Partially Observable MDPs

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Outline

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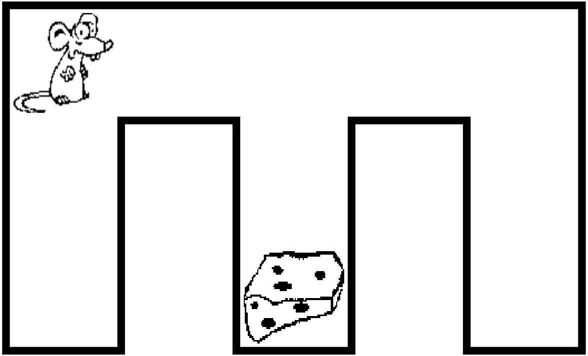
Cheese Maze



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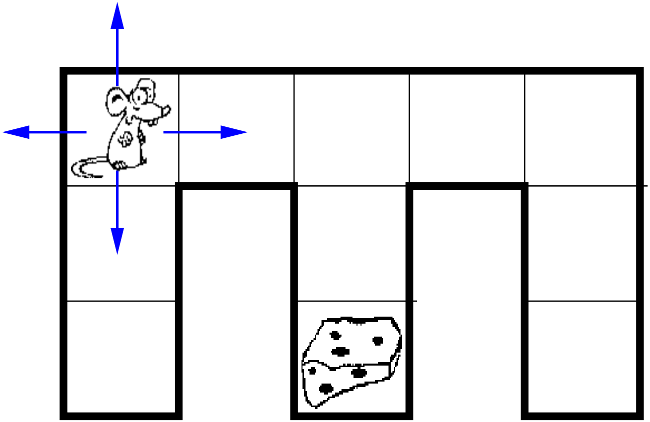
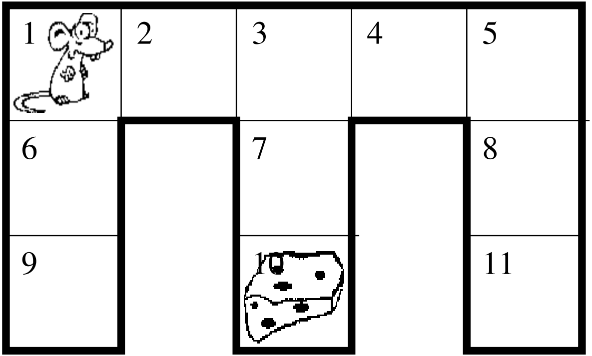


Cheese Maze

Observation : in state 1, the mouse only observes upper and left walls.



States 1–11

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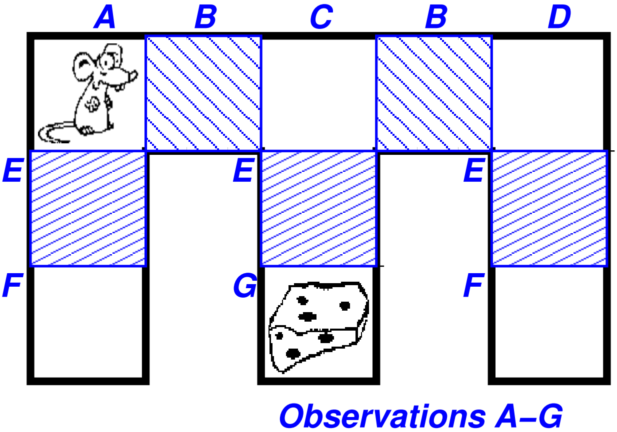

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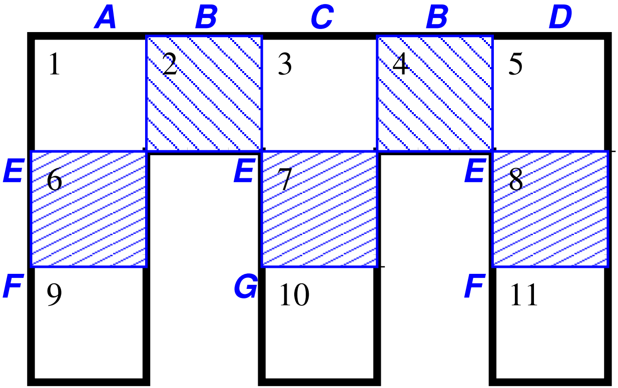
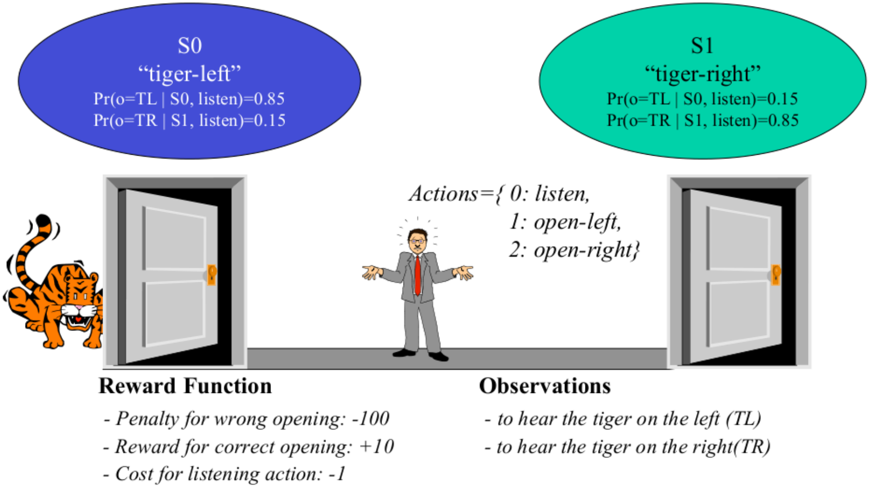


