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Analysis of bridge admittance of plucked string instruments in the high frequency range

Benjamin Elie, François Gautier
LAUM, UMR CNRS 6613, Avenue Olivier Messiaen 72085 Le Mans Cedex 9, France

Bertrand David
Institut Télécom, Télécom ParisTech, CNRS LTCI 46 rue Barrault 75634 Paris Cedex 13, France

Marthe Curtit
Itemm, Pôle national d'innovation des métiers de la musique, 71 avenue Olivier Messiaen 72000 Le Mans, France

Abstract

The differences of assessment between musical instruments of the same kind (e.g classical guitars) covers aspects linked to sounds, expressibility, or even manufacturing process. How to extract relevant information related to these aspects by measuring well chosen physical (acoustical, vibratory) parameters ? The aim of the proposed study is to identify criteria enabling the clustering of string instruments in different classes. The plucked instruments will be our application. The knowledge of these parameters would then allow to give an help to the instrument maker in the adjustments and settings likely to be made on an instrument. The modal parameters (frequency, and damping coefficient) can be estimated accurately on a large frequency range from impulse responses by using the high-resolution ESPRIT method, associated with the ESTER criterion to enumerate the signal components. Global parameters, such as the modal density, loss factors and modal overlap factor can be determined and used to estimate average mobility, derived from Skudrzyk's mean-value theorem. The application on guitars and ukulele shows a common behavior of their average mobility: it remains constant in the mid and high frequency domains. The corresponding value is used as one of the parameters enabling the characterization.

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1. Introduction

The differences of assessment between musical instruments of a same kind covers aspects linked to the resulting sound, expressibility, or even quality of the manufacturing process. The measurement of some physical parameters, acoustical or vibratory could debrief, in a way, these properties. The knowledge of relevant parameters eventually could give an help to the instrument maker in the adjustments and settings likely to be made on an instrument.

String instruments are complex assemblies and the sound produced is the result of several coupled interacting parts. It is therefore delicate to characterize string instruments. Furthermore, scientific studies enable a better understanding of the physics of string instruments, but they are mostly published for scientists, so the knowledges and the technologies are sometimes difficult to be transferable to the instrument makers. The aim of our study is to propose a mechanical characterization of string instruments using methods which requires tools affordable, easily manipulable by the instrument maker, and also transferable to their workshop. Among all the different ways of investigation available for the study of string instruments, we chose to focus on the coupling between the string and the soundboard of plucked string instruments (guitars and ukulele in this case). When the string is excited (plucked) by the musician, the vibration of the string is transmitted to the soundboard via the bridge. The measurement of the mobility (or mechanical admittance) gives information about the coupling between the string and the soundboard. The level of that coupling on plucked instruments acts on both the sound level and the sound duration: when it is strong (*i.e.* large mobility), the energy of the string is transfered more suddenly to the soundboard. It results in a more powerful sound, but with a faster decay. The trade-off between sound level and sound duration is frequently encountered by instrument makers. We propose in this study global parameters, deduced from bridge mobility measurements, which could quantify this trade-off. The method is based on an estimation of modal parameters to estimate average mobility, derived from Skudrzyk's mean-value theorem.

After a short definition of mobility, the section 2 introduces the global descriptors used to characterize the bridge mobility of plucked instruments, as well as the modal identification techniques used to compute them. Then, a few examples of experimental applications are presented in section 3. In that section, we apply the method of modal identification to guitars. Then, an example of application to the lutherie assistance is presented. It consists in comparing the average mobility of the same ukulele, first without bridge and then with bridges of different weights.

2. Bridge mobility measurements

2.1. Bridge mobility of guitars : definition

The mobility of a structure (or mechanical admittance, usually denoted $Y(\omega)$) is defined as the ratio, in the frequency domain, of the velocity $V(\omega)$ at a point of the structure to the applied excitation force $F(\omega)$. It can be written as a superimposition of modal contributions. When the observation point and the excitation point coincide, $Y(\omega)$ is written as follows:

$$Y_A(\omega) = j\omega \sum_{k=1}^{+\infty} \frac{|\Phi_k(A)|^2}{m_k(\omega_k^2 + j\eta_k\omega_k\omega - \omega^2)}, \qquad (1)$$

where A denotes the point of observation, ω_k, η_k, m_k and Φ_k are respectively the pulsation, the loss factor, the modal mass and the mode shape associated to the mode k, k being the order of the mode.

