Dehn-twisting the color code

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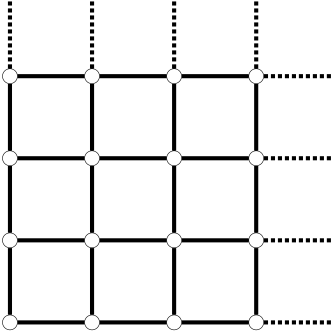
Stabilizer codes

Given n physical qubits and $n-k$ independent Pauli operators ($\mathcal{S}$):

- We encode k logical qubits.
- Code space is $+1$ common eigenspace of $\mathcal{S}$.
- Noise is modeled as iid Pauli operators on physical qubits.
- Errors anticommute with some stabilizers, flipping their measurement to -1.
- After measurements, we get a syndrome from which we compute an error-correcting operator.

Toric codes



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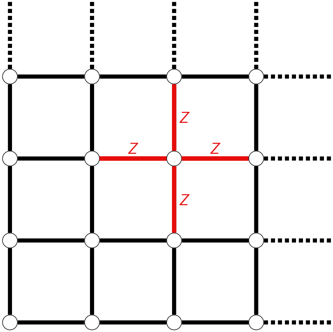

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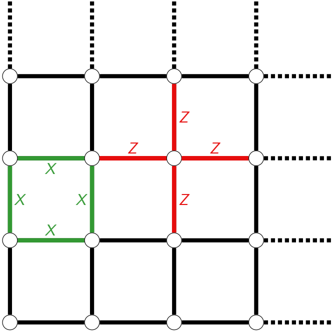

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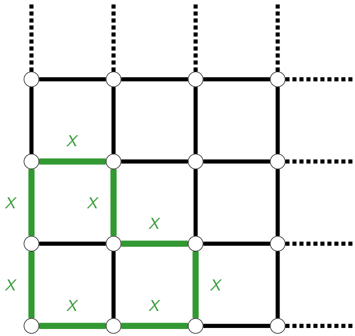

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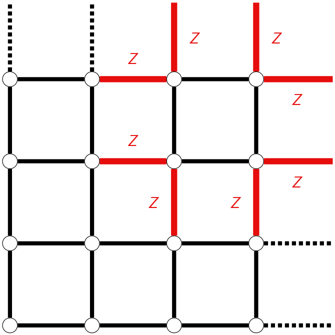

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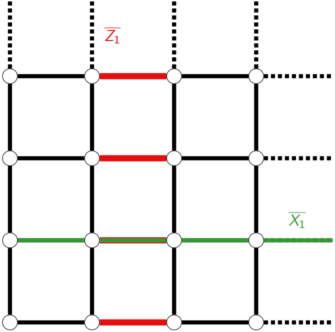

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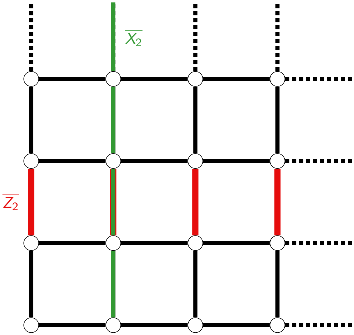
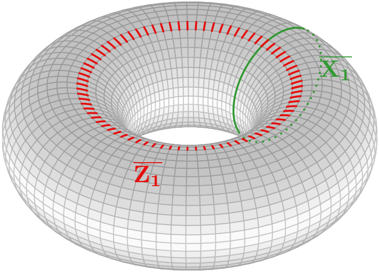
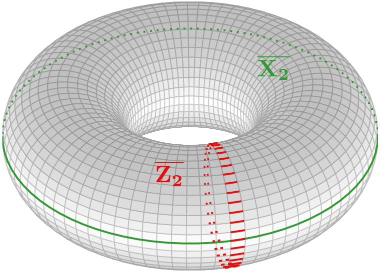


Logical operators





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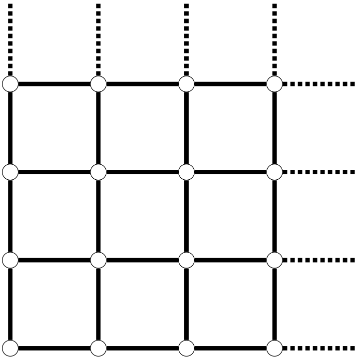

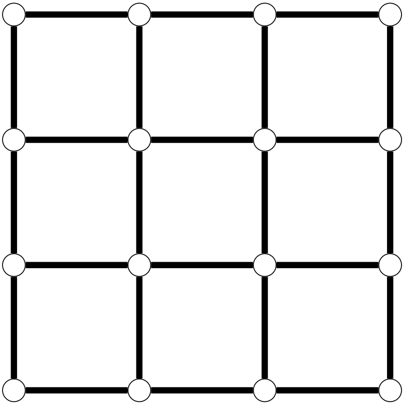

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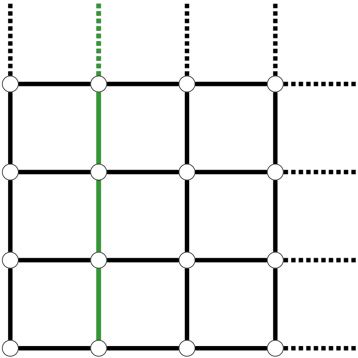