States 1–11 ***Observations A–G***

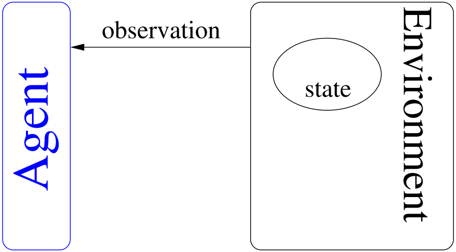
Tiger problem



Outline

Partially Observable Markov Decision Process

- $\mathcal{S}$: state space
- Ω : observation space
- $\mathcal{A}$: action space
- $T : \mathcal{S} \times \mathcal{A} \longrightarrow \Pi(\mathcal{S})$ transition function
- $O : \mathcal{S} [\times \mathcal{A} \times \mathcal{S}] \longrightarrow \Pi(\Omega)$ observation function
- $r : \mathcal{S} [\times \mathcal{A} \times \mathcal{S}] \longrightarrow \mathbb{R}$ reward function
- (b_0 : initial state distribution)
  - A POMDP describes a problem, not a solution/behavior/policy



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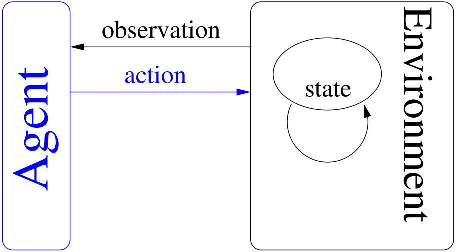

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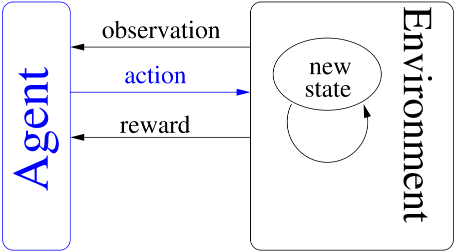

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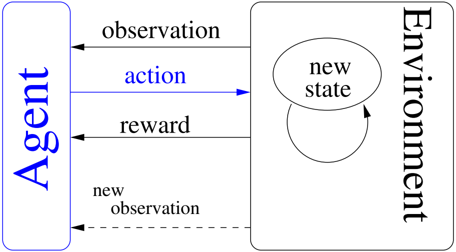
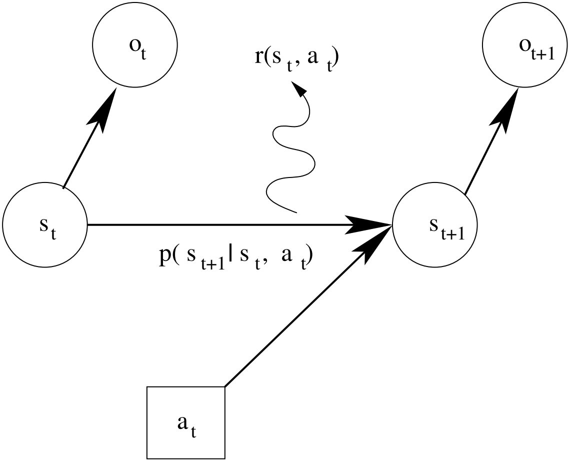


POMDP: a dynamical view



POMDP as an Influence Diagram

Outline

Solving a POMDP

Problem

find an optimal policy, *i.e.* that maximises a function of the reward. (*eg.* cumulative reward).

- non-markovian: existence of a *value function* ?
- information state: the policy is a function of what ?

Solving a POMDP

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Elements of solution

- convergence of "naïve" classical MDP algorithms
- *belief state* as valid/useful information state go direct
- Planification: when a model is known
  - WITNESS algorithm
  - INCREMENTAL PRUNNING algorithm
- Learning: when the model is unknown
  - learning useful STATE EXTENSIONS
  - learning the model
  - using PREDICTIVE STATE REPRESENTATION

Outline

POMDP
- Examples
- Formalism
- Problem
- Adapted POMDP

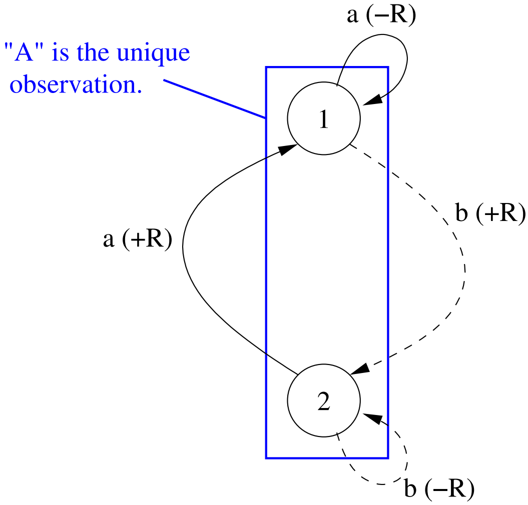
Exact resolution
- Belief states
- DP Operator
- Value Iteration
- Policy Iteration

Approximate solutions and Learning
- Approximate solutions
- Learning
- Predictive State Representation

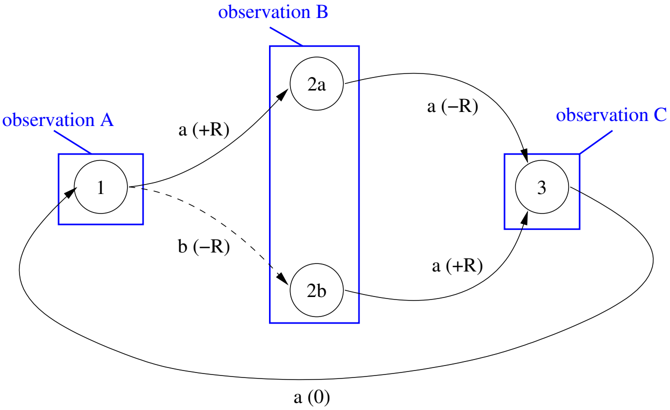
Conclusion

Stochastic Memoryless Policy

Stochastic memoryless policy can be arbitraty better than a deterministic memoryless policy.
[Singh et al., 1994]