The frequency response of a structure is commonly divided into three main frequency ranges: the *low*, *middle* and *high* frequency domains The boundaries of these domains are defined with the value of the *modal overlap factor* (or MOF). It is denoted by the symbol μ, and represents the ratio of the half power bandwidth to the difference of the eigenfrequencies of two successive modes. In the low frequency range, peaks are well-separated, therefore the modal overlap is small. The MOF tends to increase in higher frequency ranges. The boundaries of the frequency ranges are defined as follows [1]:

- low frequency range : $\mu < 30\%$
- mid frequency range : $30\% < \mu < 100\%$
- high frequency range : $\mu > 100\%$

The figure 1 shows the frequency response, as well as the modal overlap factor of a synthetic plate.

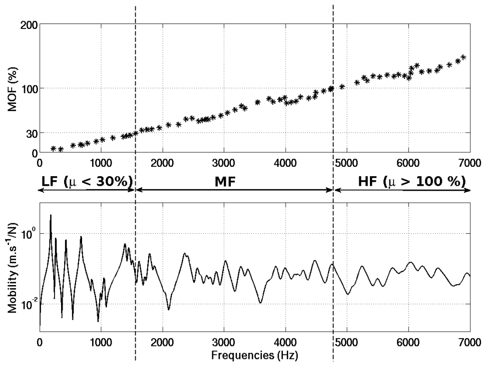


Figure 1. Modal overlap factor (MOF) and mobility of a synthetic plate. The plate is a thin rectangular isotropic plate, having the following dimensions: $L_x = 0.3$ m, $L_y = 0.4$ m, and $h = 2.5$ mm. The Young modulus is 15 GPa. The value of the modal loss factor η is 2% and is the same for all modes. The limits of the three main frequency domains are defined with the values of the modal overlap factor.

2.2. Global descriptors

2.2.1. Mean value of mobility

In the low frequency domain, the mobility is studied through the parameters of each individual mode, since they are well-separated and easily identifiable. In higher frequency ranges, we will consider that studying each individual mode is no longer relevant, since they are very fluctuating. A way to analyze mobility curves in the high frequency range could be to highlight their underlying tendency, without the disturbing individual modal contributions. This synthetic descriptor is called the mean-value.

Skudrzyk [2] presented a method to compute the mean-value of mobility. In a physical sense, it corresponds to the mobility of the structure if this latter had infinite dimensions, and so without any resonances. The main idea of Skudrzyk is to change the expression of the mechanical admittance from a discrete sum to an integral. However, the calculation of this integral is based on several assumptions:

- the structure is supposed to be homogeneous (the surface density ρ_S is constant over the whole structure),
- the local mean value of mode shapes over the whole structure is supposed to be barely dependent on the mode order,
- the structure is supposed to verify the Basile hypothesis,

then, the characteristic admittance, as defined by Skudrzyk [2], writes:

$$Y_C = G_C + jB_C, \qquad (2)$$

with

$$G_C = \frac{n(f)}{4M_{Eq}}, \qquad (3)$$

where $n(f)$ denotes the local modal density, *i.e.* the invert of the local average frequency difference between two successive modes, and M_{Eq} is a quantity with the same dimension as a mass, so it is called the equivalent mass of the system.

The imaginary part B_C vanishes when the modal density is constant [3] (*e. g.* for plates), therefore it is of less interest.

2.2.2. Plate elastic constant

The modal density profile of instruments gives information about its global behavior in a given frequency range. We propose a method to characterize the modal density profile of instruments through one parameter, called the plate elastic constant. The method is based on the assumption that the modal density profile of the instrument is similar to the one of a rectangular thin plate.

