OUTLINE

~~I. Introduction~~

~~II. Background~~

~~III. Fitting a Model~~

~~IV. Supervised Learning~~

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VI. Convolutional Neural Networks

- Definition
- Why ConvNets ?
- CNN layers
- Famous examples

Convolutional Neural Networks

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But what ***is*** (discrete) convolution?

Discrete Convolution (1D Case)

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- Given two sequences $x[n]$ and $k[n]$, the convolved sequence $a[n]$ is defined by the following linear operation:

$$a[n] = \sum_{\tau=-\infty}^{+\infty} x[\tau]k[n-\tau], \quad \text{denoted by} \quad \boldsymbol{a} = \boldsymbol{x} * \boldsymbol{k}$$

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- **Ex:** moving average, smoothing, low-pass, band-pass, etc.

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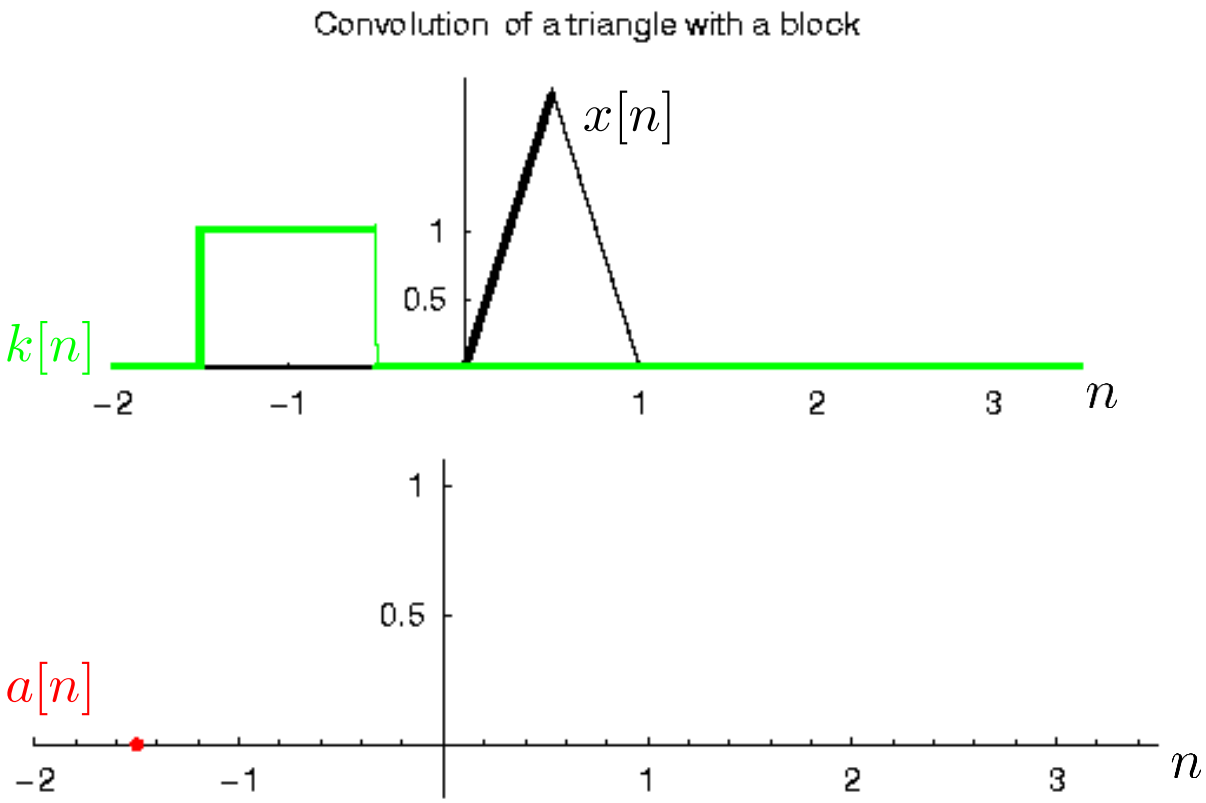

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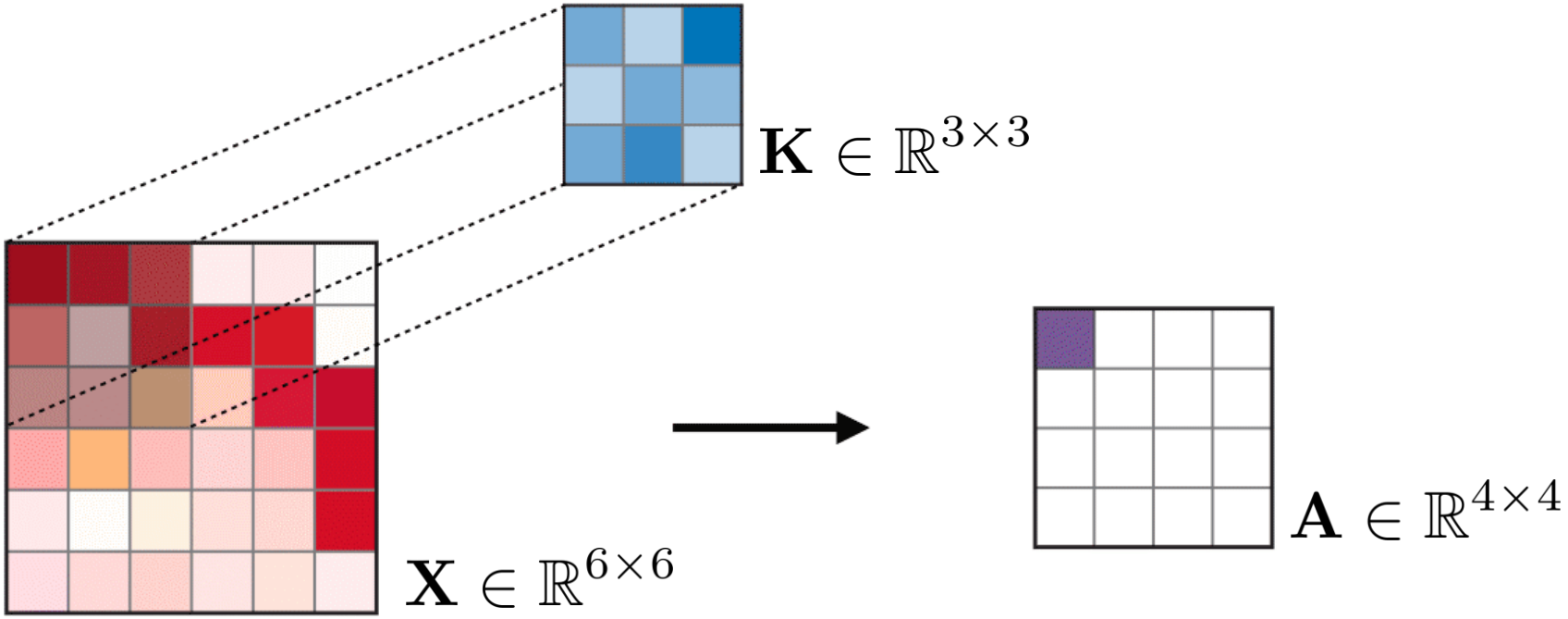

Discrete Convolution (2D Case)

- **Ex:** edge detection, sharpening, blurring, ….

$$\boldsymbol{K} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{K} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad \boldsymbol{K} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$



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$$\boldsymbol{K} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \boldsymbol{K} = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \quad \boldsymbol{K} = \frac{1}{16}\begin{bmatrix} -1 & -2 & -1 \\ -2 & 28 & -2 \\ -1 & -2 & -1 \end{bmatrix}$$

Convolution vs. Correlation

- The convolution operation (using proper ***zero-padding*** in the discrete finite case) is **associative** and **commutative**:

$$\boldsymbol{a} * \boldsymbol{b} = \boldsymbol{b} * \boldsymbol{a}, \quad \boldsymbol{a} * (\boldsymbol{b} * \boldsymbol{c}) = (\boldsymbol{a} * \boldsymbol{b}) * \boldsymbol{c}$$