No optimal memoryless policy



No memoryless policy leads to an optimal "adapted" value function. [Singh et al., 1994]

$$\vartheta^{\pi}(o) = \sum_{s \in \mathcal{S}} \Pr^{\pi}(s|o)\mathcal{V}_{\pi'}(s)$$

Convergence of "classical" algorithms [Jaakkola et al., 1994]

- TD(0)

$$\forall o \in \Omega,\ \vartheta^\pi(o) = \sum_{s \in \mathcal{S}} \Pr^\pi(s|o) \left[r(s) + \gamma \sum_{o' \in \Omega} \Pr^\pi(s,o') \vartheta^\pi(o') \right],$$

where $\Pr^\pi(s,o') = \sum_{s' \in \mathcal{S}} \Pr^\pi(s'|s) O(o'|s')$.

- Q-Learning

$$Q(o,a) = \sum_{s \in \mathcal{S}} \Pr^{\pi_{\exp}}(s|o,a) \left[r(s,a) + \gamma \sum_{o' \in \Omega} \Pr^a(s,o') max_{a' \in \mathcal{A}} Q(o',a') \right],$$

where $\Pr^{\pi_{\exp}}(s|o,a)$ is the asymptotic occupation probability and where $\Pr^a(s,o') = \sum_{s' \in \mathcal{S}} T(s'|s,a) O(o'|s')$.

Finding the best memoryless policy [Jaakkola et al., 1994]

- scalar value function: $\sum_{o \in \Omega} Pr^{\pi}(o) \sum_{s \in \mathcal{S}} Pr^{\pi}(s|o) V^{\pi}(s)$

- Monte Carlo evaluation of a policy
- Policy improvement
- loop ...

$\rightsquigarrow$ local maximum of scalar value function

Outline

Belief States : sufficient statistics of past

Belief States

distribution on states : $b_t(s) = \Pr(s_t = s)$

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distribution on states : $b_t(s) = \Pr(s_t = s)$

- Sufficient statistics : $b_t(s) = \Pr(s_t | a_t, s_{t-1}, \ldots, s_0)$

⇝ Complete information state.

- Bayesian update :

$$\begin{aligned} b_o^a(s') &= \Pr(s'|b, a, o) \\ &= \frac{O(o|s') \sum_{s \in \mathcal{S}} T(s'|s, a) b(s)}{\sum_{s \in \mathcal{S}} \sum_{s'' \in \mathcal{S}} O(o|s'') T(s''|s, a) b(s)}. \end{aligned}$$

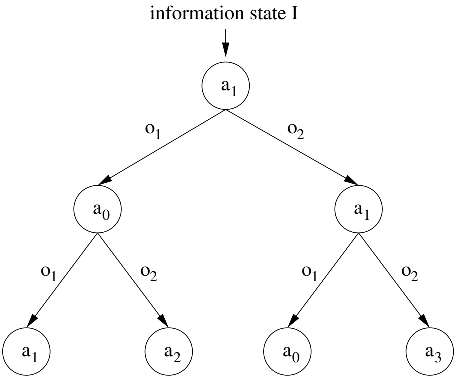
⇝ defines a (continuous) MDP which can be solved [Aström, 1965].

Policy Tree

For *belief states*

- Deterministic optimal policy
- Kind of Conditionnal Plan.

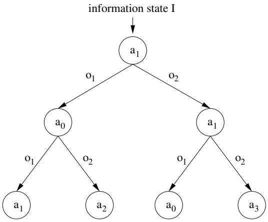
⇝ tree representation

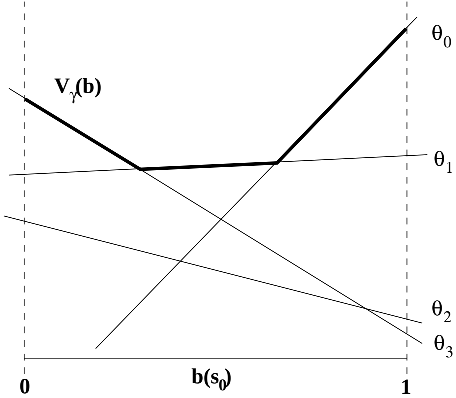


Piece-Wise Linear Convex Value Function

$$V_n^*(b) = \max_{a \in \mathcal{A}} \left[r(b,a) + \gamma \sum_{o \in \Omega} \Pr(o|b,a) V_{n-1}^*(T(b,o,a)) \right].$$

- Finite horizon n
- Vector = one policy...





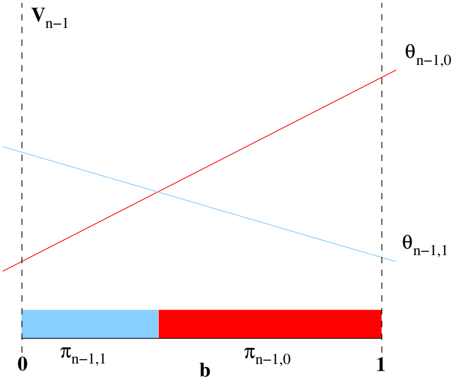
Outline

Dynamic programming on PWLC value function (1)

PWLC V_{n-1} at step $n-1$

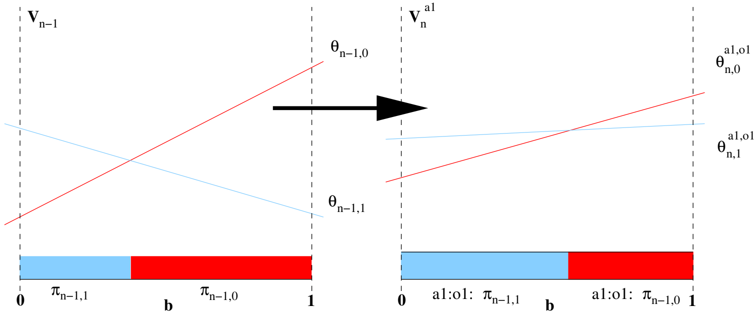
$$V_{n-1}(b) = \max_{\theta \in \Theta_{n-1}} b.\theta$$

a θ is mapped to a policy



Dynamic programming on PWLC value function (2)