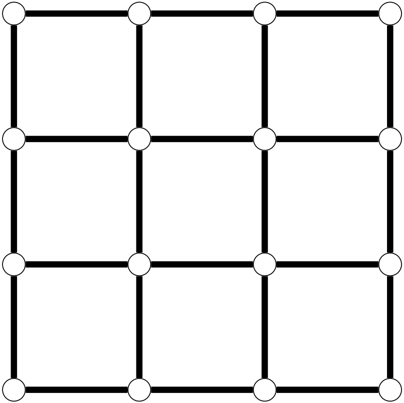

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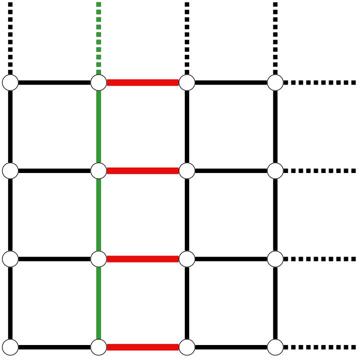

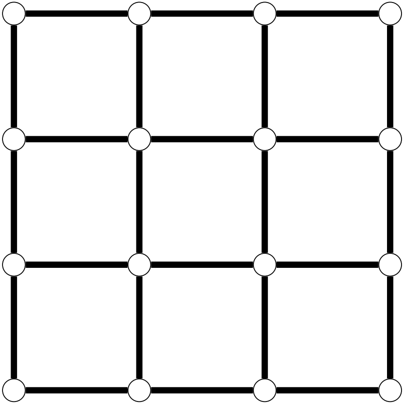

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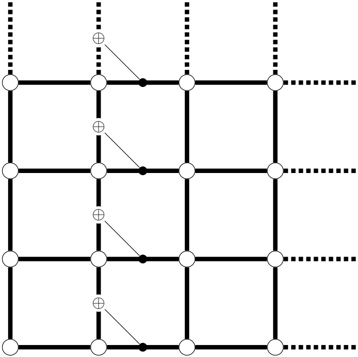

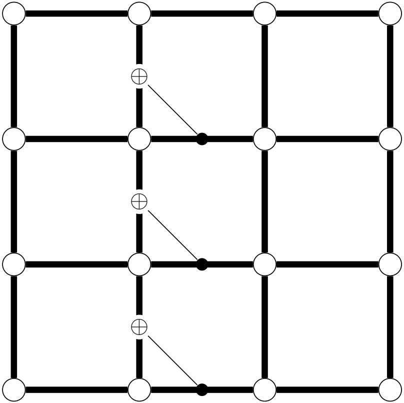

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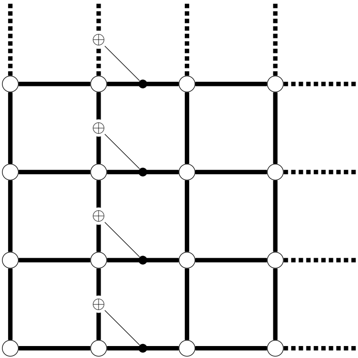

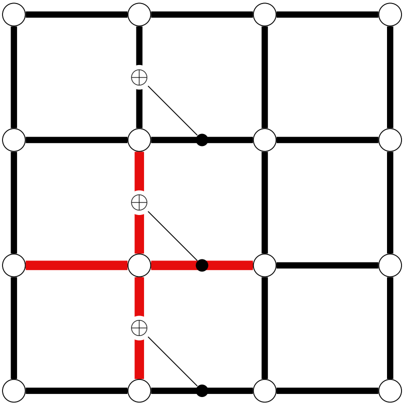

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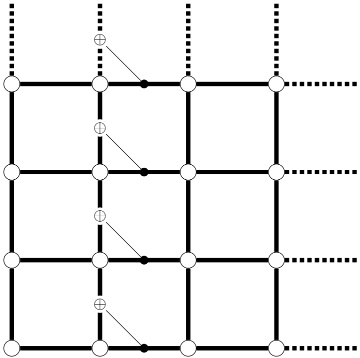

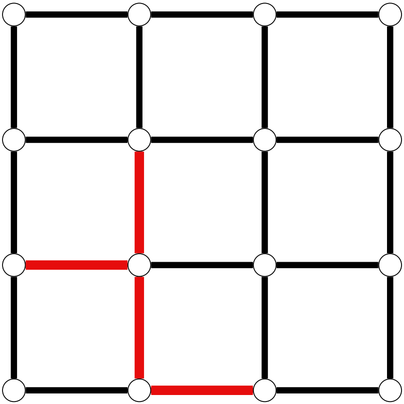

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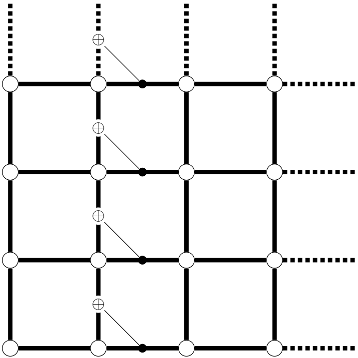

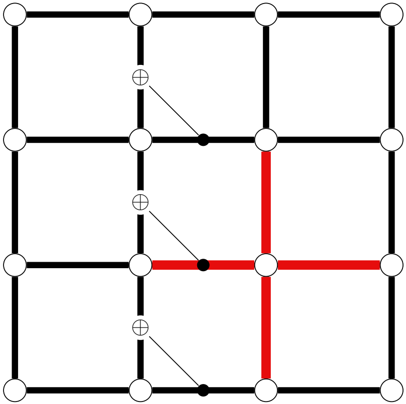

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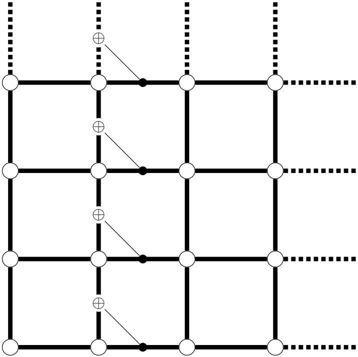

