A brief survey of 20th century logical notations

Denis Roegel

1st February 2002

We give here the logical notations used in a certain number of works of logic. Most of 20th century's work of historical importance is covered here. Although the list contains secondary work, the overview given here is not necessarily representative of the whole of contemporary logic.

An empty cell means that either there is no symbol corresponding to a concept, or that I didn't find any by skimming through the book. (Some symbols are rarely used and I may have missed them.) One must not confuse an empty cell with the mention "none." The latter means that the concept exists but that no symbol is associated to it. This is sometimes the case with "and." Some authors write pq for $p \wedge q$ (like one often writes ab for $a \times b$).

Note also that the symbol representations in this document are, in general, approximations of the symbols actually used. The latter can be of a different size, larger in width or in height, bolder or lighter, etc.

Many symbols stem from Peano who inspired himself from Gabelsberger's shorthand alphabet.

Certain works have two dates:

- the first date between parentheses represents a likely but unchecked date; this can be the case for a first edition of a book of which I only consulted a later edition; it is therefore possible that the notations changed between the editions;

- the second date between square brackets corresponds to the date where the notation has been observed; for instance, "Carnap (1934) [1951]" means that I observed the notation in a 1951 edition (actually the third printing of "The logical syntax of language") but the original German publication was in 1934.

The table gives an idea of the introduction dates of the various symbols. However, one should be careful not to draw too hasty conclusions. For instance, it can seem that Schönfinkel introduced "&", "$\rightarrow$" and "(Ex)" in 1924, but actually the author claims to borrow the symbols used by Hilbert in his classes. However, it might be the first published use of these symbols.

Only object language symbols are shown. Some authors also define other symbols for the metalanguage. For instance, Skolem in 1928 has $\rightarrow$ for the consequence relation between formulæ and $\rightleftarrows$ for their equivalence.