The expression giving the modal density of thin rectangular plates has been estimated by Courant [4]. It is given by:

$$n(f) \underset{f\to\infty}{\rightarrow} n_\infty = \frac{S}{2}\sqrt{\frac{\rho h}{D}}, \tag{4}$$

where S is the area of the plate, ρ and h denote respectively the density and the thickness of the plate, and D is the flexural rigidity modulus of the material. Basically, it is an asymptotic value toward which the modal density tends, independently of the boundary conditions. The boundary conditions changes the eigenfrquencies in the low frequency domains. The influence of boundary conditions on the modal density has been studied by Xie *et al.* [5] and are given by the following expressions:

- free:

$$n(f) = n_\infty + \frac{1}{\sqrt{2\pi}}\left(\frac{\rho h}{D}\right)^{\frac{1}{4}}(L_x + L_y)f^{-\frac{1}{2}}, \tag{5}$$

- simply supported :

$$n(f) = n_\infty - \frac{1}{\sqrt{8\pi}}\left(\frac{\rho h}{D}\right)^{\frac{1}{4}}(L_x + L_y)f^{-\frac{1}{2}}, \tag{6}$$

The term n_∞ is the asymptotic value of the modal density, independent of the boundary conditions, and L_x and L_y are the dimensions of the plate. When the plate has free vibrations, the modal density tends towards n_∞ by upper values, while it tends by lower values when its four edges are simply supported.

Expressions 5 and 6 can be written $n(f) = p\beta + q\sqrt{\beta/f}$, where $p = S/2$, q is a constant coefficient depending on boundary conditions and on the geometry of the plate, and $\beta = \sqrt{\frac{\rho h}{D}}$ is the so-called plate elastic constant, depending on the mechanical properties of the material. The plate elastic constant is determined by searching the value of β which minimizes the euclidean norm of $\mathbf{n}(f) - p\beta + q\sqrt{\beta/f}$. Hence:

$$\beta = \underset{\beta}{\operatorname{argmin}} \left\| \mathbf{n}(f) - p\beta + q\sqrt{\frac{\beta}{f}} \right\|_2^2, \tag{7}$$

where $\mathbf{n}(f)$ is the vector containing the experimental modal density, namely the invert of the difference of eigenfrequencies.

The parameter β gives an indication of the effective mass to stiffness ratio.

2.3. Experimental modal identification

In practice, the computation of these global descriptors requires an estimation of modal parameters. In the low-frequency range, Fourier based modal identification techniques [6] are sufficient to estimate modal parameters. However, in the mid and high frequency ranges, the modal identification is much more difficult, since the modal overlap becomes large [3, 1]. Nevertheless, the subspace methods [7, 8, 9] enable to extend the limits of performance of modal estimation, due to the large modal overlap in mid and high frequency ranges. We propose to apply the subspace method ESPRIT (*Estimation of Signal Parameters via Rotational Invariance Techniques*) [9], since it has been shown to be efficient in mid and high frequency ranges to estimate modal parameters of mechanical structures [10].

To apply ESPRIT, the impulse response signal $s(A,t)$ must be modeled as a sum of a finite number of exponentially damped sinusoids disturbed by an additive white Gaussian noise. Each sinusoidal component corresponding to the temporal response of a single mode.

$$s(A,t) = \sum_{k=1}^{K} a_k(A) e^{-\alpha_k t} e^{j(2\pi f_k t + \varphi_k)}, \tag{8}$$

where k, K, and A are respectively the mode order, the signal modeling order and the observation point, a_k, α_k, f_k and φ_k denote respectively, the modal amplitude, the modal damping, the modal frequency and the modal phase of the mode k.

The ESPRIT algorithm will estimate the signal parameters corresponding to modal parameters of the K sinusoidal components residing in the signal.

An important issue when applying subspace methods to composite signals is the tuning of the modeling order. The number of single components is indeed *a priori* unknown. A wrong signal modeling order can bias the estimation [11]. The ESTER (*ESTimation of ERror*) criterion, based on the assessment of the rotational invariance property that characterizes the signal subspace, has been designed by Badeau *et al.* [11]

and used by Ege [10] in the context of mechanical experiments. We thus chose to apply this technique of signal enumeration since it proves reliable and also for its compatibility with the ESPRIT algorithm which relies on the rotational invariance property.