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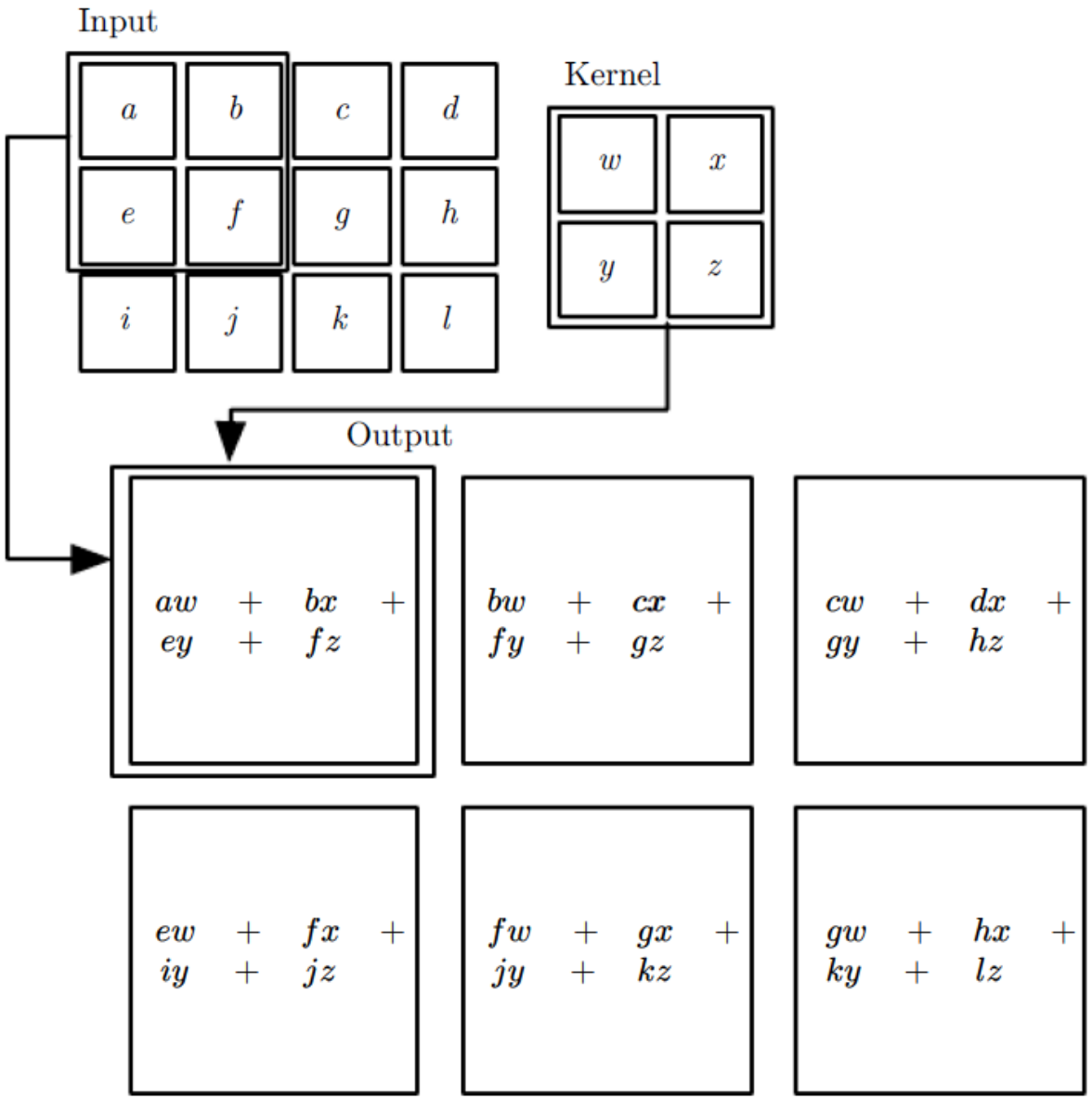
- In practice, since the kernel elements are **learned parameters**, it makes no difference (but worth keeping in mind!)

The maths:


Input
a b c d
e f g h
i j k l
Kernel
w x
y z
Output
aw + bx +
ey + fz
bw + cx +
fy + gz
cw + dx +
gy + hz
ew + fx +
iy + jz
fw + gx +
jy + kz
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Why Convolutional Neural Networks?

- Convolution leverages three key machine learning concepts:

  1) Sparse connectivity

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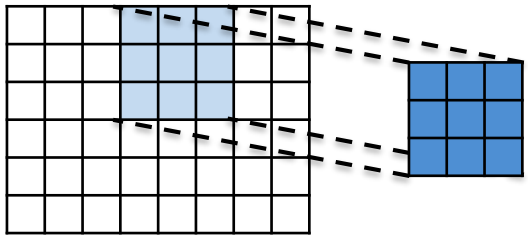
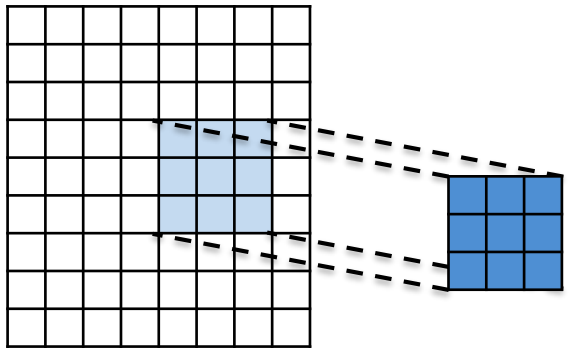
- Moreover, it provides a means for handling **variable size inputs**. The same kernel can be sled on signals/images of variable sizes:



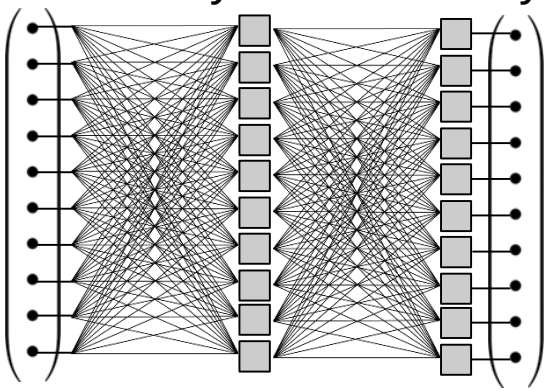


Why Convolutional Neural Networks?

1) Sparse connectivity

Two conventional fully connected layers:

$\boldsymbol{x} \in \mathbb{R}^N$



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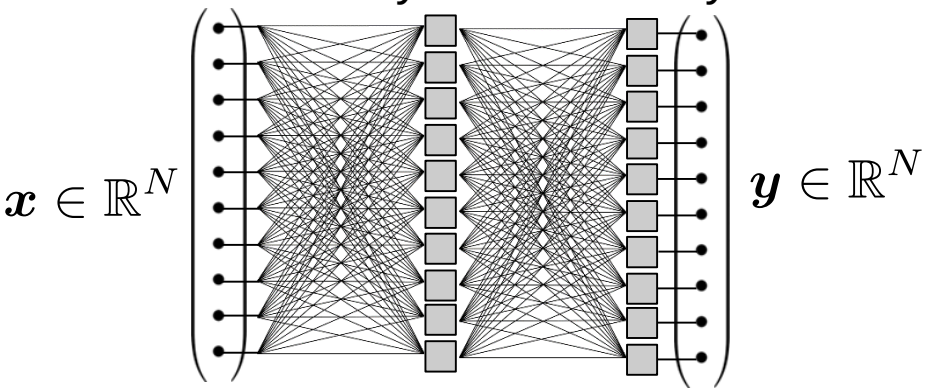

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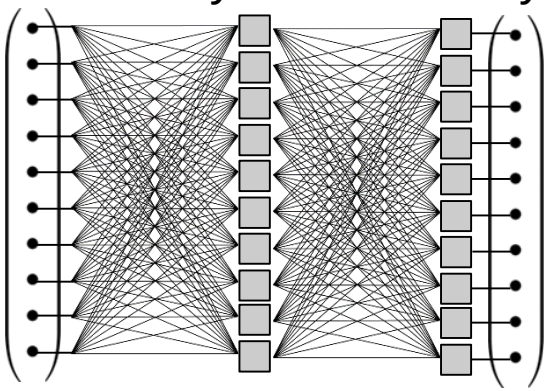
$\boldsymbol{x} \in \mathbb{R}^N$  $\boldsymbol{y} \in \mathbb{R}^N$

