Supplemental Material

The Method of Moving Frames for Surface Global Parametrization

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Contents

Co	n	teı	nt	S

1	Influence of initialization and distortion over the final result	2
2	Comparison of quad meshes	3
3	Comparison with Direct Seamless Parametrization	4
4	Quadmesh Gallery	5
5	Model Gallery	6
5.1	Failure cases	6
5.2	Models without feature edges	7
5.3	Models with feature edges	50
Refe	erences	95

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1:2 • Coiffier and Corman

1 INFLUENCE OF INITIALIZATION AND DISTORTION OVER THE FINAL RESULT

In this section, we extend the analysis of Fig. 9 and Section 8.3 to the other distortion metrics optimized through our work. For both \mathcal{D}_{ARAP} and \mathcal{D}_{LSCM} , the optimization converges to a very similar cone distribution, provided a reasonable initialization. On the other hand, \mathcal{D}_{AREA} only has a limited impact on the final cone distribution. This is due to the fact that the class of area-preserving deformation is not constraining enough for the topology to evolve during optimization.





2 COMPARISON OF QUAD MESHES

Here, we compare global statistics over quadmeshes obtained by computing an integer grid map using [Bommes et al. 2013] and by then extracting quads with [Ebke et al. 2013]. Our parametrization was obtained without optimizing for a distortion, and initialized with curvature directions. According to the histograms plotted for the angle distribution and the stretch distortion, our method leads to less stretched and more rectangular quads.



Fig. 2. Comparison of quadmeshes obtained by three concurrent algorithms with ours on the dancer model. Top row: quad meshes. Second row: angle distribution over the model. Third row: histograms of stretch distortion over elements. Our method yields minimal stretch and most orthogonal quads.

1:4 • Coiffier and Corman

3 COMPARISON WITH DIRECT SEAMLESS PARAMETRIZATION

In this experiment, we compare the stretch distortion over all triangles on 12 common models with the Direct Seamless Parametrization (DSP) method [Levi 2021]. DSP aim at minimizing the maximal stretch distortion over triangle, while our method minimizes the mean value. Overall, while the maximal stretch is higher for us in every cases (for a limited amount of triangles around singularities), we obtain a lower value of stretch distortion for a majority of the triangles.



Fig. 3. Stretch distortion histograms for 12 common models with [Levi 2021]. While our maximal value of stretch over triangles is greater for every model, a majority of triangles present less distortion with our method.

4 QUADMESH GALLERY



Fig. 4. Render of some quad models extracted from our parameterizations.

1:6 • Coiffier and Corman

5 MODEL GALLERY

In this section, we provide statistics and renders for the 276 models of our database. For each test case, we provide

- the triangle count,
- the final value of our objective function (a value >1e-4 is considered a failed optimization),
- the number of cones,
- the mean *scale* and *stretch* distortion over all triangles

for two cases of optimization. The first case is the one described in the result section of the article : simple optimization without distortion. The second case is the minimization of \mathcal{D}_{LSCM} .

Additionally, we display a render of the model with its parametrization as a checker texture, seams and feature edges as black lines and singularity cones as blue (for $-\pi/2$) or red (for index $\pi/2$) points.

On the left column, we show an histogram of the stretch distortion over all triangles.

5.1 Failure cases

Over a total of 274 models, 19 failed to produce a valid seamless parametrization without distortion. Statistics for these models as well as the broken parameterization are still presented below.

On the database without features (131 models), 8 models failed due to bad geometry : *ramses, femur, block, twirl, focal-octa, mannequin-devil, robocat-deci, sharp-sphere*. For the models *ramses* and *femur*, we provide correct results on a remeshing with a similar number of triangles and level of details (*ramses_r* and *femur_r*). For the models *sharp-sphere, block* and *focal-octa*, it is worth mentioning that our algorithm still converges when \mathcal{D}_{LSCM} is minimized.

On the database with feature edges (143 models), the 11 failed results are : *metatron*, *S22*, *S24*, *S25*, *S26*, *abca_00004035*, *vis*, *B21* and *B49*. The two latter are the two failure cases due to a limit cycle, and did not output any parametrization. 3 other models (*block*, *sharp-sphere* and *twirl*) were already counted in the no feature case and are still failing here.

For the *S24* model, we provide correct results for a remeshing of the model (*S24r*). A similar remeshing operation fixes at least models *S22*, *S25* and *S26*.

