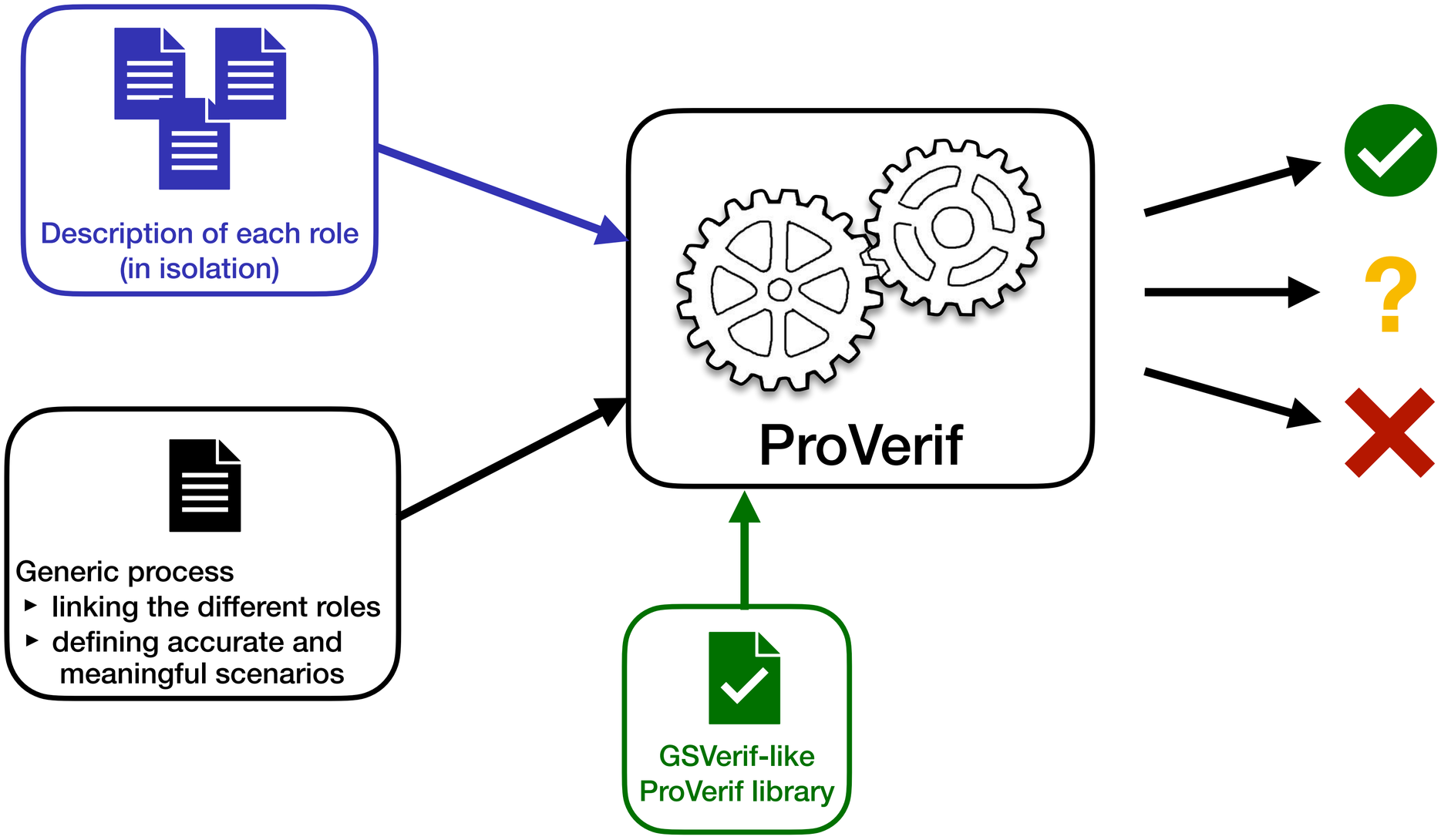

[Cheval et al - CSF'18]

