Safe Navigation in Urban Environments

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Example

After a visit to the Philadelphia Museum of Art

Rocky wants to walk home, on Warthon street.
Example

the shortest path
Example

the safest path

Problem
Example

the **shortest path** or the **safest path**?
Example

trade-offs

<table>
<thead>
<tr>
<th>Path</th>
<th>Length $\ell$ (m)</th>
<th>Risk $r \times 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3955</td>
<td>2.32</td>
</tr>
<tr>
<td>2</td>
<td>4027</td>
<td>2.02</td>
</tr>
<tr>
<td>3</td>
<td>4060</td>
<td>2.01</td>
</tr>
<tr>
<td>4</td>
<td>4922</td>
<td>1.71</td>
</tr>
<tr>
<td>5</td>
<td>5988</td>
<td>1.70</td>
</tr>
</tbody>
</table>
Road network

Exported from OpenStreetMap

Nodes  Intersections

Edges  Street segments

Weight $\ell(e)$  Physical length
### Risk model

Built from publicly available civic datasets

<table>
<thead>
<tr>
<th>DISPATCH_DATE_TIME, TEXT_GENERAL_CODE, POINT_X, POINT_Y</th>
<th></th>
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<tbody>
<tr>
<td>2010-05-05 11:34:00, Robbery No Firearm, -75.17699712, 40.0528852</td>
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<td>2010-03-31 12:47:00, Thefts, -75.22697891, 39.99847574</td>
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<td>2007-08-13 07:40:00, Theft from Vehicle, -75.24469105, 39.96263542</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>
Risk model

Built from publicly available civic datasets

1. Obtain a spatial density with Kernel Density Estimation
Risk model

Built from publicly available civic datasets

1. Obtain a spatial density with Kernel Density Estimation
2. Evaluate along street segments and normalize

Weight $r(e)$ relative crime probability
Definitions

Given a graph $G = (V, E)$, a path $P$ from $s$ to $t$.

**Length of $P$**  
\[ \ell(P) = \sum_{e \in P} \ell(e) \]
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**Risk of $P$** \( r_t(P) = 1 - \prod_{e \in P} (1 - r(e)) \)
Given a graph $G = (V, E)$, a path $P$ from $s$ to $t$.

**Length** of $P$ $\ell(P) = \sum_{e \in P} \ell(e)$

**Risk** of $P$ $r_t(P) = 1 - \prod_{e \in P} (1 - r(e))$, or $(total\ risk)$

$r_m(P) = \max_{e \in P} r(e)$. $(max\ risk)$
**Definitions**

*P dominates* $P'$ if

\[ \ell(P) \leq \ell(P') \text{ and } r(P) < r(P') \], or

\[ \ell(P) < \ell(P') \text{ and } r(P) \leq r(P') \].
Problem (SafePaths)

If $\mathcal{P}$ is the set of all possible paths from $s$ to $t$, our goal is to select a small subset of these paths $\mathcal{I} \subseteq \mathcal{P}$ such that for every path $P \in \mathcal{I}$, $P$ is neither dominated by nor equivalent to any other path $P' \in \mathcal{I}$. 
Total-Paths

Solving SafePaths with length and total risk:

\[ \ell(P) = \sum_{e \in P} \ell(e) \]

\[ r_t(P) = 1 - \prod_{e \in P} (1 - r(e)) \]
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Turn product of probabilities into sum of logs.
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**sum-sum** biobjective minimization problem
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**sum-sum** biobjective minimization problem

- Sum-Recursive
Max-Paths

Solving SafePaths with length and max risk:

\[
\ell(P) = \sum_{e \in P} \ell(e)
\]

\[
r_m(P) = \max_{e \in P} r(e)
\]
Max-Paths

Solving SafePaths with length and max risk:
\[
\ell(P) = \sum_{e \in P} \ell(e)
\]
\[
r_{\text{m}}(P) = \max_{e \in P} r(e)
\]

**sum-max** biobjective minimization problem
Max-Paths

Solving SafePaths with length and max risk:

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\ell(P) = \sum_{e \in P} \ell(e)
\]
\[
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\]