PWLC V_n at step n for first action $a1$ and observation $o1$



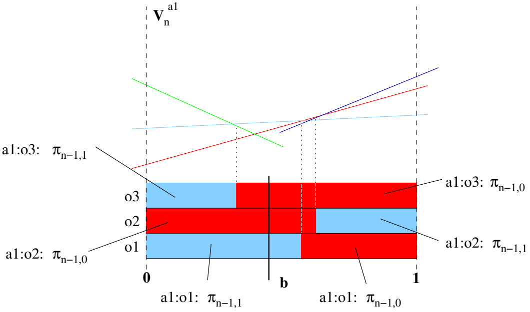
$$\theta_n^{a1,o1}(b,s) = \frac{r(s,a1)}{|\Omega|} + \gamma \sum_{s' \in \mathcal{S}} T(s,a1,s')O(s',o1)\theta_{n-1}^{a1,o1}(b^{a1,o1},s).$$

Dynamic programming on PWLC value function (3)

PWLC V_n at step n for first action $a1$

$$\theta_n^{a1}(b) = \sum_{o \in \Omega} \theta_n^{a1,o}(b).$$

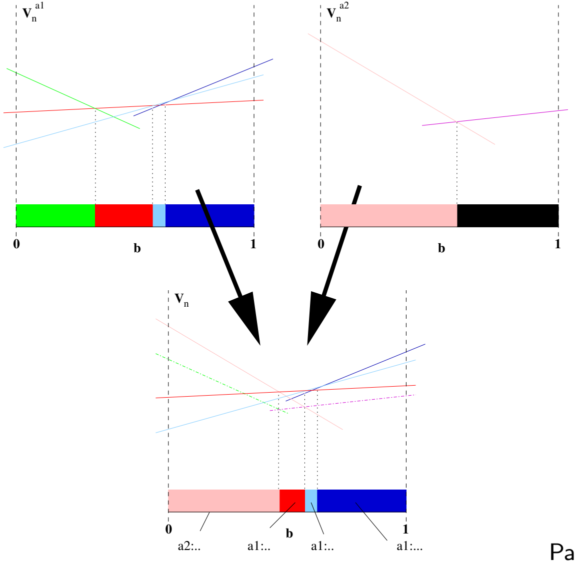
At most $|\Theta_{n-1}|^{|\Omega|}$ vectors



Dynamic programming on PWLC value function (4)

PWLC V_n at step n

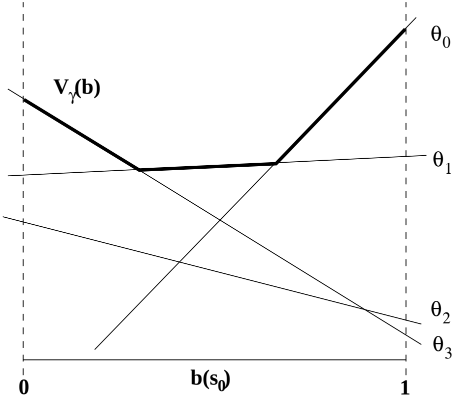
$$\theta_n(b) = \max_{a \in \mathcal{A}} \theta_n^a(b)$$



back to PI

PWLC Value Function with Belief States

- Finite Horizon POMDP
  - Optimal value function is PWLC [Smallwood and Sondik, 1973]
  - $$V_n(b) = \max_{\theta \in \Theta_n} b.\theta$$
- Infinite Horizon POMDP
  - ϵ-optimal value function is PWLC
  - Optimal only for *transient* POMDP [Sondik, 1971]



$\rightsquigarrow$ the real problem is the size of vector space.

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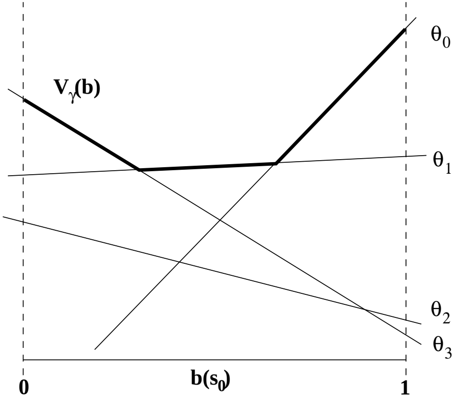
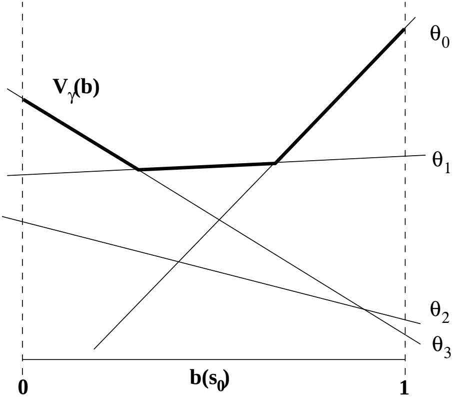


⇝ the real problem is the size of vector space.

Parcimonious representation

θ from Θ dominated : $b.\theta \leq \max_{\theta' \in \Theta} b.\theta'$.