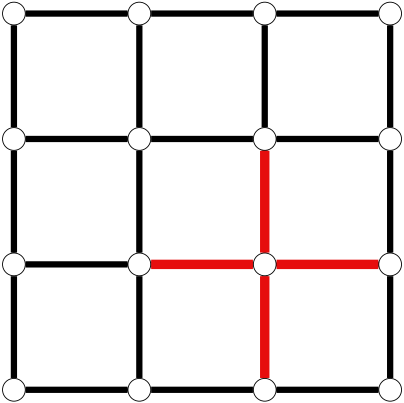

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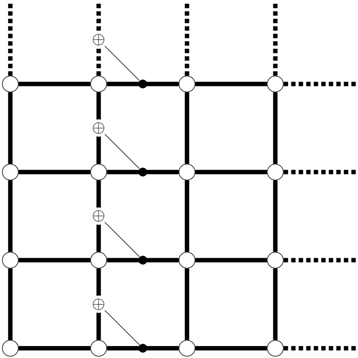

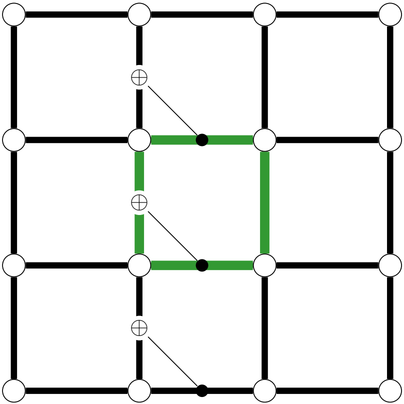

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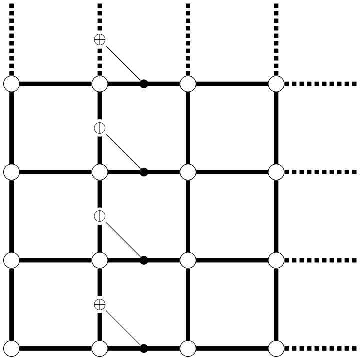

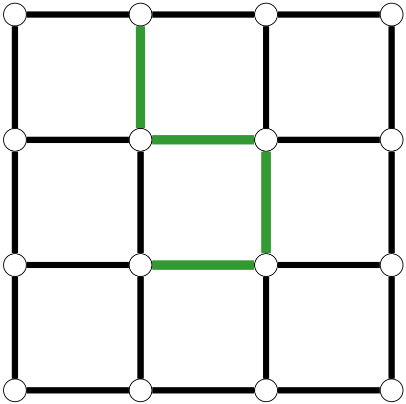

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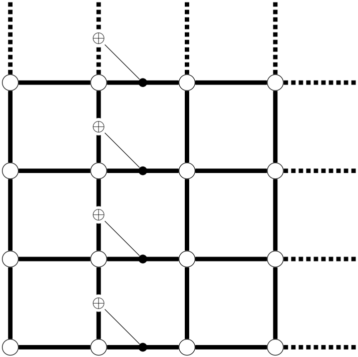

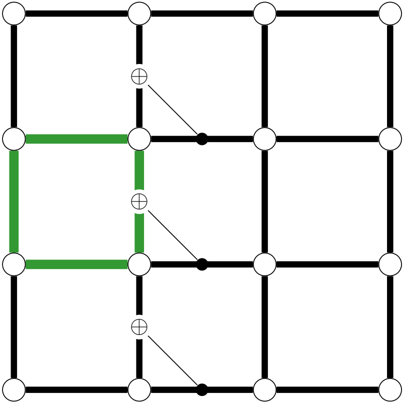

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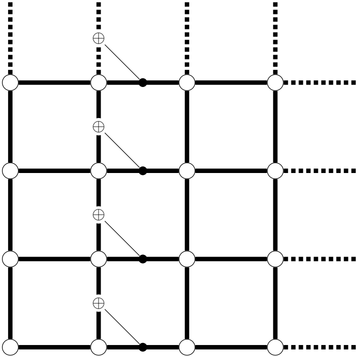

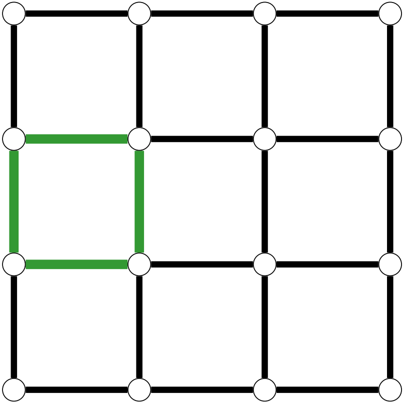

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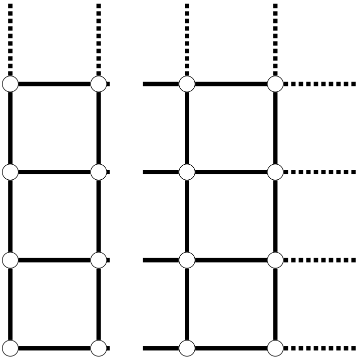

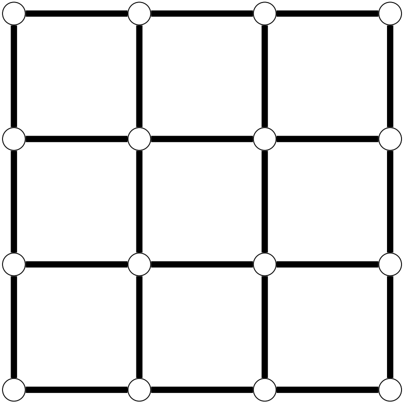

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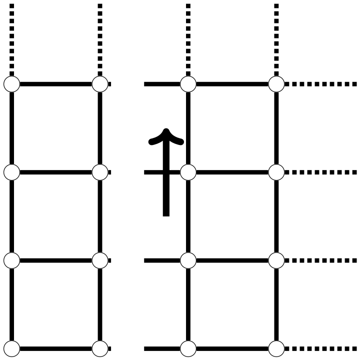