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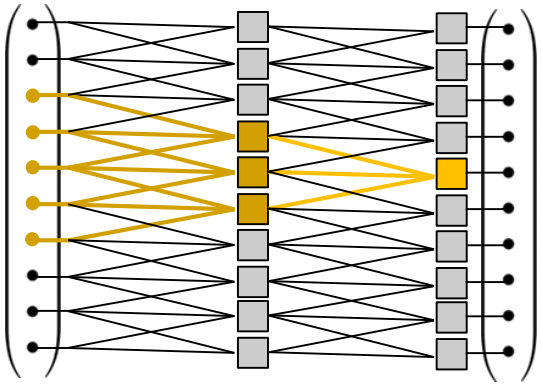
$\boldsymbol{x} \in \mathbb{R}^N$  $\boldsymbol{y} \in \mathbb{R}^N$

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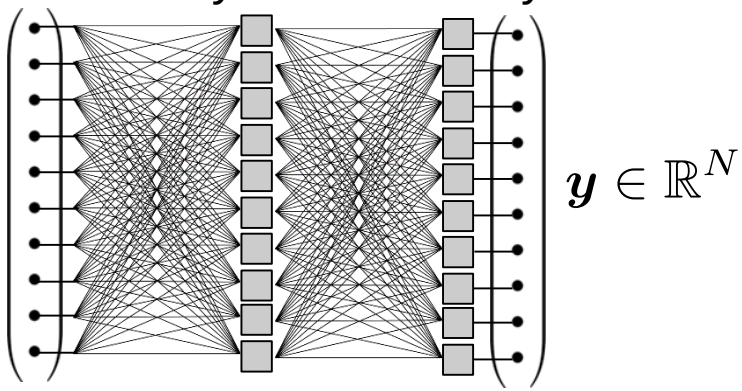

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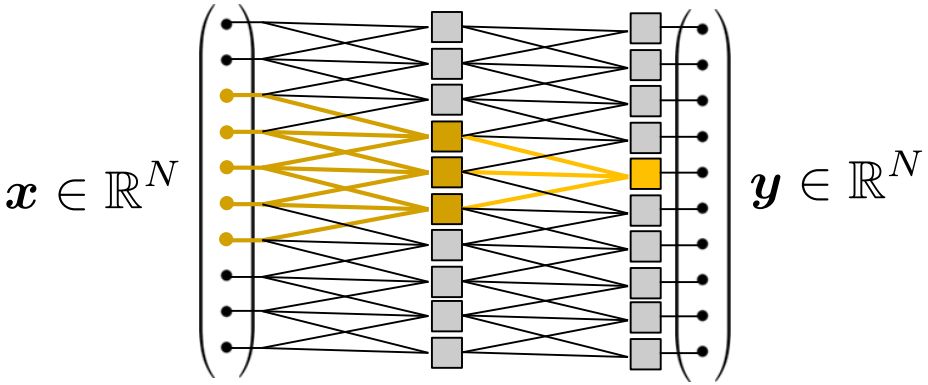

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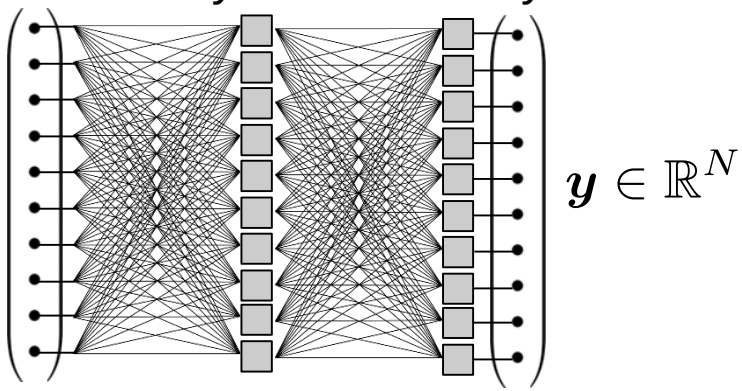

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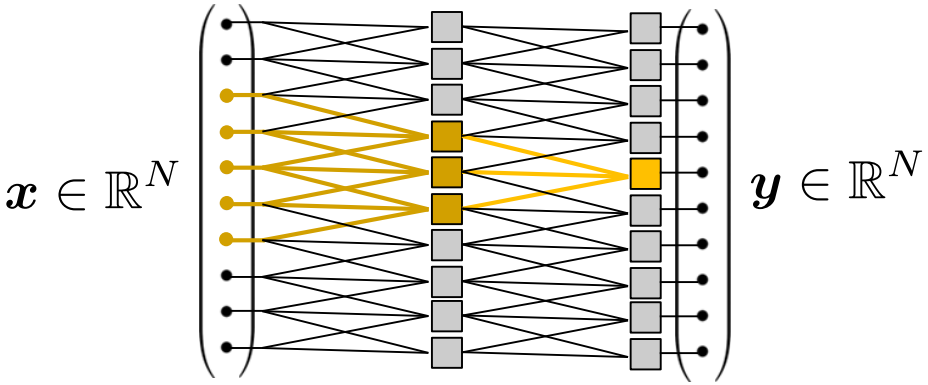

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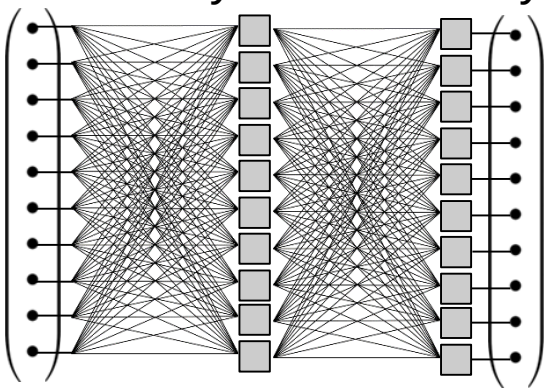
$\boldsymbol{x} \in \mathbb{R}^N$  $\boldsymbol{y} \in \mathbb{R}^N$

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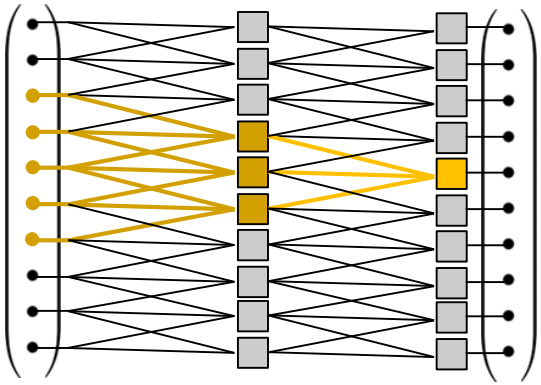
$\boldsymbol{x} \in \mathbb{R}^N$  $\boldsymbol{y} \in \mathbb{R}^N$

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Less connections** = **smaller model** = **less overfitting** & **faster computation

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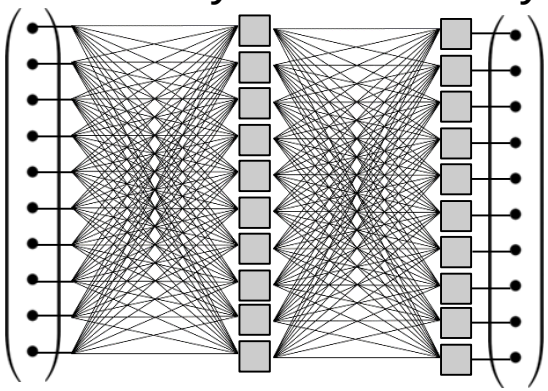
2) Parameter sharing

Two conventional fully connected layers:

$\boldsymbol{x} \in \mathbb{R}^N$  $\boldsymbol{y} \in \mathbb{R}^N$

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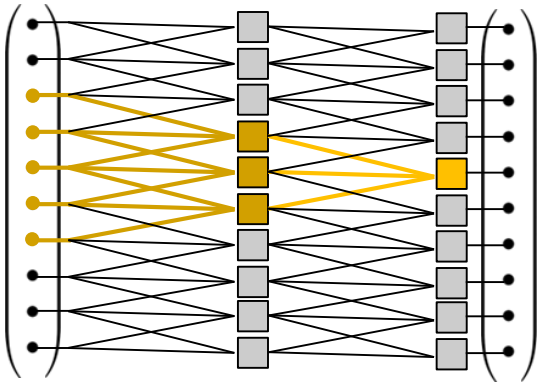
$\boldsymbol{x} \in \mathbb{R}^N$  $\boldsymbol{y} \in \mathbb{R}^N$