Author \ Concept	not	or	and	if/then	for all	exists	iff
Frege (1879)	⊤ A			B / A (conditional stroke)	⌣x⌣ $\Phi(x)$		
Peirce (1885)	$\overline{x}$			—<	Π_i	Σ_j	
Peano (1889)	$-$	∪	∩	Ɔ	Ɔ$_x$		
Schröder (1890) (to check)	$\overline{a}$	+	·	€	$\prod_i$	$\sum_i$	
Peano (ca. 1895) [1901]	$-$	∪	∩	Ɔ	Ɔ$_x$	$\exists x$	
Hilbert (1904)	$\overline{a}$	o.	u.	|			
Russell (1908)	∼	∨	.	⊃	(x)	$\exists x$	≡
Russell &Whitehead (1910–1913)	∼	∨	.	⊃	(x)	$(\exists x)$	≡
Löwenheim (1915)	$\overline{a}$	+	·	€	$\prod_i$	$\sum_i$	
Hilbert (1917/18)	$\overline{X}$	×/none	+	→	(x)		=
Skolem (1920)	$\overline{a}$	+	·		Π_i	Σ_i	
Post (1921)	∼	∨	.	⊃			≡
Tarski (1923)	∼	∨		⊃			
Schönfinkel (1924)	$\overline{a}$	∨	&	→	(x)	$(\mathrm{E}x)$	∼
Ramsey (1925)	∼	∨	.	⊃	(x)	$(\exists x)$	
Hilbert (1925)	$\overline{A}$	∨	&	→	(x)	$(\mathrm{E}x)$	
Kolmogorov (1925)	$\overline{A}$			→	(x)	$(\mathrm{E}x)$	
Hilbert (1927)	$\overline{A}$	∨	&	→	(x)	$(\mathrm{E}x)$	
Skolem (1928)	$\overline{a}$	+	·		Π_i	Σ_i	
Herbrand (1928)	∼	∨	&	⊃	(x)	$(\exists x)$	≡
Hilbert & Ackermann (1928) [1950]	$\overline{A}$	∨	&	→	(x)	$(\mathrm{E}x)$	∼
Łukasiewicz (1929)	Np	Apq	Kpq	Cpq	Px	Sx	Epq
Heyting (1929)	¬	∨	∧	⊃	(x)	$(\mathrm{E}x)$	⊃⊂
Gödel (1930)	$\overline{p}$	∨	&	→	(x)	$(\mathrm{E}x)$	∼
Heyting (1931)	$\neg p$	∨					
Gödel (1931)	$\overline{p}$ (also ∼)	∨	&	→ (also ⊃)	(x) (also $x\Pi$)	$(\mathrm{E}x)$	≡
Herbrand (1931)	∼	∨	×	→	(x)	$(\mathrm{E}x)$	≡
Gentzen (1934)	¬	∨	&	⊃	$\forall x$	$\exists x$	⊃⊂
Hilbert & Bernays (1934–39)	$\overline{A}$	∨	&	→	(x)	$(\mathrm{E}x)$	∼
Carnap (1934) [1951]	∼	∨	·	⊃	(x)	$(\exists x)$	≡
Tarski (1940)	∼	∨	∧	→	A_x	E_x	↔
Quine (ML, 1940)	∼	∨	.	⊃	(x)	$(\exists x)$	≡
Henkin (1949)	∼			⊃	(x)	$(\exists x)$	
Henkin (1950)	∼	∨	none	⊃	(x)	$(\exists x)$	
Kleene (IM, 1952)	¬	∨	&	⊃	$\forall x$	$\exists x$	∼
Carnap (1954)	∼	∨	.	⊃	(x)	$(\exists x)$	≡
Church (1956)	∼	∨	none	⊃	(x)	$(\exists x)$	≡
Suppes (IL, 1957)	$-p$	∨	&	→	(x)	$(\exists x)$	↔
Davis (C&U, 1958) [1982]	$-A$		&	⊃	(x)	$(\exists x)$	
Bernays (AST, 1958) [1968]	$\overline{p}$	∨	&	→	(x)	$(\mathrm{E}x)$	↔
Suppes (AST, 1960)	$-p$	∨	&	→	$\forall x$	$\exists x$	↔
Quine (STL, 1963)	∼			⊃	(x)	$(\exists x)$	≡
Mendelson (1964) [1968]	∼	∨	∧	⊃	(x)	$(\mathrm{E}x)$	≡
Kleene (LM, 1967) [1971]	¬	∨	&	⊃	$\forall x$	$\exists x$	∼
Smullyan (1968)	∼	∨	∧	⊃	$(\forall x)$	$(\exists x)$	↔
Blanché (1968)	∼	∨	.	⊃	(x)	$(\exists x)$	≡
Putnam (1971)	∼	∨	∧	⊃	(x)	$(\exists x)$	≡
Boolos & Jeffrey (1974) [1996]	$-p$	∨	&	→	$\forall x$	$\exists x$	↔
Copi (SL, 1979)	∼	∨	·	⊃	(x)	$(\exists x)$	≡
Quine (1982)	$-p$	∨	none	→	$\forall x$	$\exists x$	↔
Vax (1982)	∼	∨	∧	⇒	$\forall x$	$\exists x$	⇔
Rivenc (1989)	¬	∨	∧	→	$\forall x$	$\exists x$	↔
Gochet & Gribomont (1990)	¬	∨	∧	⊃	$\forall x$	$\exists x$	≡
Lalement (1990)	¬	∨	∧	⇒	$\forall x$	$\exists x$	⇔
Ruyer (1990)	¬	∨	∧	→	$\forall x$	$\exists x$	↔
Smullyan (1992)	∼	∨	∧	⊃	$\forall x$	$\exists x$	≡
Largeault (1992)	¬	∨	&	→	$\forall x$	$\exists x$	↔
Cori & Lascar (1993)	¬	∨	∧	⇒	$\forall x$	$\exists x$	⇔
Largeault (1993)	¬	∨	&	→	$\forall x$	$\exists x$	↔
Chazal (1996)	¬	∨	∧	⊃	$\forall x$	$\exists x$	⬭
Gauthier (1997)	∼		∧	⊃	$\forall x$	$\exists x$	
Leroux (1998)	¬	∨	∧	⊃	$\forall x$	$\exists x$	≡
Encyclop. Britannica (1999)	∼	∨	none/·	⊃	$(\forall x)$	$(\exists x)$	≡