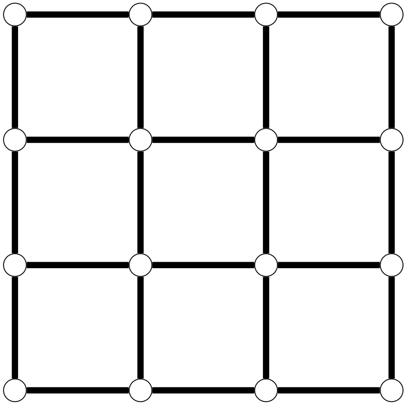

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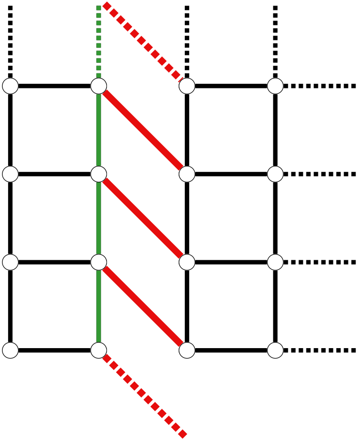

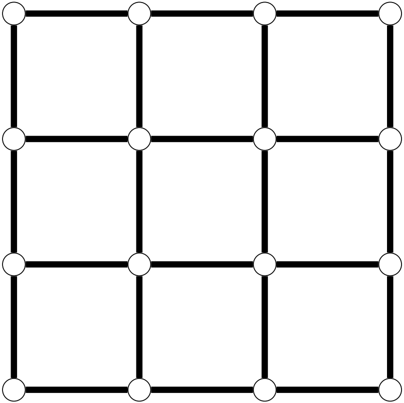

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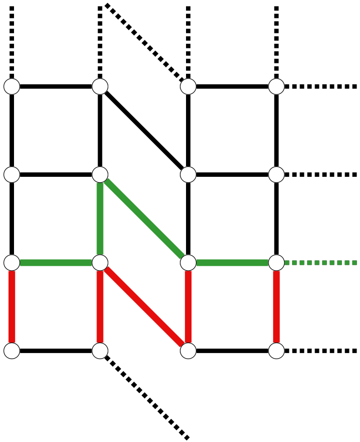

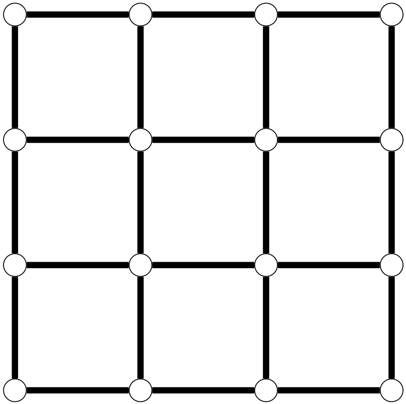

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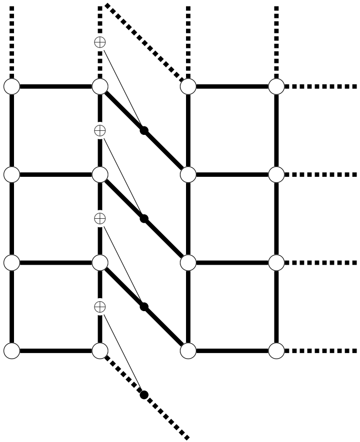

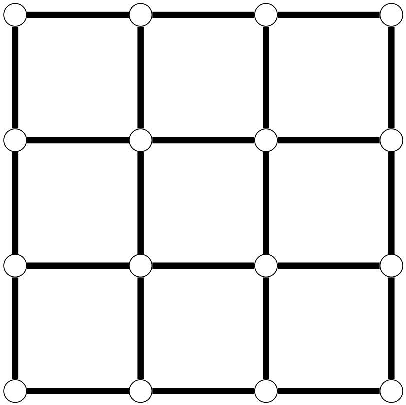

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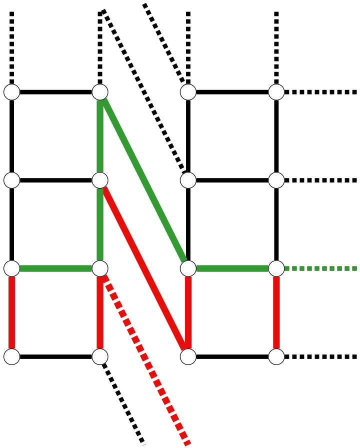

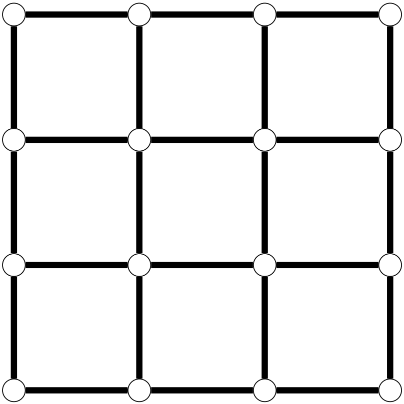

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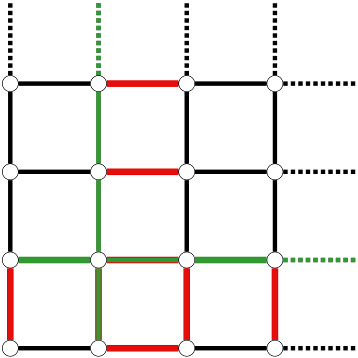