- Exists a minimal representation [Littman and Szepesvári, 1996]



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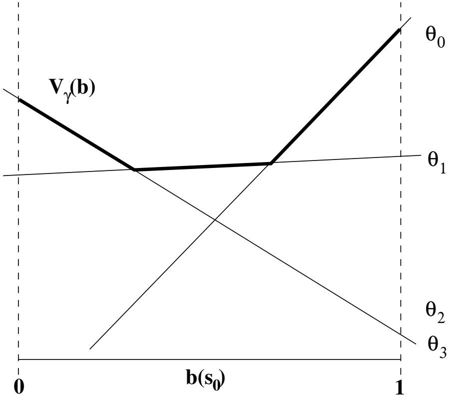

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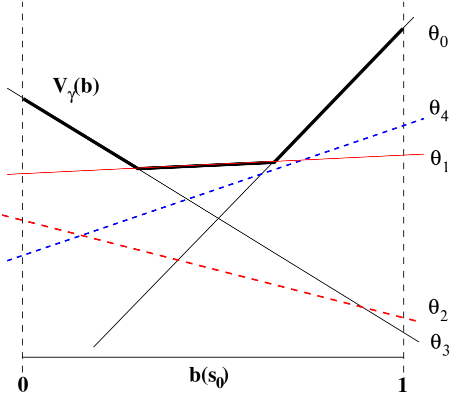
Parcimonious representation

θ from Θ dominated : $b.\theta \leq \max_{\theta' \in \Theta} b.\theta'$.

- Exists a minimal representation [Littman and Szepesvári, 1996]
- θ_2 : entirely dominated
- θ_4 : needs PRUNING



Outline

Algorithm WITNESS: concepts [Cassandra et al., 1994]

- Incremental build of parcimonious representation.

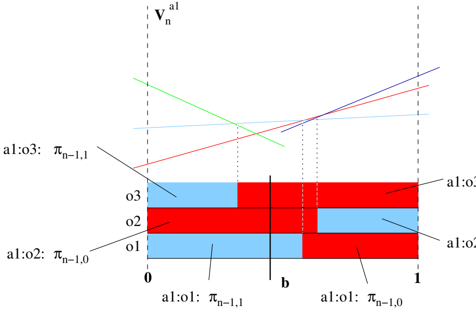
1. Start from belief state b
2. Look for the best vector of its region
3. Add all "neighbors" to the agenda Υ
   - Either remove from it (dominated vector)
   - Or add best vector from region to V and its neihbors to Υ.
4. Loop

Algorithm WITNESS(Θ_{n-1}, a)

Input: A parsimonious representation Θ_{n-1} of V^*_{n-1}, an action a

Output: A parsimonious representation $V_n^{*,a}$

$b \leftarrow$ a belief state of $\mathcal{B}$
$\hat{\Theta} \leftarrow \{\theta_n^a(b)\}$
$\Upsilon \leftarrow \mathcal{N}(\theta_n^a(b))$
while $\Upsilon \neq \emptyset$ **do**
  $v \leftarrow$ `RemoveElement`(Υ)
  if $v \in \hat{\Theta}$ **then**
    $b \leftarrow$ **null**
  else
    $b \leftarrow$ `FindVectInRegion`$(v, \hat{\Theta})$
  end
  if $b \neq$ **null then**
    $\hat{\Theta} \leftarrow \hat{\Theta} \cup \{\theta_n^a(b)\}$
    $\Upsilon \leftarrow \Upsilon \cup \{v\}$
    $\Upsilon \leftarrow \Upsilon \cup \mathcal{N}(\theta_n^a(b))$
  end
end
$\Theta_n^a \leftarrow \hat{\Theta}$
return Θ_n^a

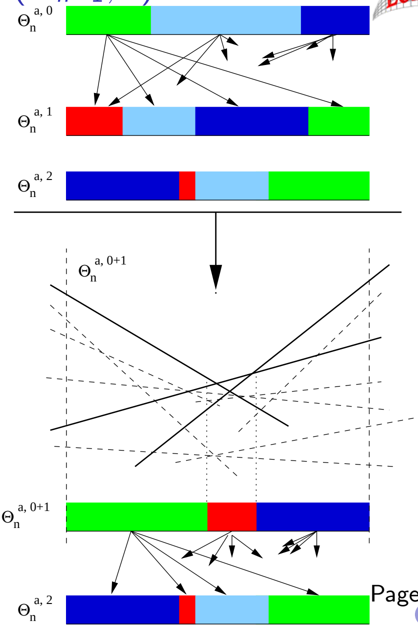


IncrementalPruning: concept [Zang and Lio, 1996]

- Lots of small pruning vs global final pruning.

1. With the set Ψ of all sets of $\Theta_n^{a,o}$ vectors.
2. Take two sets from it and prune them
3. Add new prunned set to Ψ, loop.

Algo IncrementalPruning(Θ_{n-1}, a)



Input: A parsimonious representation Θ_{n-1} of V^*_{n-1}, an action a

Output: A parsimonious representation $V_n^{*,a}$

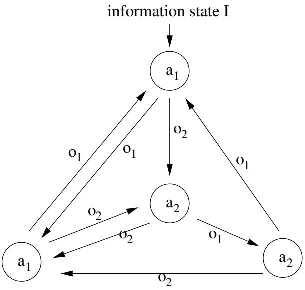
$\Psi \leftarrow \bigcup_o \{\Theta_n^{a,o}\}$
while $|\Psi| > 1$ **do**
  $A \leftarrow$ `RemoveElement`(Ψ)
  $B \leftarrow$ `RemoveElement`(Ψ)
  $D \leftarrow$ Prune$(A \oplus B)$
  $\Psi \leftarrow \Psi \cup \{D\}$
end
return Ψ

Outline

Policy Iteration: concepts [Hansen, 1998]

- Grow an ϵ-optimal FSA controler

1. From a given FSA δ compute all new vectors
2. For each new vector
   2.1 If exists in δ, added to $\hat{\delta}$
   2.2 Else modify same but dominated node *i* to $\hat{\delta}$
   2.3 Else add new node to $\hat{\delta}$
3. Loop



- A FSA policy has PWLC Value Function
- One node = One vector

Algorithm Policy Iteration(δ, ϵ)