Sources

- Frege (1879): *Begriffsschrift* (Halle: Louis Nebert, translated in "From Frege to Gödel")
- Peirce (1885): *On the Algebra of Logic; A Contribution to the Philosophy of Notation* (American Journal of Mathematics 7: 180–202).
- Peano (1889): *Arithmetices principia, nova methodo exposita* (Augustæ Taurinorum, Ed. Fratres Bocca. — XVI, 20 p, translated in "From Frege to Gödel" as "The principles of arithmetic")
- Schröder (1890): *Vorlesungen über die Algebra der Logik (exakte Logik)* (volume 1, Leipzig: Teubner)
- Peano (ca. 1895): *Formulaire de mathématiques* (according to Cajori, vol. 2, page 300, Peano used ∃ in volume II, number 1, of his *Formulaire de mathématiques*, which was published in 1897)
- Hilbert (1904): *Über die Grundlagen der Logik und der Arithmetik* (Verhandlungen des Dritten Internationalen Mathematiker-Kongresses in Heidelberg vom 8. bis 13. August 1904, 174–185) (in van Heijenoort, "u." is replaced by "a.") (translated as "On the foundations of logic and arithmetic" in "From Frege to Gödel")
- Russell (1908): *Mathematical logic as based on the theory of types* (American Journal of Mathematics 30: 222–262, reproduced in "From Frege to Gödel")
- Russell & Whitehead (1910–1913): *Principia Mathematica* (Cambridge: At the University Press)
- Löwenheim (1915): *Über Möglichkeiten im Relativkalkül* (Mathematische Annalen 76: 447–470, translated as "On possibilities in the calculus of relatives" in "From Frege to Gödel")
- Hilbert (1917/1918): *Prinzipien der Mathematik* (Lecture notes by Paul Bernays, Winter-Semester 1917–1918, Unpublished typescript. Bibliothek, Mathematisches Institut, Universität Göttingen) (personal communication by Richard Zach)
- Skolem (1920): *Logisch-kombinatorische Untersuchungen über die Erfüllbarkeit und Beweisbarkeit mathematischen Sätze nebst einem Theoreme über dichte Mengen.* (Videnskabsakademiet i Kristiania, Skrifter I, No. 4, 1919, 1–36, translated as "Logico-combinatorial investigations in the satisfiability or provability of mathematical propositions: A simplified proof of a theorem by L. Löwenheim and generalizations of the theorem" in "From Frege to Gödel")
- Post (1921): *Introduction to a general theory of elementary propositions* (American Journal of Mathematics 43: 163–185, also in "From Frege to Gödel")
- Tarski (1923): *On the Primitive Term of Logistic* (in Tarski: "Logic, Semantics, Metamathematics", Oxford University Press, 1956)
- Schönfinkel (1924): *Über die Bausteine der mathematischen Logik* (Mathematische Annalen 92: 305–316, translated as "On the building blocks of mathematical logic" in "From Frege to Gödel")
- Ramsey (1925): *The Foundations of Mathematics* (Proceedings of the London Mathematical Society, series 2, 25: 338–384, also in F. P. Ramsey: "Philosophical Papers", Cambridge University Press, 1990)
- Hilbert (1925): *Über das Unendliche* (Mathematische Annalen (1926) 95: 161–190, translated as "On the infinite" in "From Frege to Gödel")
- Kolmogorov (1925): О принципе *tertium non datur* (Математический сборник, 32: 646–667, translated as "On the principle of excluded middle" in "From Frege to Gödel")
- Hilbert (1927): *Die Grundlagen der Mathematik* (Abhandlungen aus dem mathematischen Seminar der Hamburgischen Universität 6: 65–85, translated as "The foundations of mathematics" in "From Frege to Gödel")
- Skolem (1928): *Über die mathematische Logik* (Norsk matematisk tidsskrift 10: 125–142, translated as "On mathematical logic" in "From Frege to Gödel")
- Herbrand (1928): *Sur la théorie de la démonstration* (Comptes rendus hebdomadaires des séances de l'Académie des sciences, Paris, 186 : 1274–1276, translated as "On proof theory" in "Logical Writings")
- Hilbert & Ackermann (1928): *Grundzüge der Theoretischen Logik* (translated as "Principles of Mathematical Logic") (Berlin: Springer-Verlag)
- Łukasiewicz (1929): *Elementy logiki matematycznej* (Warsaw) (translated as "Elements of mathematical logic", New York: Macmillan, 1963)
- Heyting (1929): *Die formalen Regeln der intuitionistischen Logik* (Sitzungsberichte der Preußischen Akademie der Wissenschaften zu Berlin, Physikalisch-mathematische Klasse, 42–56, 57–71, 158–169, 1930)
- Gödel (1930): *Die Vollständigkeit der Axiome des logischen Funktionenkalküls* (Monatshefte für Mathematik und Physik 37: 349–360, translated as "The completeness of the axioms of the functional calculus of logic" in "From Frege to Gödel")
- Heyting (1931): *Die intuitionistische Grundlegung der Mathematik* (Erkenntnis, 2: 106–115)
- Gödel (1931): *Über formal unentscheidbare Sätze der Principia mathematica und verwandter Systeme I* (Monatshefte für Mathematik und Physik 38: 173–198, translated as "On formally undecidable propositions of Principia Mathematica and related systems I" in "From Frege to Gödel")
- Herbrand (1931): *Sur la non-contradiction de l'arithmétique* (Journal für die reine und angewandte Mathematik 166: 1–8, translated as "On the consistency of arithmetic" in van Heijenoort's "From Frege to Gödel", where the correct symbols are given, which is not the case in the "Logical Writings" (1971, edited by Goldfarb) which use more standard symbols)