The case where a model provides a valid parametrization without distortion but fails when optimized for \mathcal{D}_{LSCM} also happens 8 times for some high triangle count models, namely *brain100K*, *grayloc*, *igea100k*, *rgb_dragon*, *B31*, *M7*, *M9* and *S9*. This is due to the triggering of one of the early stopping of the optimization : either the maximal number of iterations was reached, or the energy gained in the last few iterations was too small.























































riangles:	100132		25%	No dist.
	No dist.	min LSCM	2070	min LSCI
Energy:	1.120E-08	1.012E-07	₫ ^{20%}	
Time:	2363s	4172s	0° 15% -	
#Singus:	56	66	80 E E 10%	
Scale:	1.012	1.042	E 1070	
Stretch:	1.142	1.029	5% -	
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Energy: Time: #Singus: Scale: Stretch:	No dist. 6.176E-08 4503s 215 1.009 1.143	min LSCM 3.740E-07 5514s 217 1.063 1.053		12% - 10% - 10% - 8% - 8% - 4% - 2% -		No dist.
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Triangles:	19324			25%	No dist.
	No dist.	min LSCM		20%	min LSCM
Energy:	2.175E-08	4.352E-07	3 F	a tr	
Time:	345s	598s		රි 15% - ප	
#Singus:	24	24		10%	
Scale:	1.01	1.034		F	
Stretch:	1.144	1.03		5% -	































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Time	3551c	3187e	- EN F	Com 0				
#Singus:	155	161		1% angle		N		
Scale:	1.022	1.017		۲. ۵%		1.		
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	I	1		0%				
			-	1.0) 1.2	1.4 1 Stretch: $\sigma_{1/}$.6 1.8 σ_2	3 2.0







































No dist. min LSCM Energy: 2.965E-08 4.742E-05 Time: 1868s 4002s #Singus: 20 20 Scale: 1.007 1.044 Stretch: 1.119 1.008	Triangles:	99769		70%	No dist.
Energy: 2.965E-08 4.742E-05 Time: 1868s 4002s #Singus: 20 20 Scale: 1.007 1.044 Stretch: 1.119 1.008		No dist.	min LSCM	60% -	min LSCM
Time: 1868s 4002s #Singus: 20 20 Scale: 1.007 1.044 Stretch: 1.119 1.008	Energy:	2.965E-08	4.742E-05	₫ ^{50%}	
#Singus: 20 20 Scale: 1.007 1.044 Stretch: 1.119 1.008	Time:	1868s	4002s	° 40%.	
Scale: 1.007 1.044 Stretch: 1.119 1.008 10%	#Singus:	20	20	ше 30% -	
Stretch: 1.119 1.008	Scale:	1.007	1.044	20% -	
	Stretch:	1.119	1.008	10% -	
			Į.		






















riangles:	100096		25%	No dist.
	No dist.	min LSCM	20%	min LSCM
Energy:	6.366E-08	1.136E-07	ti and the second secon	
Time:	1840s	4086s	S 15% -	
#Singus:	52	48		
Scale:	1.015	1.089	E	
Stretch:	1.18	1.027	5% -	



































Energy: Time: #Singus: Scale: Stretch:	No dist. 1.380E-08 1862s 22 1.007 1.13	min LSCM 1.920E-07 4381s 24 1.07 1.018		30% 25% 20% 10% 5% 0%	No dist. min LSCM
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riangles:	12370		80%	No dist.
	No dist.	min LSCM	0070	min LSCM
Energy:	2.181E-08	9.009E-08	貫 60% -	
Time:	91s	441s	e Co	
#Singus:	10	10	50 40% -	
Scale:	1.003	1.005	E	
Stretch:	1.049	1.012	20%	

















uu-mem Triangles: :	ento100k 200182			30% No dist.	
	No dist.	min LSCM		2507	J
Energy:	4.459E-08	1.198E-07		23%	
Time:	7323s	12176s		3 20% -	
#Singus:	98	98	11 19 G	15%	
Scale:	1.013	1.064		[↓] 10% -	
Stretch:	1.162	1.025		5% -	
				0% 1.0 1.2 1.4 1.6 1.8 2.0 Stretch: σ_1/σ_2	0