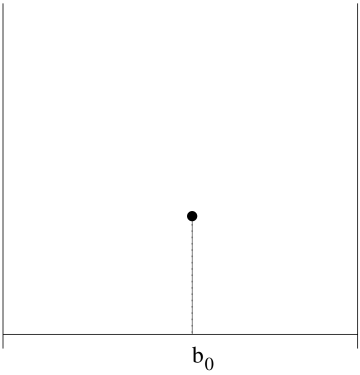
Input: A finite state controller δ and a positive real ϵ
Output: A finite state controller δ^* which is ϵ-optimal

repeat
 Compute V^δ from δ by solving equations (**??**)
 Build $\hat{V}^\delta \leftarrow \texttt{DynamicProgOperator}(V^\delta)$ details on DP
 $\hat{\delta} \leftarrow \emptyset$
 foreach $\hat{\theta}^j \in \hat{V}^\delta$ **do**
  if *there exists a node i of δ associated with $\hat{\theta}^j$ with identical action and links* **then**
   add i to $\hat{\delta}$
  else if *there exists a node i such that $\hat{\theta}^j$ dominates θ^i* **then**
   add i to $\hat{\delta}$, with the action and the links of $\hat{\theta}^j$
  else
   add a *new* node to $\hat{\delta}$ with the actions and links of $\hat{\theta}^j$
 end
 Add to $\hat{\delta}$ all the other nodes of δ that are reachable from $\hat{\delta}$
 $\delta \leftarrow \hat{\delta}$
until $\|\hat{V}^\delta - V^\delta\| \leq \epsilon(1-\gamma)/\gamma$
return $\hat{\delta}$

Outline

Point Based Value Iteration [Pineau et al., 2003]

- Start: set of belief states



See also [Spaan and Vlassis, 2005], [Seuken and Zilberstein, 2007]...

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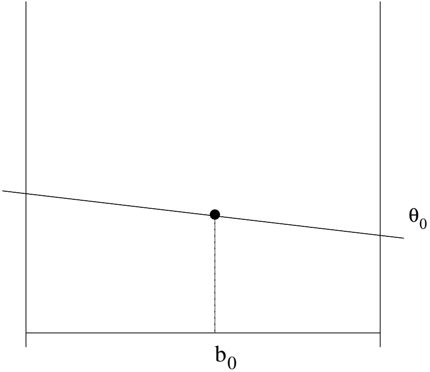

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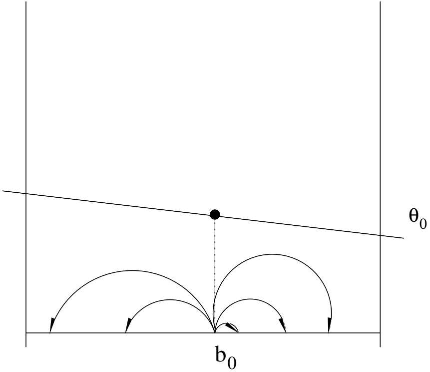

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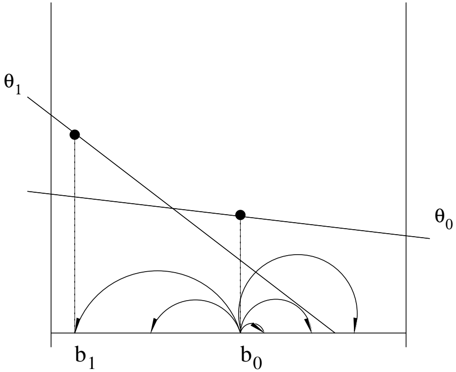

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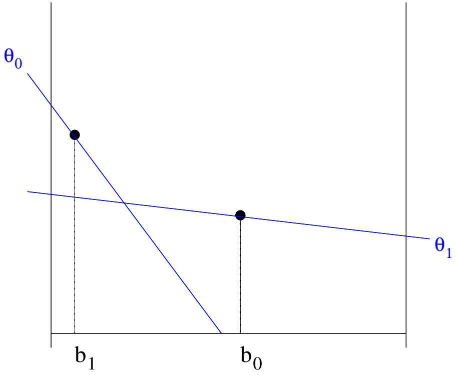

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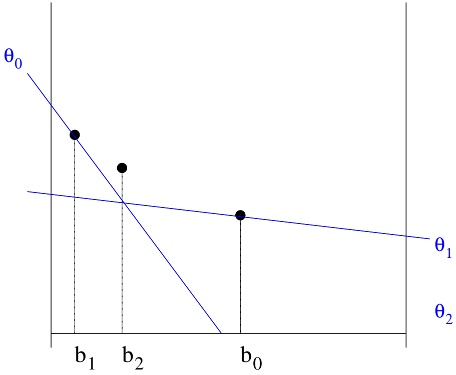

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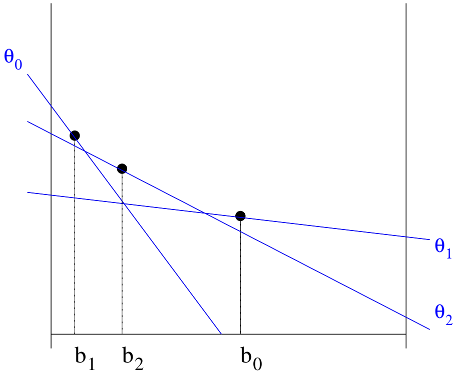

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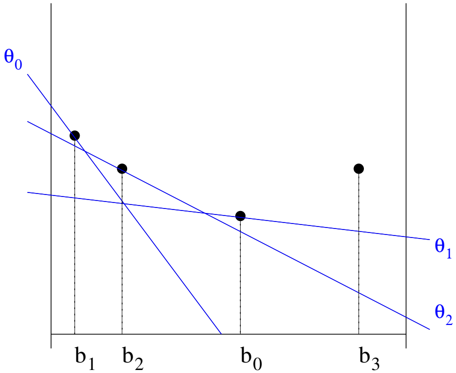
Point Based Value Iteration [Pineau et al., 2003]