3. Experimental applications

3.1. Analysis process

The figure 2 represents in a bloc-diagram the analysis process applied in this study to compute the global parameters defined in the section 2.

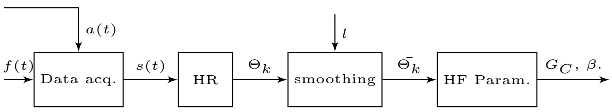


Figure 2. Bloc diagram of the process .

The signal $s(t)$ of the impulse response is obtained by a simultaneous measurement of the excitation signal, by means of a small impact hammer (PCB 086E80), and the acceleration signal, by means of a small accelerometer (PCB 352C23). The instruments are hanged by their headstock and the strings are damped. The accelerometer is put on the bridge, in the vicinity of the base of the lowest string. The force is applied just next to the observation point, as shown in figure 3.

Figure 3. Experimental set-up .

Then $s(t)$ is analyzed with the subspace technique ESPRIT . This allows us to estimate the signal parameters Θ_k, corresponding to the modal parameters in output. A smoothing, which consists in averaging each modal parameters with the values of its 2 l closest neighbors, is applied to the modal parameters Θ_k. This step aims to reduce the important local variations that might occur in the values of modal parameters. For instance, since the study focuses on two-dimensional structures, double modes might occur and locally increase the modal density. One get thus the local averaged modal parameters $\bar{\Theta}_k$. Finally, the values of modal parameters are used to compute the global descriptors defined in section 2.

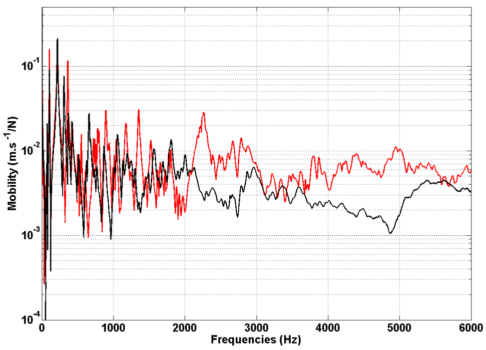
3.2. Analysis of guitar frequency responses

Mobility, modal density and averaged mobility are presented in figure 4 for two classical guitars. The modal density is growing in the low-frequency domain, then it becomes constant in the mid frequency domain. In the high-frequency domain, it slightly decreases. The behavior of the estimated modal density in the mid and high frequency ranges is proper to rectangular plates with four free edges. This result suggests that the guitar can be assimilated to an equivalent rectangular thin plate in these frequency domains. Since the dimensions of every classical guitar are very similar, the dimensions of the equivalent plate are set to the same value for every guitar. The equivalent plate is thus a square plate (0.3×0.3 m^2), since it roughly corresponds to the dimensions of the lower bout of the soundboard, which is the most mobile part. The values of parameters p and q in expression 7 are $p = 9.0 \times 10^{-2}$ m^2 and $q = 0.24$ m. The theoretical modal density of the equivalent plate is plotted in figure 4 (a).

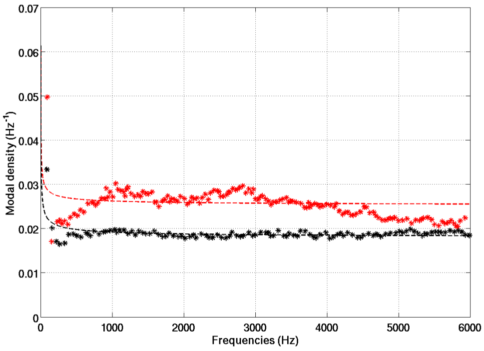
The interest of such a result for the characterization of guitars is to reduce their characteristics to those of a rectangular thin plate which would be equivalent to the guitar. These parameters define the *character* of the instrument. It is helpful for the luthier, notably to objectively quantify the *performance* of the instrument wished by the musician.