1:50 • Coiffier and Corman

5.3 Models with feature edges











riangles:	4262		25%	No dist.
	No dist.	min LSCM	20% -	min LSC
Energy:	2.190E-08	2.013E-06	a a a a a a a a a a a a a a a a a a a	
Time:	27s	73s	ပိ 15% -	
#Singus:	8	8		
Scale:	1.006	1.084	E III	
Stretch:	1.133	1.046	5% -	





























riangles:	2798		20%	No dist.
	No dist.	min LSCM		min LSCM
Energy:	1.391E-07	6.911E-07	15% -	
Time:	24s	57s	Ŭ ₹ 1007	
#Singus:	14	8		
Scale:	1.017	1.131	5%	
Stretch:	1.211	1.074		





riangles:	4624		1507	No dist.
	No dist.	min LSCM	19%	min LSC
Energy:	2.907E-08	1.930E-06	12/0 1007	
Time:	32s	87s	e 10%	
#Singus:	16	16	8% -	
Scale:	1.004	1.18	F 5% -	
Stretch:	1.162	1.167	2% -	



































inangies.				4%	No dist.
	No dist.	min LSCM		3% -	min LSCM
Energy:	5.592E-08	1.476E-07	$h \rightarrow h$	불 2% -	
Time:	19s	43s	105 - 1	0° 2% -	
#Singus:	8	8		10 2% -	
Scale:	1.023	1.031		⊢ 1%·	
Stretch:	1.31	1.321		0% -	







B50 Triangles: Energy: Time: #Singus: Scale: Stretch:	1990 No dist. 1.821E-08 21s 12 1.033 1.463	min LSCM 4.900E-07 45s 8 1.089 1.363		$\begin{array}{c} 6\% \\ 5\% \\ 5\% \\ 6\% \\ 7\% \\ 1\% \\ 1\% \\ 1\% \\ 1\% \\ 1\% \\ 10 \\ 1.2 \\ 1.4 \\ 1.6 \\ 1.6 \\ 1.8 \\ 2.0 \\ \text{Stretch: } \sigma_1/\sigma_2 \end{array}$
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riangles:	2758			14%	No dist.
	No dist.	min LSCM		12% -	min LSCM
Energy:	3.786E-08	2.197E-07		Ħ 10%	
Time:	16s	56s		ပိ _{8%} -	
#Singus:	8	8	Y	.ee 6% -	
Scale:	1.017	1.08		4%	
Stretch:	1.128	1.074		2% -	




























































































































abca_000 Triangles:	03957 1060			14% No dist.
	No dist.	min LSCM		12% - min LSCM
Energy:	3.173E-08	1.177E-05	1200	10% -
Time:	7s	25s		Ŭ 8% -
#Singus:	12	12		
Scale:	1.012	1.019		
Stretch:	1.131	1.106		2%
				$0\% \begin{bmatrix} 1.0 & 1.2 & 1.4 & 1.6 & 1.8 & 2.0 \\ 1.0 & 1.2 & 1.4 & 1.6 & 1.8 & 2.0 \\ Stretch: \sigma_1/\sigma_2 \end{bmatrix}$





Tangies.	/04			No dist.
	No dist.	min LSCM	5% -	min LSCM
Energy:	2.301E-08	1.679E-08	1 4% -	
Time:	11s	55s	ပိ ച 3% -	
#Singus:	28	26	liang	
Scale:	1.018	1.045	E 2%	
Stretch:	1.35	1.341	1% -	











riangles:	012		100% -	No dist.
	No dist.	min LSCM	80% -	min LSC!
Energy:	3.007E-12	4.111E-12	ant	
Time:	2s	2s	ට 60% - ප	
#Singus:	0	0	-ie 40%	
Scale:	1.0	1.0	F	
Stretch:	1.0	1.0	20% -	























REFERENCES

David Bommes, Marcel Campen, Hans-Christian Ebke, Pierre Alliez, and Leif Kobbelt. 2013. Integer-grid maps for reliable quad meshing. ACM Transactions on Graphics (TOG) 32, 4 (2013), 1–12.

Olga Diamanti, Amir Vaxman, Daniele Panozzo, and Olga Sorkine-Hornung. 2015. Integrable polyvector fields. ACM Transactions on Graphics (TOG) 34, 4 (2015), 1–12.

Hans-Christian Ebke, David Bommes, Marcel Campen, and Leif Kobbelt. 2013. QEx: Robust quad mesh extraction. ACM Transactions on Graphics (TOG) 32, 6 (2013), 1–10.

Xianzhong Fang, Hujun Bao, Yiying Tong, Mathieu Desbrun, and Jin Huang. 2018. Quadrangulation through morse-parameterization hybridization. ACM Transactions on Graphics (TOG) 37, 4 (2018), 1–15.

Zohar Levi. 2021. Direct Seamless Parametrization. ACM Transactions on Graphics (TOG) 40, 1 (2021), 1-14.

Ryan Viertel and Braxton Osting. 2019. An Approach to Quad Meshing Based on Harmonic Cross-Valued Maps and the Ginzburg–Landau Theory. SIAM Journal on Scientific Computing 41, 1 (2019), A452–A479.