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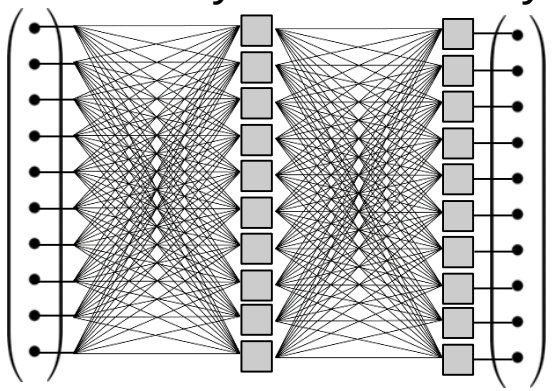

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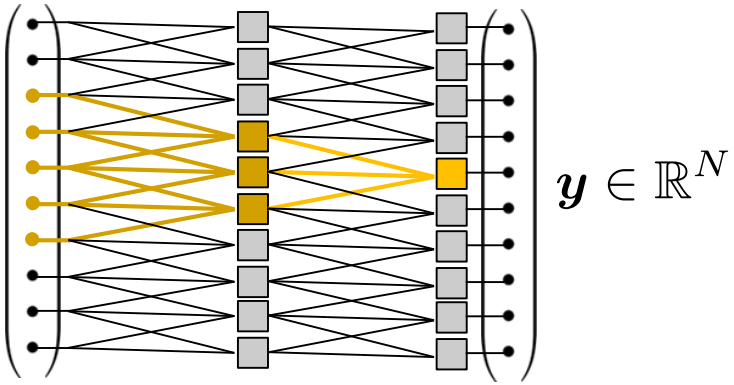

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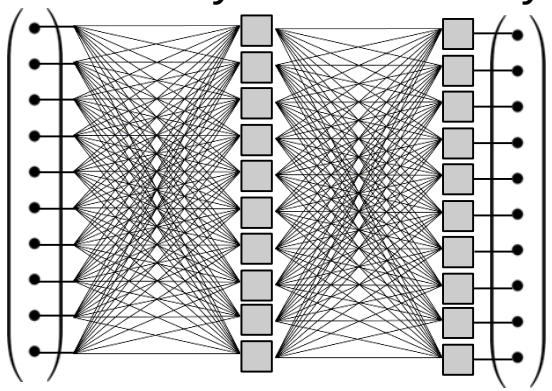

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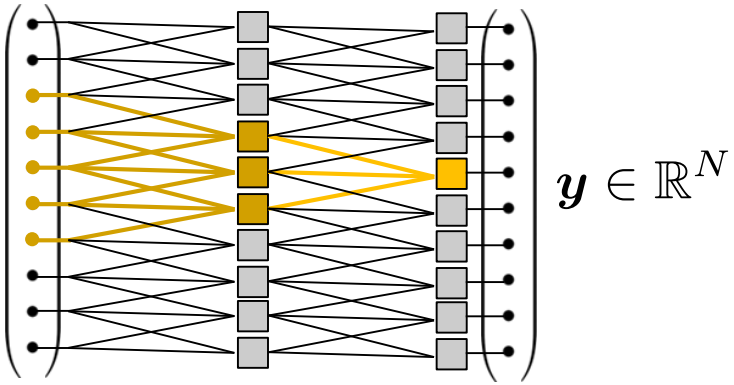
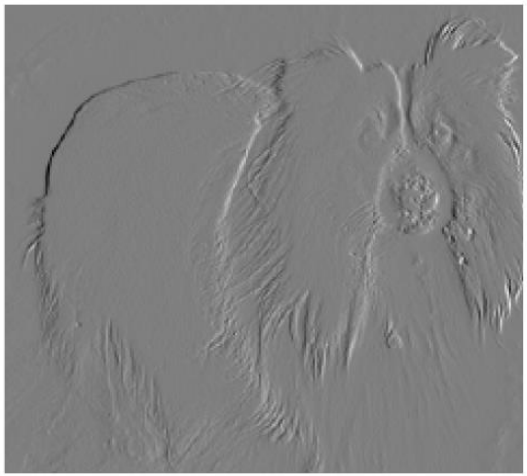


- Parameters **shared** (or ***tied***) across every position of the input
- ➡ $\mathcal{O}(LS)$ parameters
- Even **smaller model**, even **less overfitting**

Why Convolutional Neural Networks?

Computational efficiency



$$\mathbf{K} = \begin{array}{|c|c|}\hline -1 & 1 \\ \hline \end{array}$$

- Example: horizontal edge detection (280 x 320 pixels)
  - **Convolutional:** 1 x 2 kernel, 280 x 319 ≈ 2 x 10^5 operations
  - **Fully connected**: 280 x 320 x 280 x 319 ≈ 8 x 10^9 weights ≈ 16 x 10^9 operations

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- Hence, the **feature map** can be interpreted as indicating **where** some features (matching the kernels) appear in the input

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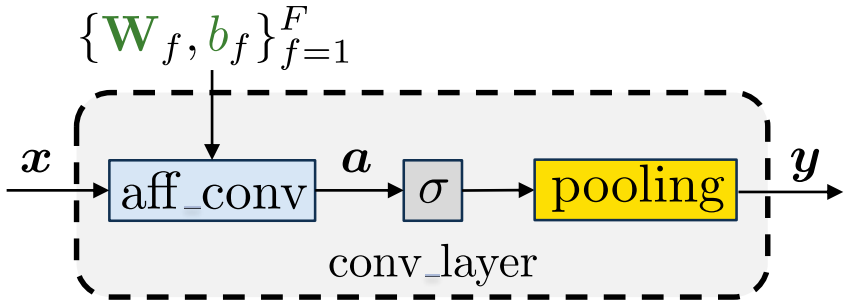
- **Note**: Convolution is **not** equivariant to rotation or scaling
- **Exercise:** Supposed my input signal is a list of incomes in a given city, sorted in ascending order? Supposed my input is a 2D map of the humidity of the soil around a given location?

CNN layers

- A **CNN layer** typically consists of 3 stages



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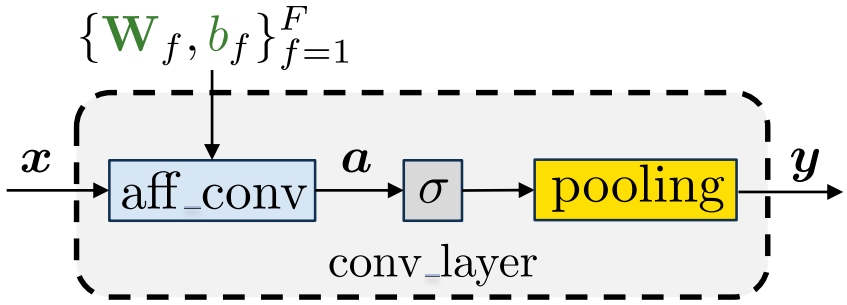

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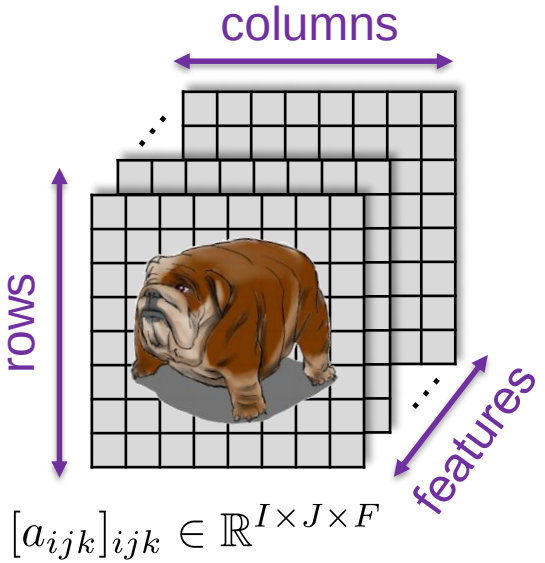