3.3. Influence of the bridge on the ukulele frequency response

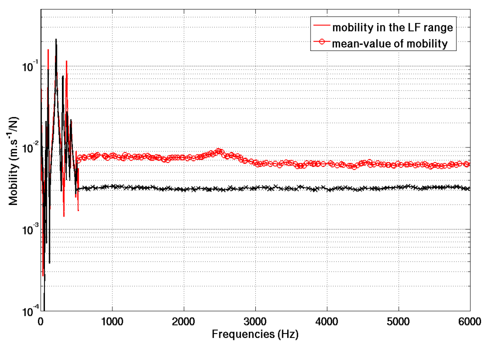
This section gives an example of application of this study to problematics encountered by luthiers. The bridge of guitars, or ukulele in this case, is known to have a large influence on the frequency response, in particular because of its mass which is in the same range than the one of the soundboard. The choice of the material used to make the bridge is one of the problematics encountered by the luthier. This part of the study aims to compare the parameters defined in section 2.2 for different configurations of the same ukulele. First, the admittance without the bridge is measured, then it is measured with bridges, made with different woods (mahogany and rosewood). The configuration with the rosewood bridge is first analyzed in its normal configuration, then the rosewood



(a)



(b)



(c)

Figure 4. Bridge mobility measurements of two different classical guitars. The guitar represented by the black line is a basic industrial guitar. The red line represents an high quality guitar from a luthier. (a) Bridge mobility without processing. (b) Modal density estimation and modal density of the equivalent plate. $L_x = L_y = 0.3$ m, $p = 9.0 \times 10^{-2}$ m^2 and $q = 0.24$ m. (c) mean-value of the mobilty in the mid and high frequency ranges computed for both guitars.

is sawn-off in order to make the bridge less rigid. The masses of the bridges and the soundboard are presented in table I

The modal density profile is similar to the one of the plate, but in this case, the equivalent plate has its four edges simply-supported. The equivalent plate is a square plate (0.135×0.135 m^2), corresponding

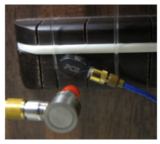
Elements	Mass (g)	Picture
Soundboard	42	
Mahogany bridge	3	
Rosewood bridge	7	
Modified rosewood bridge	6	
Saddle	1	

Table I. Masses and pictures of the different elements of the ukulele

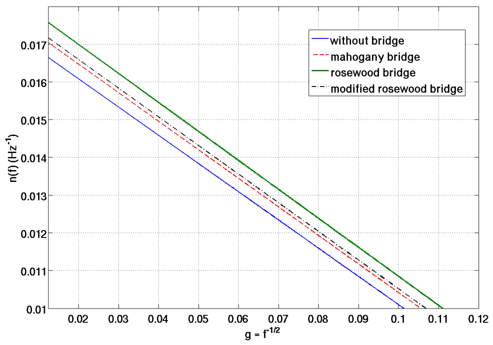
roughly to the dimensions of the lower bout of the soundboard. The values of parameters p and q in expression 7 are $p = 9.1 \times 10^{-3}$ m^2 and $q = -0.054$ m. The figure 5 shows the modal density of the equivalent plates, and the mean-value of mobility of the ukulele for the different configurations.

Since the bridge adds effective mass to the system, the mobility of the instrument with bridge should be less than the mobility of the instrument without bridge. The figure 5 confirms such a tendency. One can notice that the heavier the bridge, the less the mobility. However, the modal density profiles are similar and do not change a lot. It means that the added mass, due to the bridge, is compensated by the added stiffness, the mass to stiffness ratio for the different configurations remaining in the same range. The choice of the wood used for the bridge is therefore more influent on the average mobility.

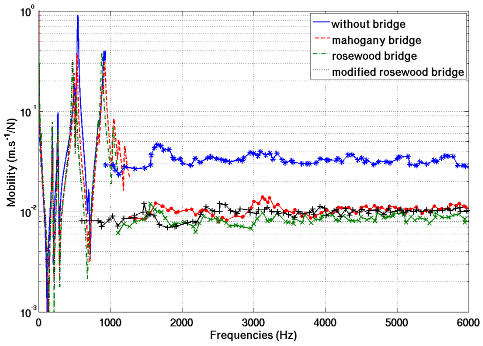
4. Conclusions

The developed approach for this study enables the global description of the mechanical behavior, in the middle and high frequency domains, of guitar and ukulele soundboards. The ESPRIT method, when associated with the signal enumeration technique ESTER enables the estimation of modal parameters with good precision in the mid and high frequency ranges.