- Start: set of belief states
- Alternate : update / expand
- Only approximation



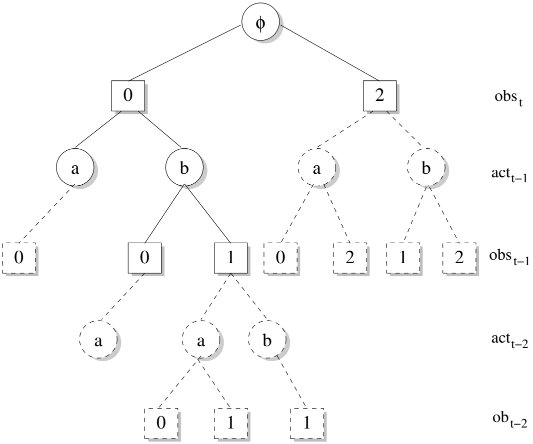
See also [Spaan and Vlassis, 2005], [Seuken and Zilberstein, 2007]...

Outline

Learn State Extensions

POMPD $\equiv$ variable n-Markov Decision Process

- ext. states = (o, a) histories
- Start with 'obs' as "histories"
- Extend *ambiguous* states
- heuristics (Q variations)
- statistical diff. in probability distributions.
- See [McCallum, 1995], [Dutech, 2000]



GPOMDP algorithm [Baxter and Bartlett, 2000]

- randomized policy: $\{\mu(\theta,.)\}_{\theta \in \mathbb{R}^k}$
- Gradient estimate

$$z_{t+1} = \gamma z_t + \frac{\nabla \mu_{a_t}(\theta, o_t)}{\mu_{a_t}(\theta, o_t)}$$

$$\Delta_{t+1} = \Delta_t + \frac{1}{t+1}[r_{t+1} z_{t+1} - \Delta_t]$$

- Interlaced with policy improvement with gradient ascent.

$$\theta_{t+1} = \theta_t + \alpha \Delta_{t+1}$$

$\rightsquigarrow$ local optimum

Outline

Predictive State Representation

- a test: $t_i = o_1 a_1 o_2 \dots o_n$
- prediction. history h: $\Pr(o_1, \dots, o_n | h, a_1, \dots, a_{n-1})$
- set of tests: $\mathcal{Q} = \{t_i\}_{i=1,\dots,q}$

Predictive State Representation

$(1 \times q)$ prediction vector $p(h) = \{\Pr(t_1|h), \Pr(t_2|h), \dots, \Pr(t_q|h)\}$ iff $\forall h,\ \Pr(t|h) = f_t(p(h))$

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- linear PSR : $\Pr(t|h) = p(h) m_t^T$
- Update : $p_i(hao) = \Pr(t_i|hao) = \frac{\Pr(aot_i|h}{\Pr ao|h} = \frac{p(h)m_{aot_i}^T}{p(h)m_{ao}^T}$

Theorem

For any environment that can be represented by a finite POMDP model, there exists a linear PSR with number of tests no larger than the number of states in the minimal POMDP model.

matrix dyn. sys. | skip details

Learning PSRs [Singh et al., 2003]

- How to maintain correct predictions for the tests
  $\rightsquigarrow$ m_{aot_i} and m_{ao}
- Gradient of the error
- $E(t) = \sum_{x \in X_t} [p(x|h_{t-1}) - \hat(p)(x|h_{t-1})]^2$
  where X_t is the set of all extension tests possible from time t
- indirect solution, local optimum, huge iterations

Discovering PSRs [James and Singh, 2004]

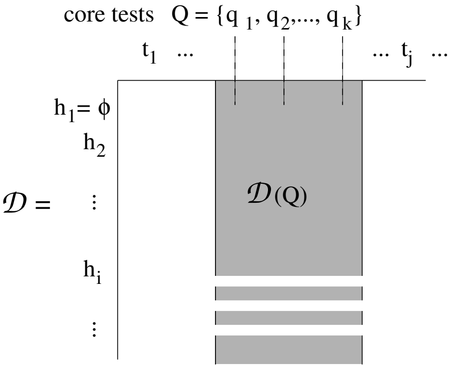
- for histories and tests of size 1
- build the empirical system-dynamics matrix $\mathcal{D}$
- look for independant columns $\rightsquigarrow$ core-tests $\mathcal{Q}_{\mathcal{T}_1}$
- look for independant rows $\rightsquigarrow$ core-histories $\mathcal{Q}_{\mathcal{H}_1}$
- build new $\mathcal{D} = (\mathcal{Q}_{\mathcal{T}_1} \bigcup \mathcal{Q}_{\mathcal{T}_1}^{+ao}) \bigotimes (\mathcal{Q}_{\mathcal{H}_1} \bigcup \mathcal{Q}_{\mathcal{H}_1}^{+ao})$
- loop

- (uses rank estimation of unknown matrix, need reset action, can learn PSR in parallel)

System-dynamics Matrix [Singh et al., 2004]

PSRs : set of k columns for syst-dyn of linear dimension k.