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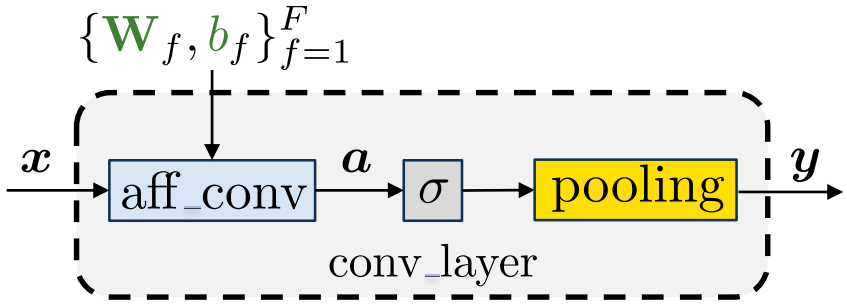

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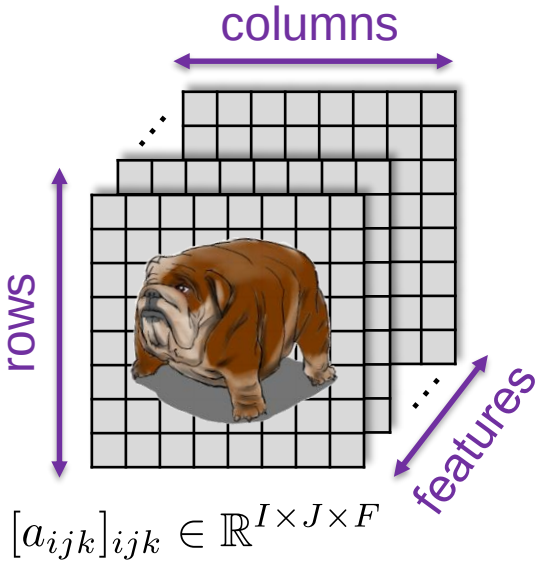

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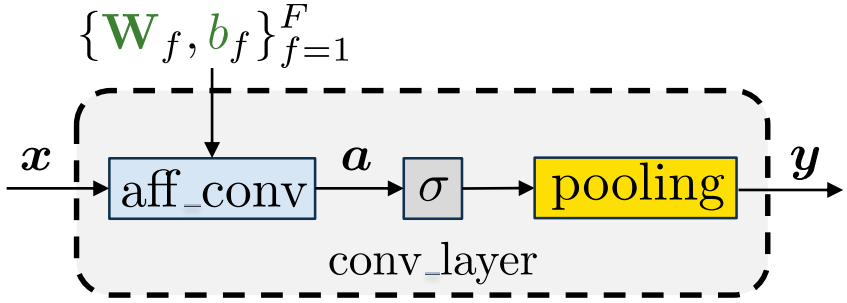

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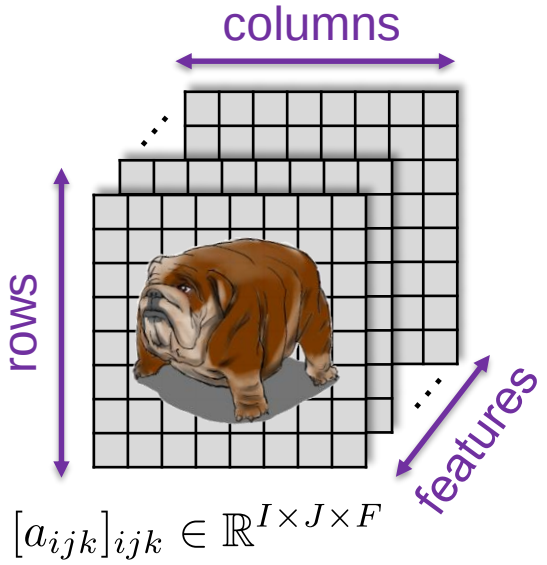

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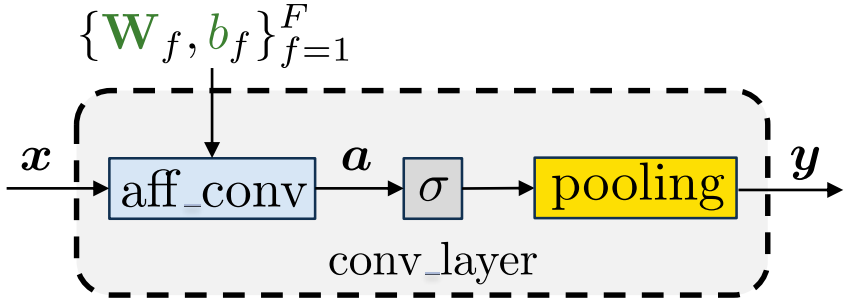

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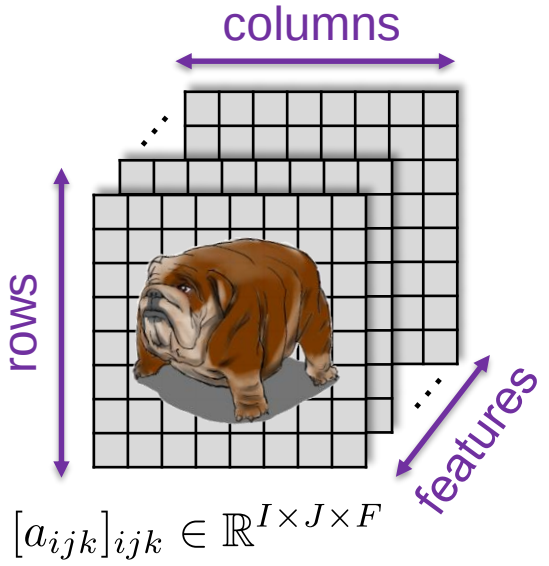
- There is then a **pooling** stage, that replaces every **neighborhood** in a given feature map by a **summary statistics** (ex: mean or max)



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- **Counter-example:** to denoise an image, we must preserve the location of the features. In that situation pooling is not desirable..

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- Also useful when dealing with **variable size** input: adjust the pooling and stride to obtain a fixed-size output

