Proof theory for quantified monotone modal logics

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Non-normal modal logics allow us to deal with many interpretations of modal operators where the schema $\Box(A\&B) \leftrightarrow (\Box A\&\Box B)$ and/or the necessitation rule $\vdash A/\vdash \Box A$ don't seem to hold – e.g., with epistemic [9], deontic [10], and 'high-probability' interpretations [5]. In recent years, the development of analytic sequent calculi for non-normal propositional modal logics has been the object of active investigation, see [4, 7, 8]. In particular [7] extends the applicability of labelled sequent calculi [6] to non-normal propositional modal logics through the internalization of neighbourhood semantics. We extend this approach to the first-order case by internalizing the semantics of monotone neighbourhood frames with constant domains introduced in [1, 2]. Then, neighbourhood frames with varying domains and non-normal logics with free quantification will be considered; these logics has been studied only in the unpublished [3]. Finally, the role of the Barcan Formulas in monotone neighbourhood frames both with constant and varying domains will be analyzed.

This paper provides the first proof-theoretic study of quantified non-normal modal logics. It introduces labelled sequent calculi for the first order extension, both with free and with classical quantification, of all the monotone non-normal modal logics, as well as of some interesting extensions thereof, and it studies the role of the Barcan Formulas in these calculi. It will be shown that the calculi introduced have good structural properties: they have invertibility of the rules, height-preserving admissibility of weakening and contraction, and syntactic cut elimination. It will also be shown that each of the calculi introduced is sound and complete with respect to the appropriate class of neighbourhood frames with either constant or varying domains. In particular the completeness proof constructs a formal proof for derivable sequents and a countermodel for underivable ones, and it gives a semantic proof of the admissibility of cut. Finally, some preliminay results on the extension of our approach to the non-monotonic case are discussed.

References

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