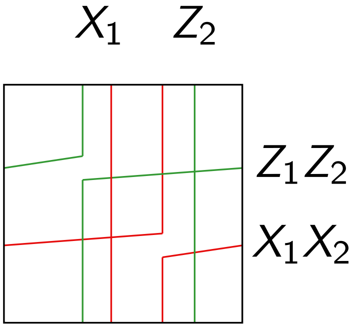

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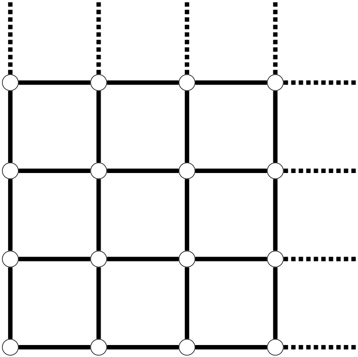

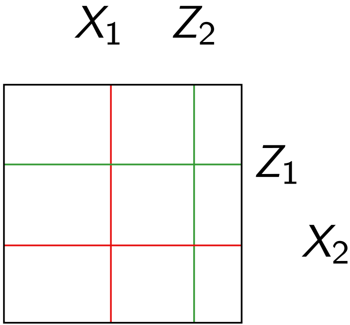

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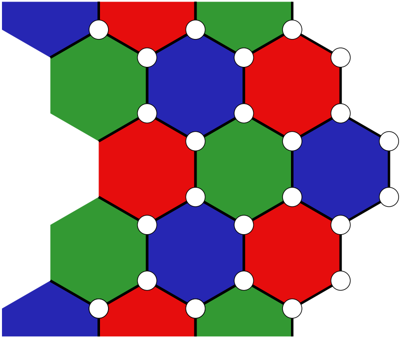

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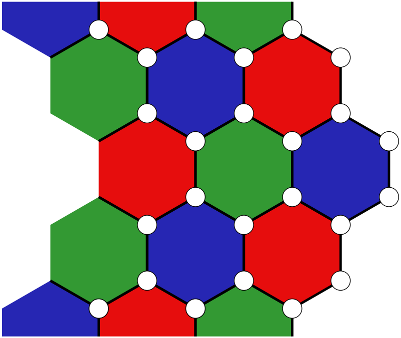

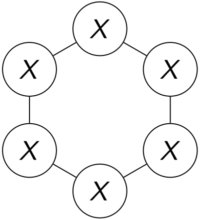

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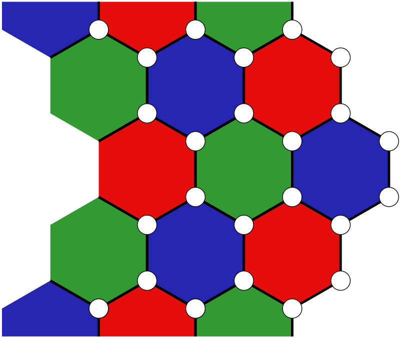

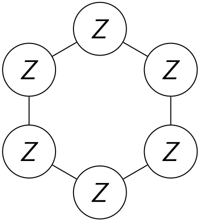

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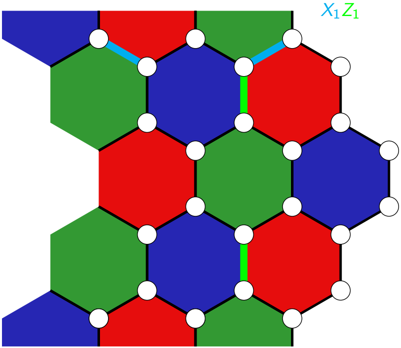

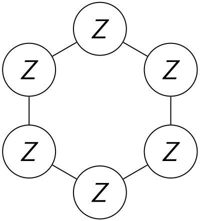

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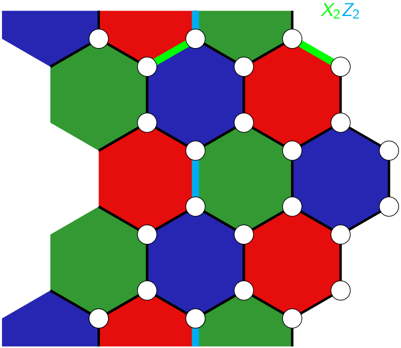

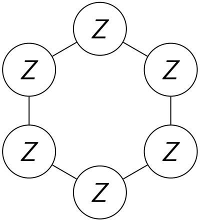

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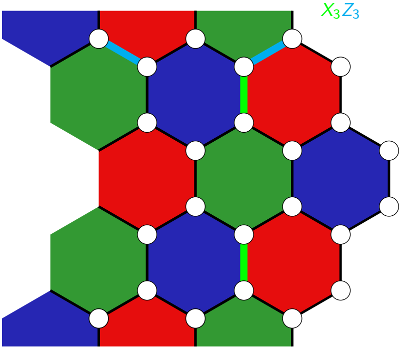

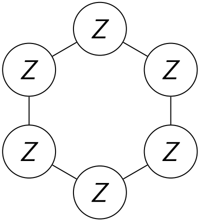

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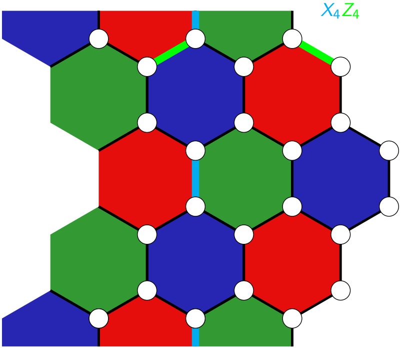