Pooling

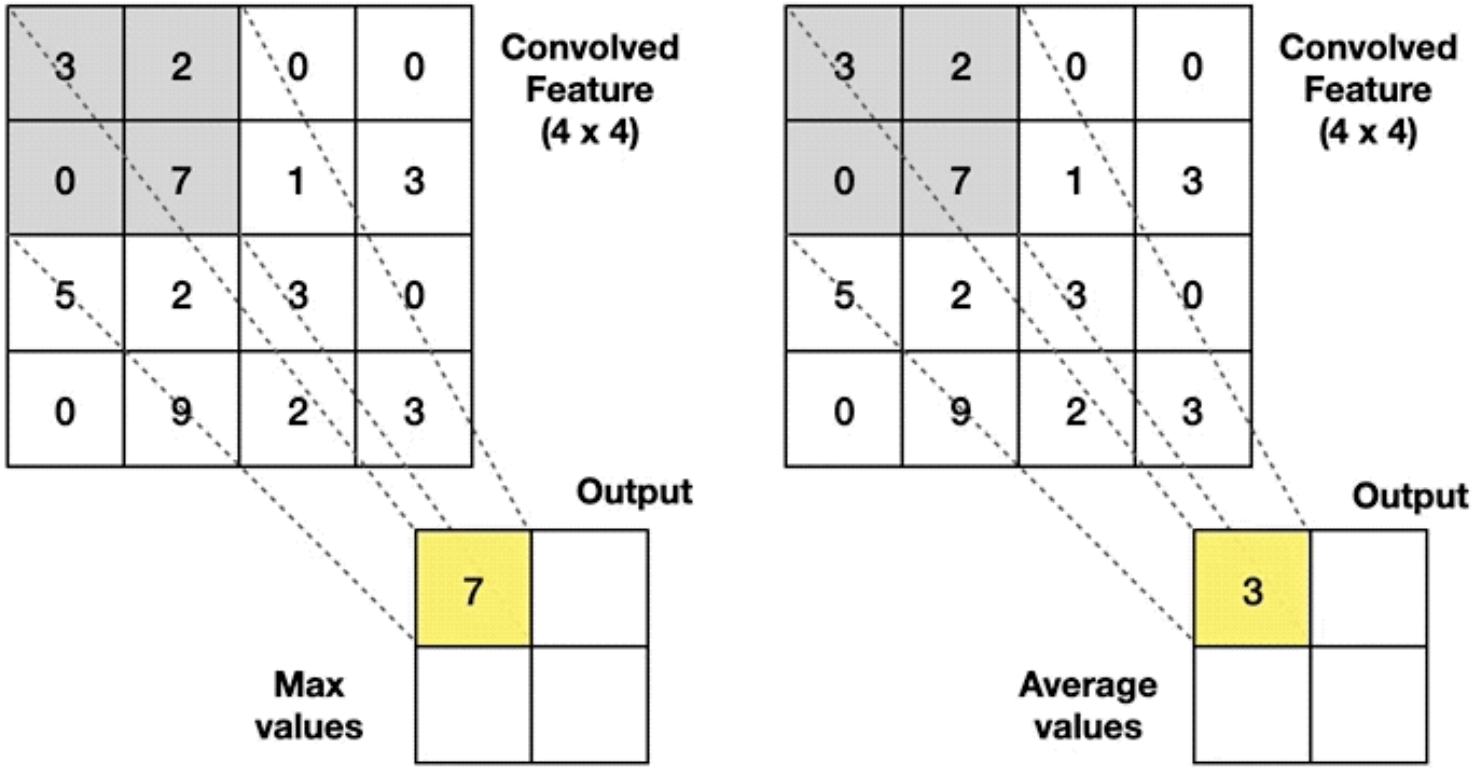
Max Pooling

Take the **highest** value from the area covered by the kernel

Average Pooling

Calculate the **average** value from the area covered by the kernel

Example: Kernel of size 2 x 2; stride=(2,2)



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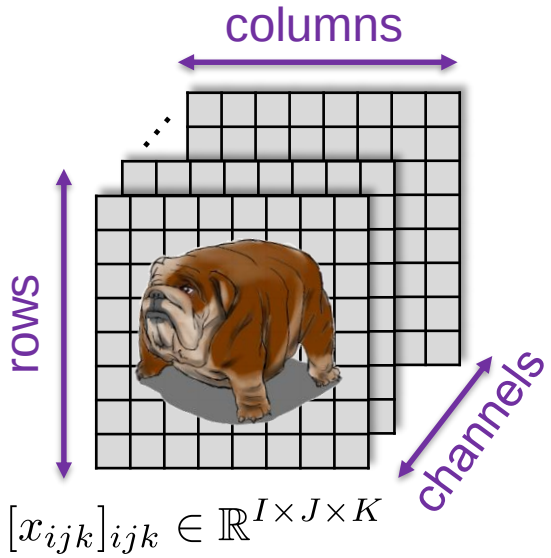

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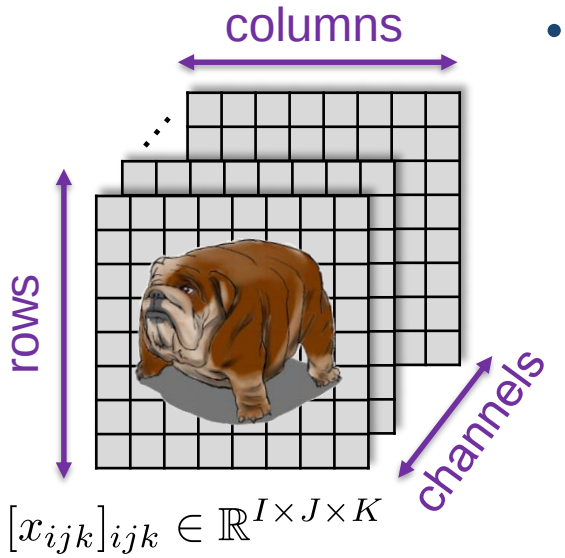

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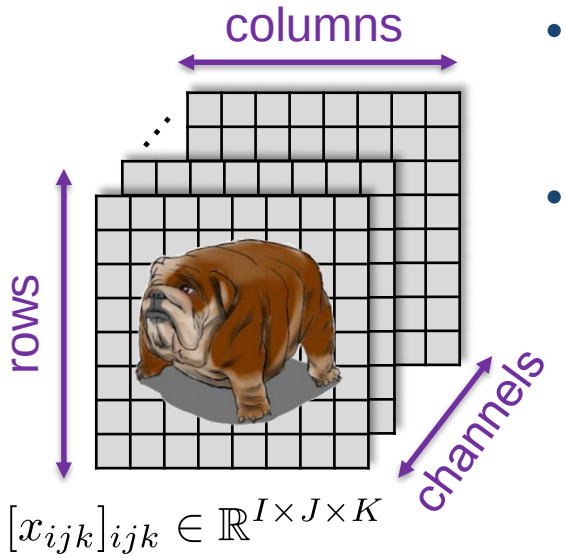

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$$a_{ijf} = (\mathbf{X} *_{1,2} \mathbf{W})_{ijf} = \sum_k \sum_m \sum_n x_{ijk} w_{i+m,j+n,f,k}$$

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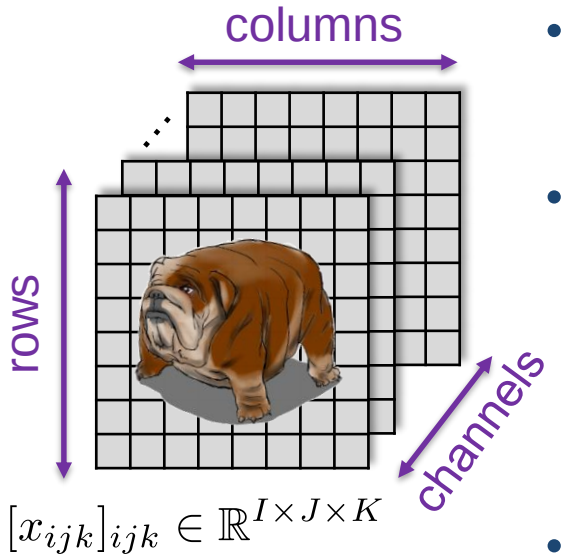

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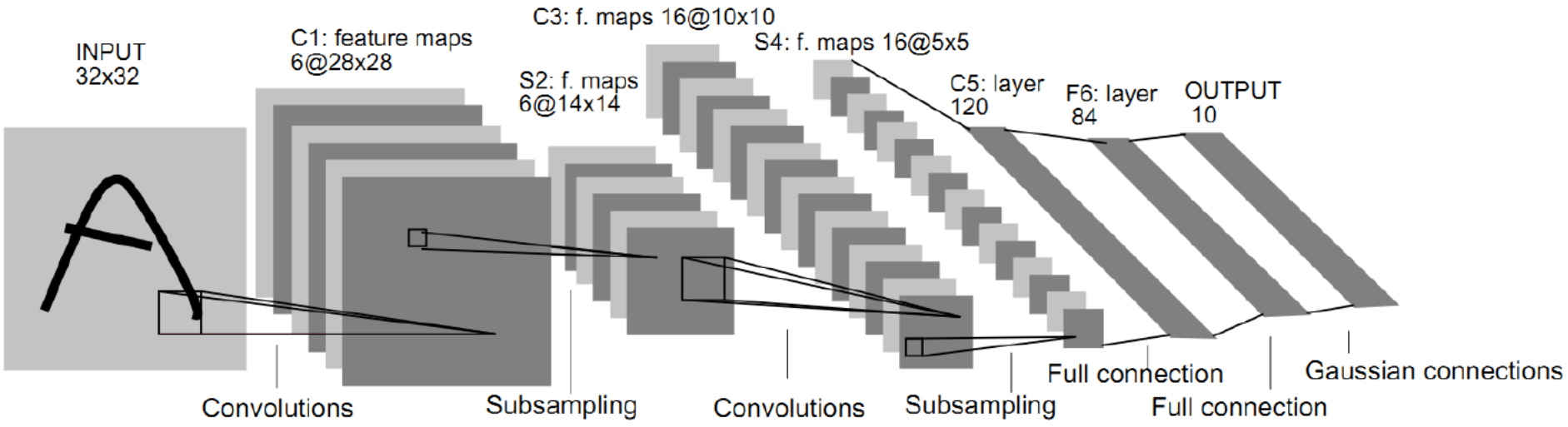
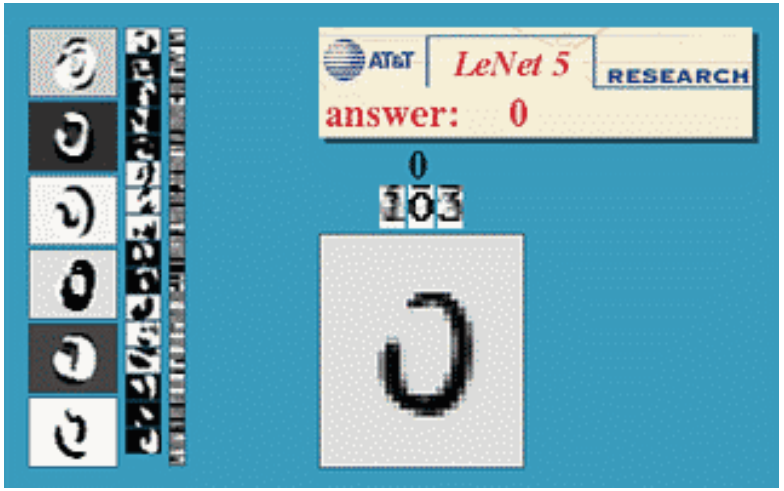
- Each **entry** of the kernel acts as a $F \times K$ matrix: the list of kernels $\mathbf{W}$ form a **4-way** tensor