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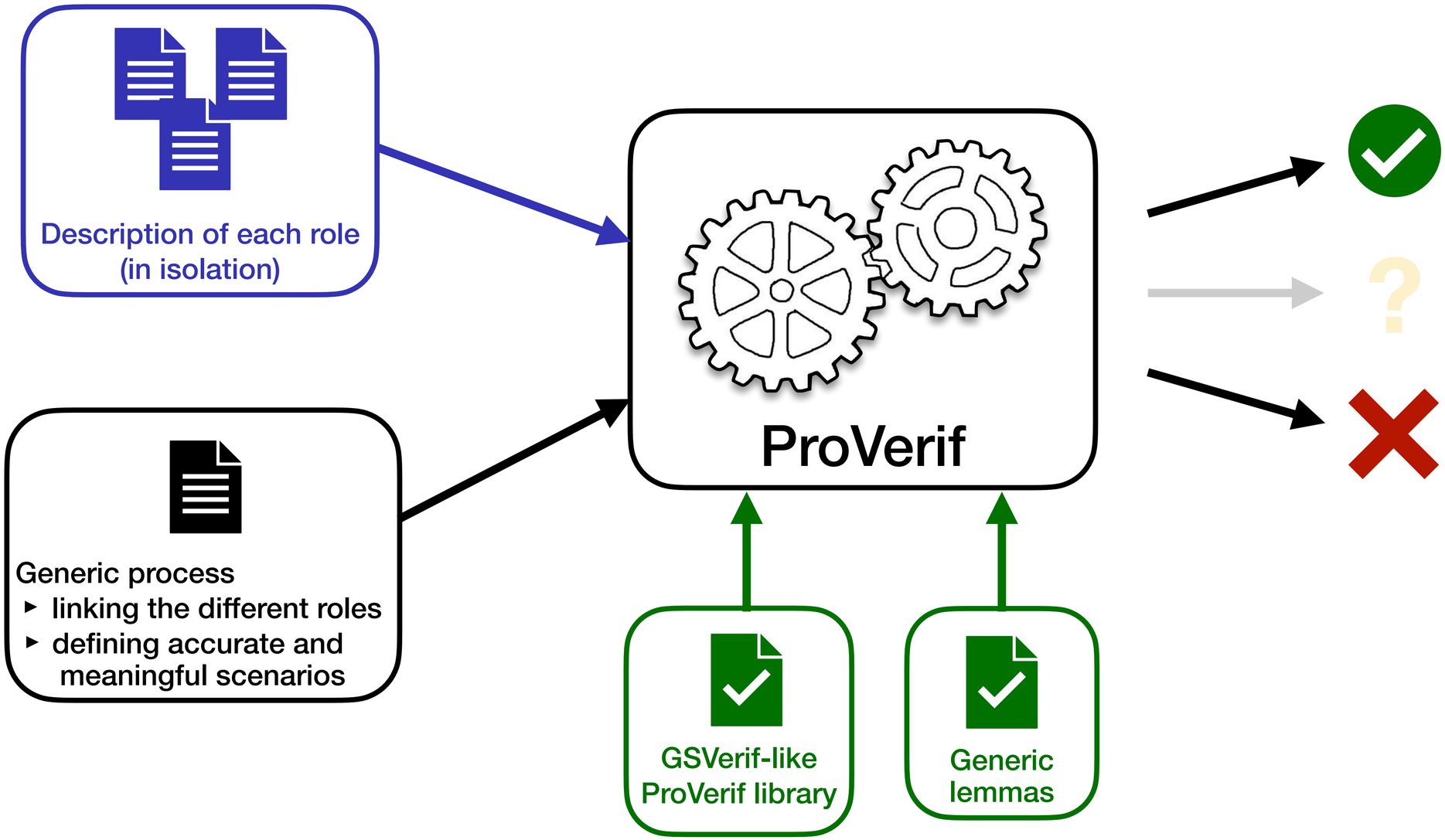

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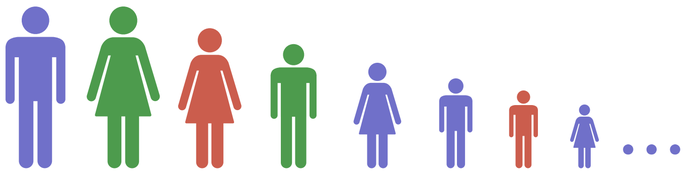
Which scenarios?

A scenario as general as possible

- An arbitrary number of voters…
- who can freely revote…
- with a private bulletin board…
- and a counting function equivalent to the multiset of votes.



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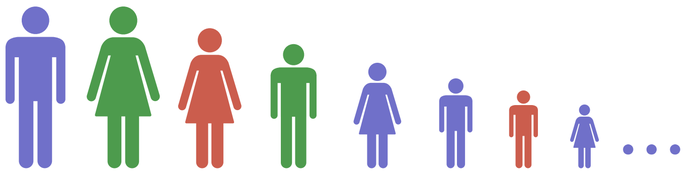

How to extend/adapt this scenario?

- Bound the number of voters → **define a restriction**
- Prevent revote → **define a restriction**
- Model a public bulletin board → **add an output on a public channel**
- **…**

Details

Protocol specific processes

- **12 processes**
- **Setup phase:** 4 processes (how voting data are generated, how they are received by voters, what are their initial knowledge, what is a valid vote)
- **Voting phase:**
  - **Voter:** 2 processes (how a voter casts a vote, how they verify)
  - **Bulletin board:** 5 processes (how to update the bulletin board, what is a valid ballot, how voters are publicly identified)
- **Tally:** 1 process (how to open a ballot)
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Generic processes and libraries

- **8 processes defining the generic scenario**
  (voter registration, voting process, tally, main system…)
- GSVerif-like axioms to manipulate cells, counters, etc
  ➡ 2 new axioms for nested counters and emphasize term freshness
- 8 well-crafted lemmas (27 queries) to improve termination and accuracy

Applications

Protocol	Origin of the files	Voter	Registrar (setup)	Server (1 CCR/M)	E2E verifiability
Helios (toy)	(new files)	😇	—	😇	✔ 16s
Belenios (tally)	(existing personal files)	😇	😇 😈	😈 😇	✔ 24s
Belenios (last)	(existing personal files)	😇	😇	😇	✘ 5s
Belenios-counter (last)	(existing personal files)	😇	😇	😇	✘ 8s
Belenios-hash[1] (last)	(new files)	😇	😇 😈	😈 😇	✔ 62s
Swiss Post	(Swiss Post gitlab[2])	😇	😇	😇	✔ 58s
CHVote	[Bernhard et al - 2018]	😇	😇	😇	✔ 17s

[1]inspired by [Baloglu et al - EVoteID 2021]

[2]https://gitlab.com/swisspost-evoting/e-voting/e-voting-documentation/-/tree/master/Symbolic-models

Conclusion

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Future work

- Extend the framework to analyze vote secrecy
- Extend GSVerif with the new invariants introduced in this work
- Improve the modeling of the tally:
  - consider counting functions different from the multiset of votes (e.g., Condorcet, Single Transferable Vote, d'Hondt method)
  - provide a more accurate model of the homomorphic or mixnet tally