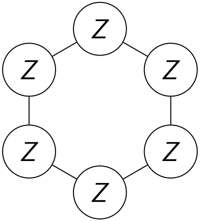

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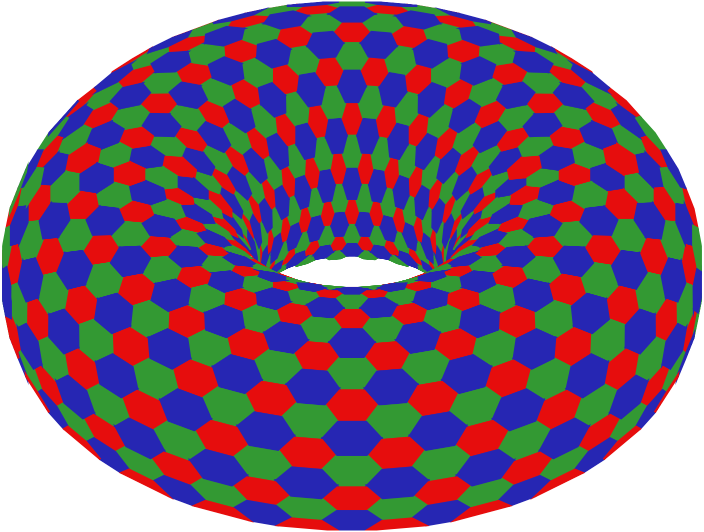
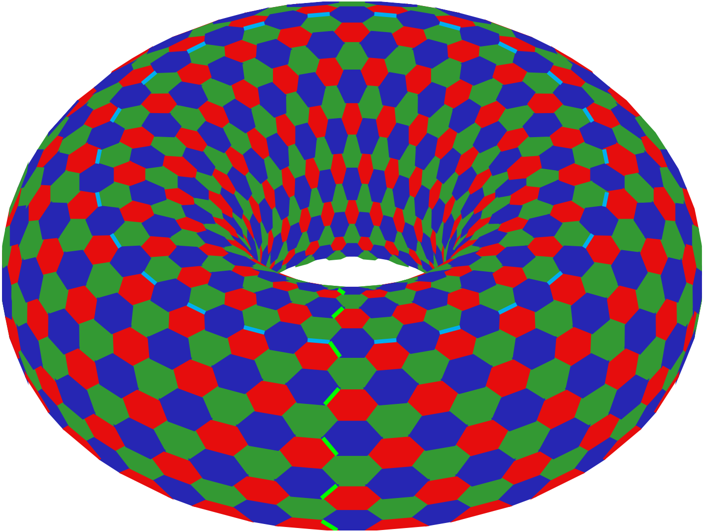
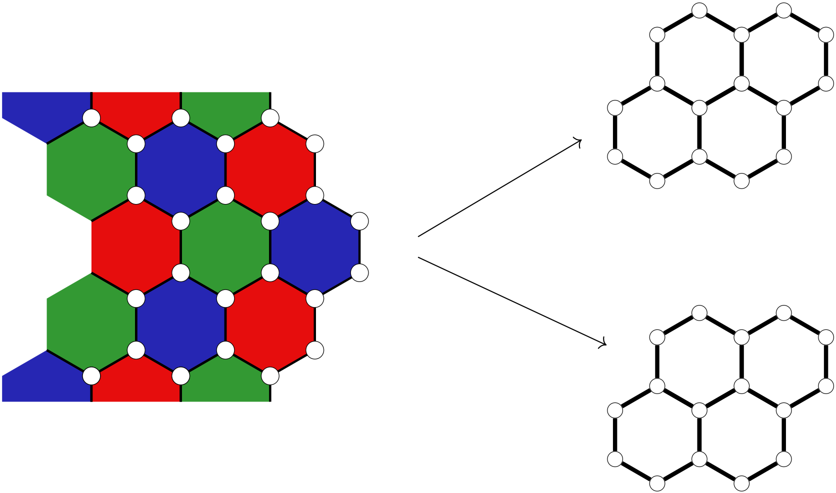


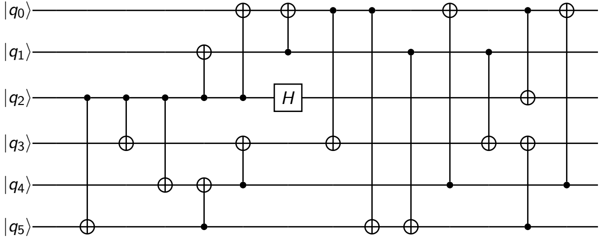
Logical Pauli operators

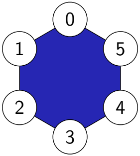


Code equivalence[3]

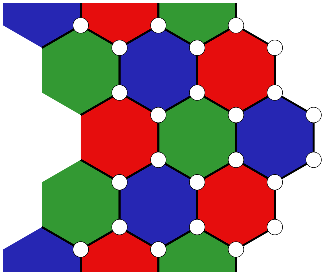


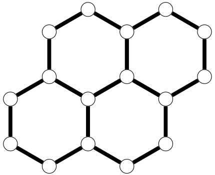
Unfolding procedure[3]

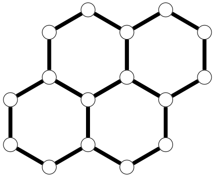




Dehn twists







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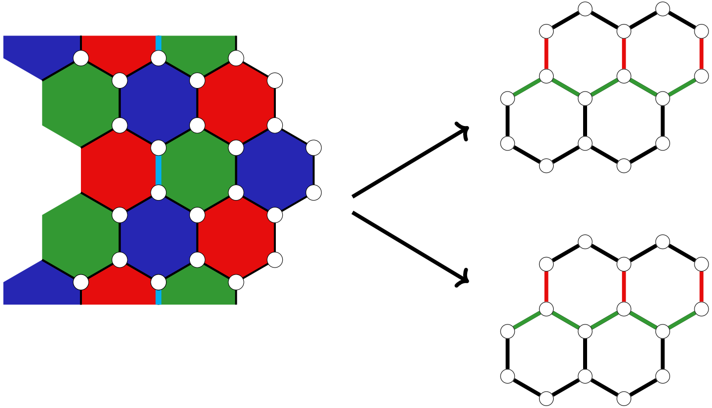