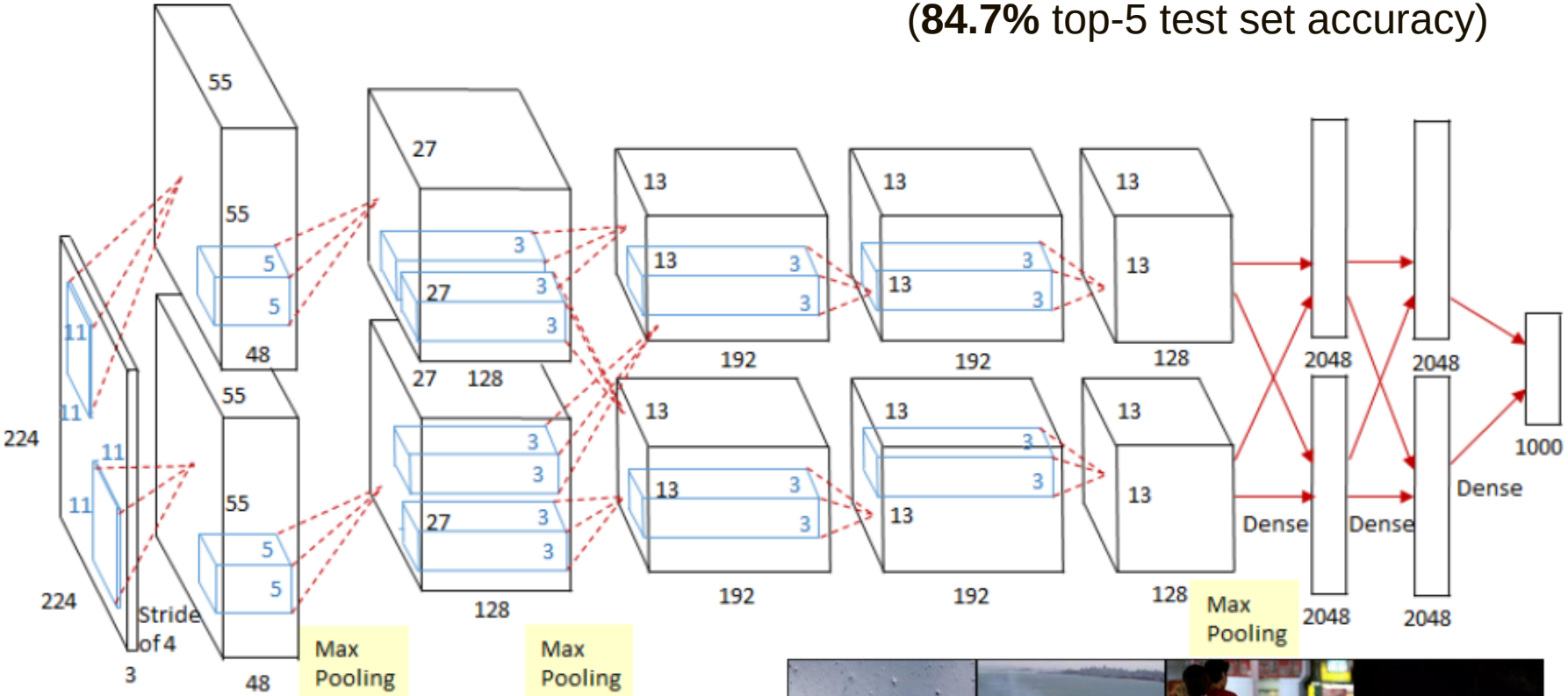
Famous example: LeNet-5 for digit recognition (1989)





Famous example: AlexNet for ImageNet classification (2012)

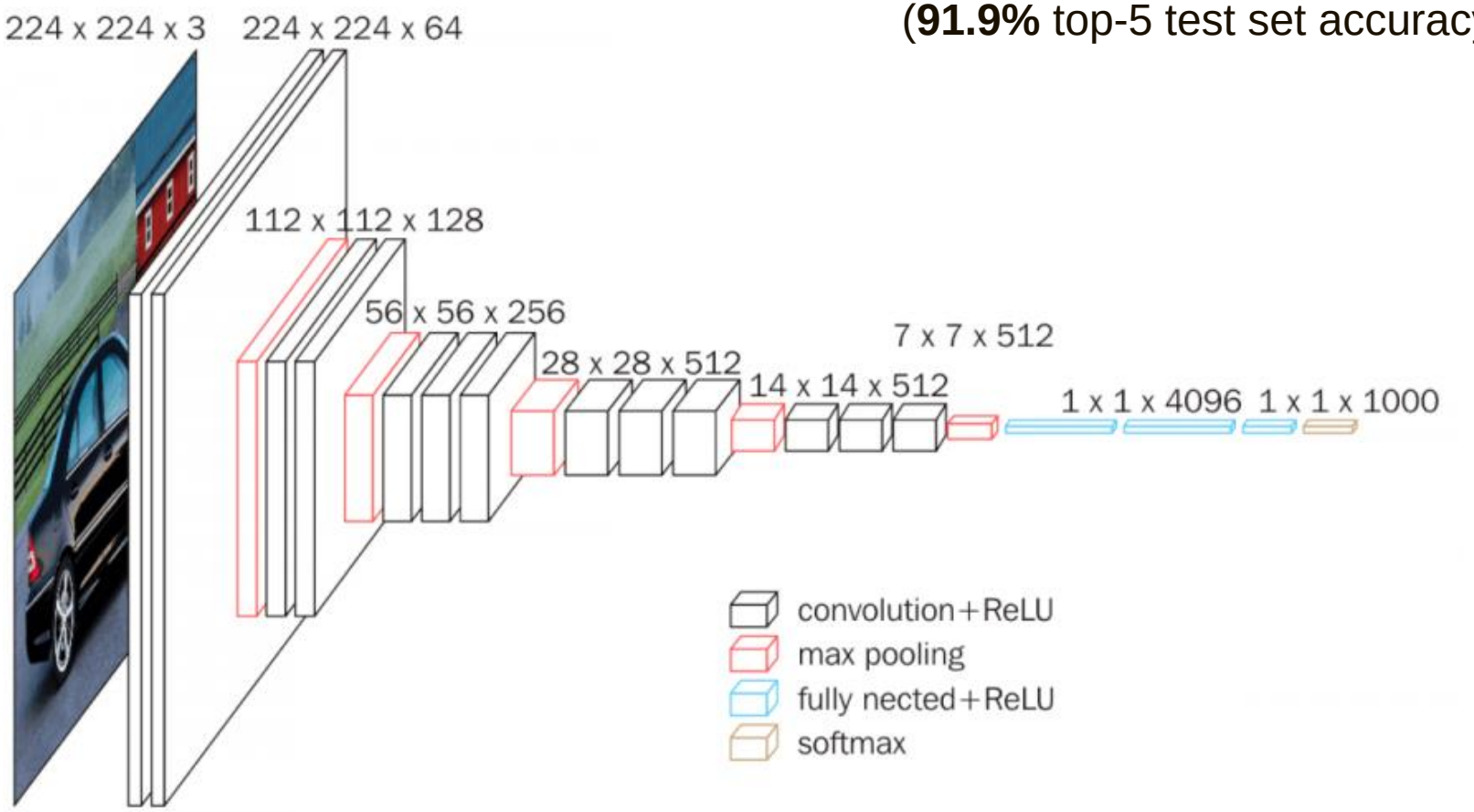
(**84.7%** top-5 test set accuracy)