- n-MDP $\rightsquigarrow (|\mathcal{A}||\Omega|)^n$
- POMDP,HMM $\rightsquigarrow$ $< |\mathcal{S}|$
- POMDP $\subset$ PSR



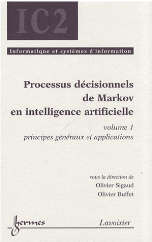
Conclusion

- What was here
  - formalization of POMDPs
  - memoryless policies
  - belief states and PWLC value function
  - value iteration: WITNESS, INCREMENTAL PRUNING
  - policy iteration
  - others: state extension, GPOMDP, PSR

Conclusion

- What was here
  - formalization of POMDPs
  - memoryless policies
  - belief states and PWLC value function
  - value iteration: WITNESS, INCREMENTAL PRUNING
  - policy iteration
  - others: state extension, GPOMDP, PSR
- What was left
  - complexity results (from *bad* to *worst*)
  - applications (robotics, H/C dialog, H/R interactions, ??)
  - cognitive aspects (how *good* representations are build)

Some starting references



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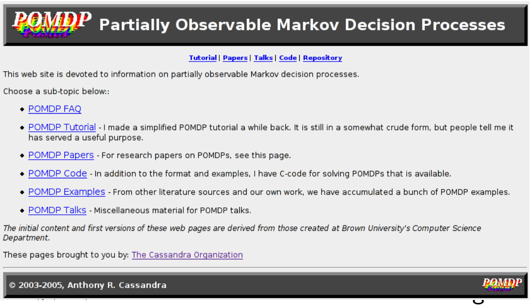
(a translation is about to be puvlished)

http://www.pomdp.org/



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Neighbor vectors

Step (3) of DP: $\theta_n^{a,o} = \frac{r(a)}{|\Omega|} + \gamma P^{a,o} \theta_{n-1}^{a,o}(b^{a,o})$
With *any* θ_{n-1} instead of *THE BEST* $\theta_{n-1}^{a,o}(b^{a,o})$

$$\widetilde{\theta}^{a,o} = \frac{r(a)}{|\Omega|} + \gamma P^{a,o} \theta_{n-1},$$

$\rightsquigarrow$ Family of vector

Neighbor of $\theta_n^a = \sum_{o \in \Omega} \theta_n^{a,o}$

$\nu = \tilde{\theta}_n^{a,o'} + \sum_{o \neq o'} \theta_n^{a,o}$ where $\tilde{\theta}_n^{a,o'} \neq \theta_n^{a,o'}$

Theorem

For a belief state b, there exists a "best" vector iff it is also the case for one of its neighbor.

Back to WITNESS

Find WITNESS vectors

Algorithm 1: FindVecInRegion(θ, Θ)

Input: A representation Θ, a vector $\theta \in \Theta$

Output: A point of the region or **null**

```
LP ← SetUpLinearProgram ( θ, Θ )
SolveLinearProg (LP)
if NoSolution (LP) then
    return null
end
if val(LP) ≤ 0 then
    return null
end
return Solution (LP)
```

Back to WITNESS

Algorithm 2: SetUpLinearProgram(θ, Θ)

Input: A representation Θ, a vector $\theta \in \Theta$

Output: A Linear Program Problem

solve

$\max_{\mathbb{R}} \epsilon$

with

$x.(\theta - \tilde{\theta}) \geq \epsilon, \ \forall \tilde{\theta} \in \Theta, \ \tilde{\theta} \neq \theta$

$x \in \Pi(\mathcal{S})$

Find dominated vectors

Algorithm 3: CheckDomination(Θ)

Input: A representation Θ

Output: A representation without any entirely dominated vector

if $|\Theta|$ ¡ *2* **then**
    return Θ
end
$\tilde{\Theta} \leftarrow \emptyset$
repeat
    $\theta \leftarrow \texttt{RemoveElement}(\Theta)$
    if $\nexists \theta' \in \tilde{\Theta}$ *t.q.* $\theta' \geq \theta$ **then**
        $\tilde{\Theta} \leftarrow \{\theta' | \theta' \in \tilde{\Theta},\ \theta \not\geq \theta'\}$
        $\tilde{\Theta} \leftarrow \tilde{\Theta} \cup \{\theta\}$
    end
until $\Theta = \emptyset$
return $\tilde{\Theta}$

Back to INCREMENTAL PRUNNING

Algorithm 4: Pruning($\tilde{\Theta}$)

Input: A representation $\tilde{\Theta}$ of V

Output: A parsimonious representation Θ of V

$\hat{\Theta} \leftarrow \emptyset$
while $\tilde{\Theta} \neq \emptyset$ **do**
    $\theta \leftarrow \texttt{RemoveElement}(\tilde{\Theta})$
    $b \leftarrow \texttt{FindVectInRegion}(\theta, \hat{\Theta})$
    if $b \neq$ **null then**
        $\tilde{\Theta} \leftarrow \tilde{\Theta} \cup \{\theta\}$
        $\theta^* \leftarrow \texttt{BestVector}(\tilde{\Theta}, b)$
        $\tilde{\Theta} \leftarrow \tilde{\Theta} - \{\theta\}$
        $\hat{\Theta} \leftarrow \hat{\Theta} \cup \{\theta^*\}$
    end
end
$\Theta \leftarrow \hat{\Theta}$
return Θ

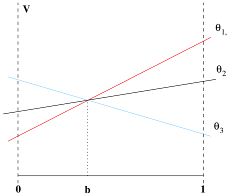
Check one vector

Algorithm 5: BestVector(Θ, b)

Input: A representation Θ, a belief state b

Output: The best vector of Θ for this state

$v^* \leftarrow -\infty$
foreach $\theta \in \Theta$ **do**
  $v \leftarrow b.\theta$
  if $v = v^*$ **then**
    $v^* \leftarrow$ `LexicographicMaximum`(θ^*, θ)
  end
  if $v > v^*$ **then**
    $v^* \leftarrow v$
    $\theta^* \leftarrow \theta$
  end
end
return θ^*



Algorithm 6: LexicographicMaximum(θ, $\tilde{\theta}$)

Input: Two vectors θ and $\tilde{\theta}$ from Θ

Output: The lexicographic maximum of the two vectors

foreach $s \in \mathcal{S}$ **do**
  if $\theta(s) > \tilde{\theta}(s)$ **then**
    return θ
  end
  if $\theta(s) < \tilde{\theta}(s)$ **then**
    return $\tilde{\theta}$
  end
end
return θ