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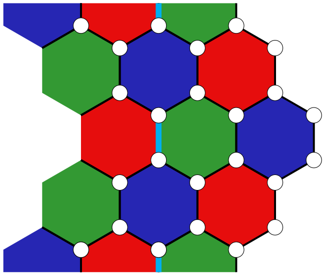

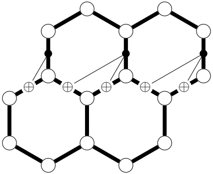

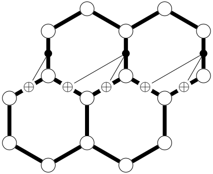

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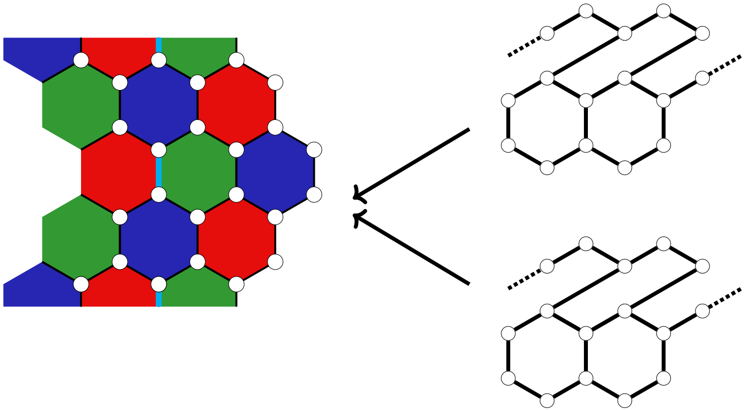

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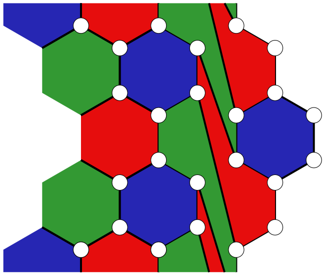

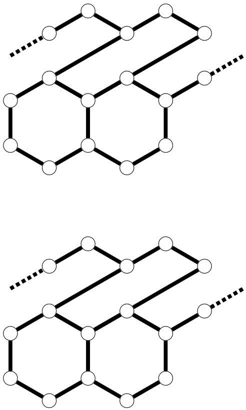

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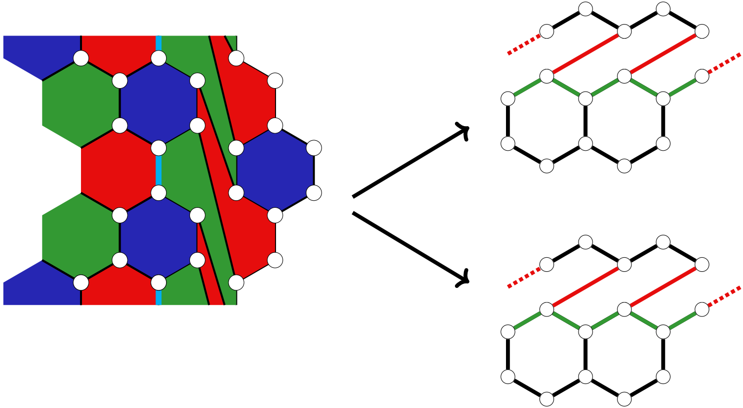

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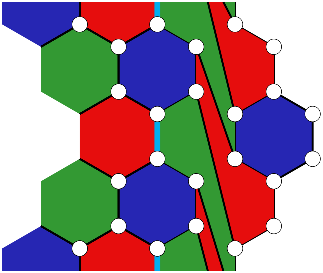

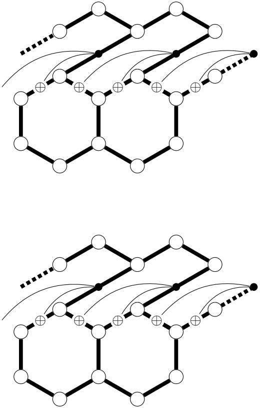

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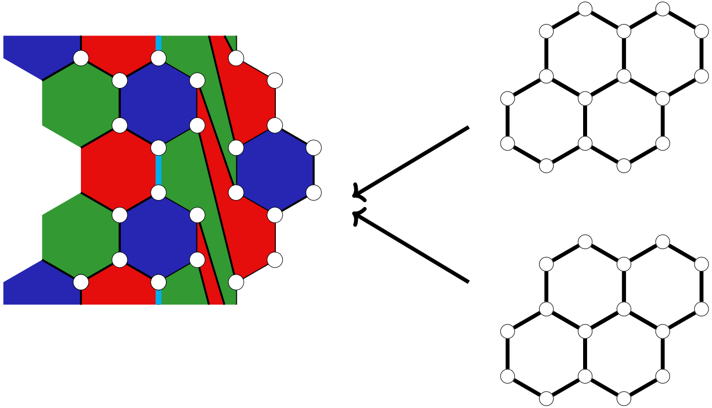
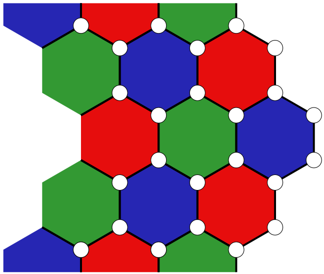
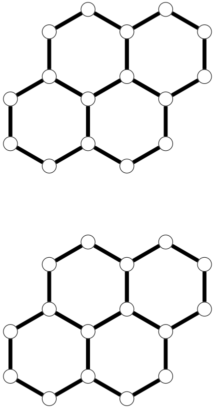
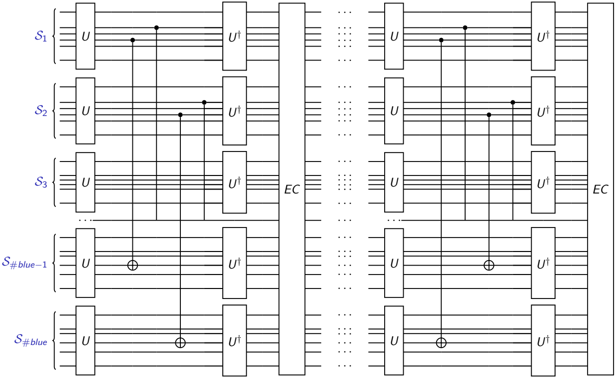