- Gentzen (1934): *Untersuchungen über das logische Schließen* (Mathematische Zeitschrift 39: 176–210, 405–431) (in the French translation “Recherches sur la déduction logique”, 1955, the symbols are ⫟, $\vee$, &, $\supset$, $\forall x$, Ǝx, $\supset\subset$)
- Hilbert & Bernays (1934–1939): *Grundlagen der Mathematik* (Berlin: Springer-Verlag, translated in French as “Fondements des mathématiques,” Paris: L’Harmattan, 2001)
- Carnap (1934): *Logische Syntax der Sprache* (Wien: Julius Springer, translated as “The Logical Syntax of Language”, 1937)
- Tarski (1940): *Introduction to logic and to the methodology of deductive sciences* (New York: Oxford university press)
- Quine (1940): *Mathematical Logic* (New York: W. W. Norton & company, inc.)
- Henkin (1949): *The completeness in the first order functional calculus* (Journal of Symbolic Logic, 14: 150–166)
- Henkin (1950): *Completeness in the theory of types* (Journal of Symbolic Logic, 15(2): 81–91)
- Kleene (1952): *Introduction to Metamathematics* (Princeton, NJ: D. van Nostrand)
- Carnap (1954): *Einführung in die symbolische Logik* (translated as “Introduction to Symbolic Logic and its Applications”, 1958) (Wien: Springer)
- Church (1956): *Introduction to Mathematical Logic* (Princeton: Princeton University Press)
- Suppes (1957): *Introduction to Logic* (New York: Van Nostrand Reinhold Company)
- Davis (1958): *Computability and Unsolvability* (New York: McGraw Hill)
- Bernays (1958): *Axiomatic Set Theory* (Amsterdam: North-Holland)
- Suppes (1960): *Axiomatic Set Theory* (Princeton, N.J.: Van Nostrand)
- Quine (1963): *Set Theory and its Logic* (Cambridge: Belknap Press of Harvard University Press)
- Mendelson (1964): *Introduction to Mathematical Logic* (Princeton, NJ: Van Nostrand)
- Kleene (1967): *Mathematical Logic* (New York: John Wiley)
- Smullyan (1968): *First-order Logic* (New York: Springer-Verlag)
- Blanché (1968): *Introduction à la logique contemporaine* (Paris : A. Colin)
- Putnam (1971): *Philosophy of Logic* (New York: Harper and Row)
- Boolos & Jeffrey (1974): *Computability and Logic* (Cambridge University Press)
- Copi (1979): *Symbolic Logic* (New York: Macmillan)
- Quine (1982): *Methods of logic* (previous editions had Russell and Whitehead’s symbols) (Cambridge, Mass.: Harvard University Press)
- Vax (1982): *Lexique logique* (Paris : Presses Universitaires de France)
- Rivenc (1989): *Introduction à la logique* (Paris : Payot)
- Gochet & Gribomont (1990): *Logique, volume 1 : méthodes pour l’informatique fondamentale* (Paris : Hermès)
- Lalement (1990): *Logique, réduction, résolution* (Études et recherches en informatique, Paris : Masson) (translated as “Computation as logic”, Englewood Cliffs, NJ: Prentice-Hall, 1993)
- Ruyer (1990): *Logique* (Paris : Presses Universitaires de France)
- Smullyan (1992): *Gödel’s Incompleteness Theorems* (New York: Oxford University Press)
- Largeault (1992): *L’intuitionisme* (Que sais-je ? 2684, Paris : Presses Universitaires de France)
- Cori & Lascar (1993): *Logique mathématique. Cours et exercices* (Paris : Masson, translated in English as “Mathematical logic: A Course with Exercises”, 2 volumes, New York: Oxford University Press, 2000 and 2001)
- Largeault (1993): *La logique* (Que sais-je ? 225, Paris : Presses Universitaires de France)
- Chazal (1996): *Éléments de logique formelle* (Paris : Hermès)
- Gauthier (1997): *Logique et fondements des mathématiques* (Paris : Diderot Éditeur)
- Leroux (1998): *Introduction à la logique* (Bibliothèque des sciences, Paris : Diderot Éditeur)
- Encyclop. Britannica (1999):

Collections mentionned:

- Jean van Heijenoort: “From Frege to Gödel”, Cambridge, Mass.: Harvard University Press, 1967
- Jacques Herbrand: “Logical Writings”, Cambridge, Mass.: Harvard University Press, 1971
- F. P. Ramsey: “Philosophical Papers”, Cambridge University Press, 1990
- Alfred Tarski: “Logic, Semantics, Metamathematics”, Oxford University Press, 1956