Proven not so useful in the end

Famous example: VGG-16 for ImageNet classification (2014)

(**91.9%** top-5 test set accuracy)



- Uses only 3x3 kernels, but more depth (16 vs. 7 layers)

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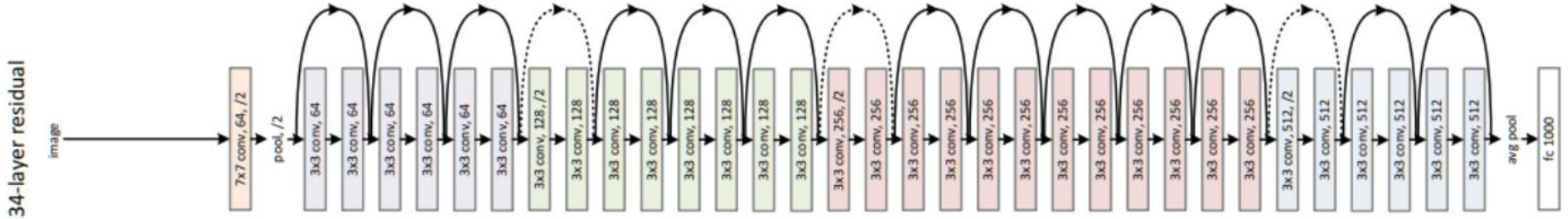
94.3% top-5 for ResNet-152 / **95.2%** for ResNet-200 (2016)



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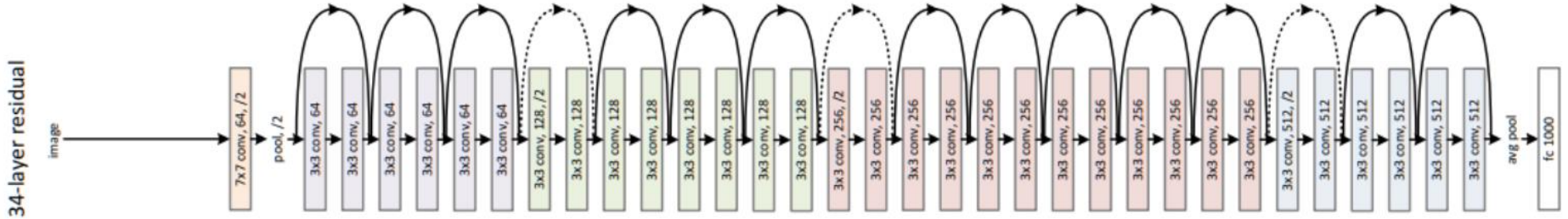

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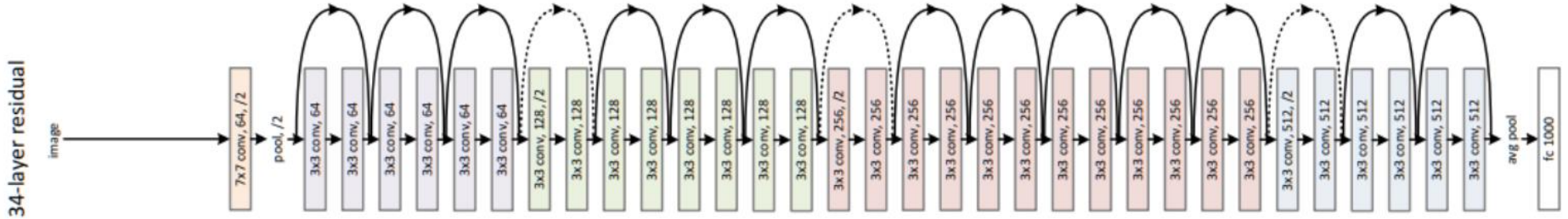

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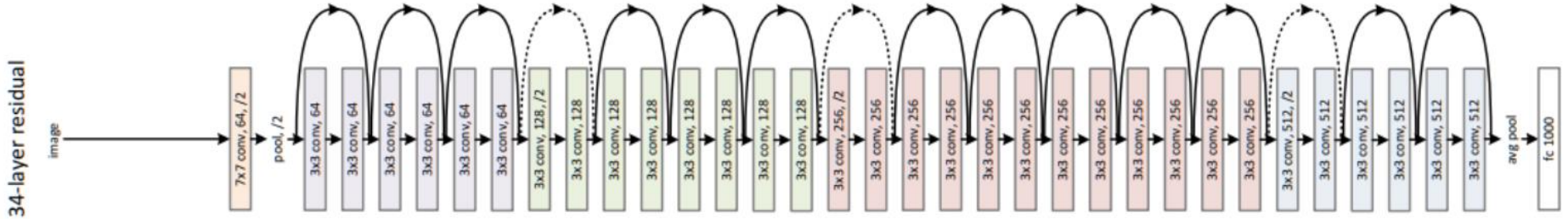

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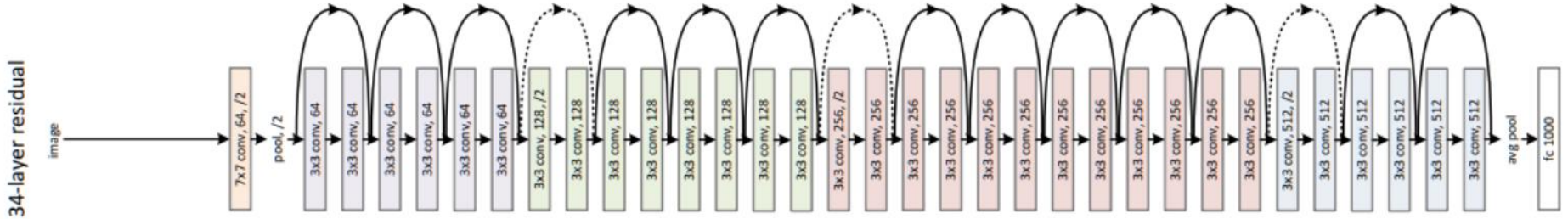

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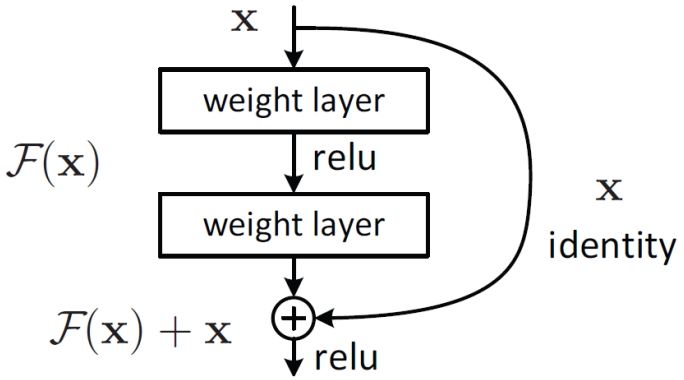
- **Recall:**



$$G_{b^1} = \sigma'(a_t^1) \odot \mathbf{W}^{2\top} \sigma'(a_t^2) \odot \mathbf{W}^{3\top} \ldots \sigma'(a_t^3) \odot \mathbf{W}^{L\top} \sigma'(a_t^L) \odot \nabla_{\boldsymbol{x}^L} \ell(\boldsymbol{x}_t^L, \boldsymbol{y}_t)$$

OUTLINE

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A.I., Machine Learning, Deep Learning: What, How, Why and When

II. Background
Tensors and Multivariate Calculus

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Optimization techniques, Backpropagation, Gradient Descent, PyTorch

IV. Supervised Learning
Regression, Classification, Loss Design, Over & Underfitting

V. Unsupervised Learning
From K-means and PCA to Deep Clustering and Autoencoders

VI. Convolutional Neural Networks
Definition, Why, Layers, Famous Examples