**sum-max** biobjective minimization problem

- Max-Exhaustive
Max-Paths

Solving SafePaths with length and max risk:

\[
\ell(P) = \sum_{e \in P} \ell(e)
\]

\[
r_m(P) = \max_{e \in P} r(e)
\]

**sum-max** biobjective minimization problem

- Max-Exhaustive
- Max-Recursive
Speed-ups

- Approximation via early stopping
Speed-ups

- Approximation via early stopping
- Ellipse pruning
Speed-ups

- Approximation via early stopping  \((\text{subset of paths})\)
- Ellipse pruning  \((\text{same set of paths})\)
Setting

- Sample \((s, t)\) pairs

Experiments
Setting

- Sample \((s, t)\) pairs
- Focus on short distances
Setting

- Sample \((s, t)\) pairs
- Focus on short distances
Setting

- Sample \((s, t)\) pairs
- Focus on short distances
- Run algorithms with/without approximation and pruning
Setting

- Sample \((s, t)\) pairs
- Focus on short distances
- Run algorithms with/without approximation and pruning
- Compare solution quality and speed
Results

- Fairly accurate approximations
Results

- Fairly accurate approximations
- Significant speed-ups
Conclusions

- Built a risk model from civic datasets
- Formalized a problem of safe urban navigation
  *Two variants*: total-risk and max-risk
- Proposed practical algorithmic solutions
Conclusions

- Built a risk model from civic datasets
- Formalized a problem of safe urban navigation
  - Two variants: total-risk and max-risk
- Proposed practical algorithmic solutions

Thanks!
Sum-Recursive($G, \ell(), r()$)

1: $P_\ell^* \leftarrow \text{Dijkstra}(G, \ell())$
2: $P_r^* \leftarrow \text{Dijkstra}(G, r())$
3: $\mathcal{S} \leftarrow \{P_\ell^*, P_r^*\}$
4: $\text{sum-rec}(P_\ell^*, P_r^*, \mathcal{S})$
5: return $\mathcal{S}$
Routine \text{sum-rec}(P_u, P_l, \mathcal{S})

1: $\lambda \leftarrow (r(P_u) - r(P_l))/(\ell(P_u) - \ell(P_l))$
2: $\forall e \in E : f(e) = r(e) - \lambda \ell(e)$
3: $P_i \leftarrow \text{Dijkstra}(G, f())$
4: if $P_i \neq P_u$ and $P_i \neq P_l$ then
5: $\mathcal{S} \leftarrow \mathcal{S} \cup \{P_i\}$
6: $\text{sum-rec}(P_u, P_i, \mathcal{S})$
7: $\text{sum-rec}(P_i, P_l, \mathcal{S})$
Routine \texttt{max-rec}(P_u, P_l, \mathcal{I})

1: \( \rho \leftarrow (r(P_u) + t(P_l))/2 \)
2: if \( \exists e \in E, t(P_l) < r(e) < \rho \) then
3: \( P_i \leftarrow \text{Dijkstra}(G_{r(e)<\rho}, \ell()) \)
4: if \( P_i \neq P_l \) then
5: \( \mathcal{I} \leftarrow \mathcal{I} \cup \{P_i\} \)
6: \( \text{max-rec}(P_i, P_l, \mathcal{I}) \)
7: \( \text{max-rec}(P_u, P_i, \mathcal{I}) \)
<table>
<thead>
<tr>
<th></th>
<th>Total-Paths</th>
<th>Max-Paths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Chicago</td>
<td>Philadelphia</td>
</tr>
<tr>
<td>Total</td>
<td>2400</td>
<td>2400</td>
</tr>
<tr>
<td>$</td>
<td>S</td>
<td>= 1$</td>
</tr>
<tr>
<td>$</td>
<td>S</td>
<td>= 2$</td>
</tr>
<tr>
<td>Ident. $\mathbb{A}_5$</td>
<td>520</td>
<td>476</td>
</tr>
<tr>
<td>Ident. $\mathbb{A}_{10}$</td>
<td>420</td>
<td>407</td>
</tr>
<tr>
<td>Ident. $\mathbb{A}_{20}$</td>
<td>337</td>
<td>327</td>
</tr>
</tbody>
</table>