Dehn twists

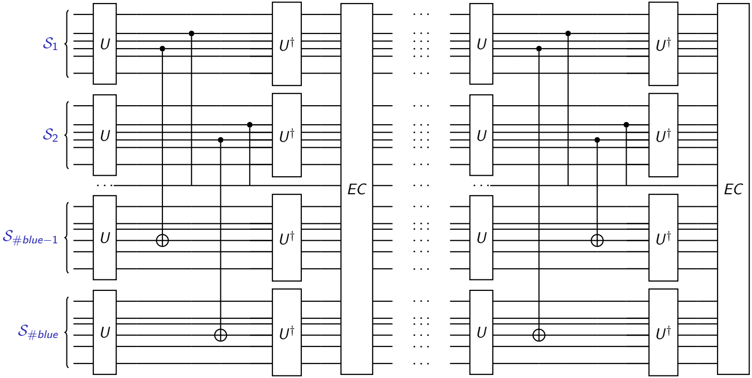




Fault-tolerant CNOT



Fault-tolerant CNOT



We expect $d_{eff} = \frac{d}{12}$ $\left(d_{eff} = \frac{d}{2p(\lfloor \frac{q}{2} \rfloor + 1)} \text{ for a } (2p, 2q, 2q) \text{ lattice} \right)$.

Summary and further prospects

What we have seen:

- We explicitly compute the unfolding operators
- We go back and forth between color code and surface codes
- Dehn twists in surface codes translate to constant depth CNOT for color codes

Further prospects:

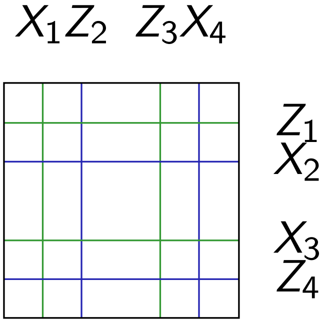
- Phenomenological noise simulation and threshold estimation
- Color code with other layouts (hyperbolic, 3D...)
- Larger constant-depth gate set

R. Koenig, G. Kuperberg, and B. W. Reichardt, “Quantum computation with turaev–viro codes,” *Annals of Physics*, vol. 325, pp. 2707–2749, dec 2010.

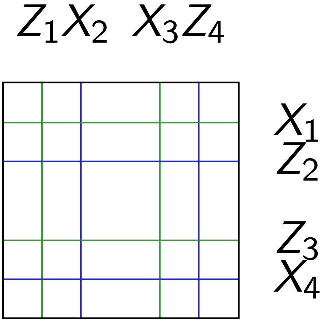
N. P. Breuckmann, C. Vuillot, E. Campbell, A. Krishna, and B. M. Terhal, “Hyperbolic and semi-hyperbolic surface codes for quantum storage,” *Quantum Science and Technology*, vol. 2, p. 035007, aug 2017.

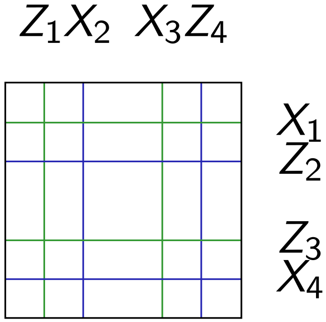
A. Kubica and M. E. Beverland, “Universal transversal gates with color codes: A simplified approach,” *Physical Review A*, vol. 91, mar 2015.

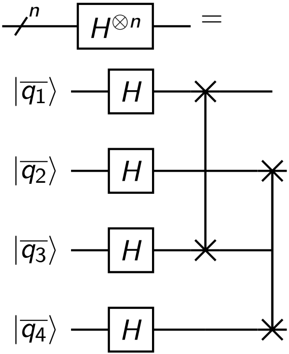
Logical operators



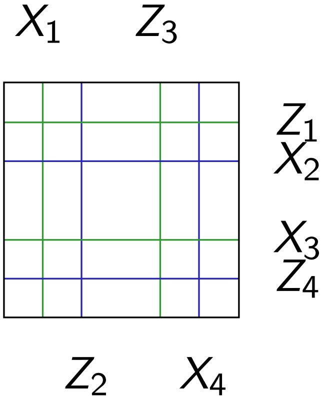
Logical operators



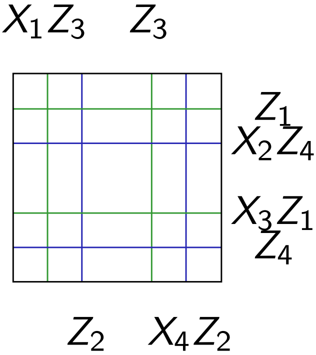

Z_1X_2 X_3Z_4
X_1
Z_2
Z_3
X_4

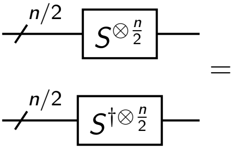

n
$H^{\otimes n}$
$=$
$|\overline{q_1}\rangle$
$|\overline{q_2}\rangle$
$|\overline{q_3}\rangle$
$|\overline{q_4}\rangle$
H
H
H
H

Logical operators



Logical operators





Logical operators