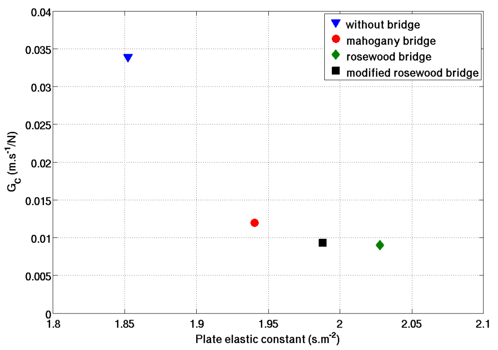
Applications on guitars showed a common feature of their average mobility, this latter presenting a plate-like behavior in the mid and high frequency domains. Thus, the guitar soundboard can be considered as a plate. From the values of modal parameters estimated by ESPRIT and some geometrical assumptions, it is possible to determine the mechanical properties of an equivalent plate (rigidity, surface density, ...), allowing a simple characterization of guitars through these parameters.



(a)



(b)



(c)

Figure 5. (a) Modal density of the equivalent plate. $L_x = L_y = 0.135$ m, $p = 9.1 \times 10^{-3}$ m^2 and $q = -0.054$ m. (b) Mean-value of mobility in the mid and high frequency ranges and (c) comparison of the values of plate elastic constant and average mobility for the different configurations of the ukulele.

Applications on ukulele highlighted the same behavior, except the fact that the plate has now its four edges simply supported. However, the parameters are still relevant to characterize the instrument. A study via these parameters on different configurations of an ukulele permitted to show the influence of the bridge on the frequency response of the instrument.

Thus, the luthier has the possibility to objectively check the adjustments and the settings made on his instrument. These parameters are, *in fine*, intended to be determined from a large corpus of instruments to allow a clustering of instruments.

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References

[1] J. Berthaut, M. N. Ichchou, and L. Jézéquel, "Piano soundboard: Structural behaviorand numerical and experimental study in the modal range", Applied Acoustics **64**, 1113–1136 (2003).

[2] E. Skudrzyk, "The mean value method of predicting the dynamic response of complex vibrators", J. Acoust. Soc. Am. **67(4)**, 1105–1135 (1980).

[3] R. H. Lyon, *Statistical Energy Analysis of Dynamical Systems: Theory and Applications* (MIT Press, Cambridge) (1975).

[4] R. Courant and D. Hilbert, *Methods of Mathematicals Physics and Vol. 1* (Wiley Classic edition) (1989).

[5] G. Xie, D. J. Thompson, and C. J. C. Jones, "Mode count and modal density of structural systems : Relationships with boundary conditions", J. of Sound and Vibration **274(3)**, 621–651 (2004).

[6] N. M. M. Maia and J. M. M. Silva, "Modal analysis identification techniques", Phil. Trans. R. Soc. Lond. **359**, 29–40 (2001).

[7] R. O. Smith, "Multiple emitter location and signal parameter estimation", in *Proc. RADC Spectrum Estimation Workshop*, 243–258 (1979).

[8] Y. Hua and T. K. Sarkar, "Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise", IEEE Trans. Acoust. Speech Sig. Process. **38(5)**, 814–824 (1990).

[9] R. Roy and T. Kailath, "Esprit – estimation of signal parameters via rotational invariance techniques", IEEE Transactions on Acoustics Speech and Signal Processing **37(7)**, 984–995 (1989).

[10] K. Ege, D. Boutillon, and B. David, "High-resolution modal analysis", J. of Sound and Vibration **325**, 852–869 (2009).

[11] R. Badeau, B. David, and G. Richard, "A new pertubation analysis for signal enumeration in rotational invariance techniques", IEEE Transactions on Signal Proceeding **54(2)** (2006).