Safe Navigation in Urban Environments

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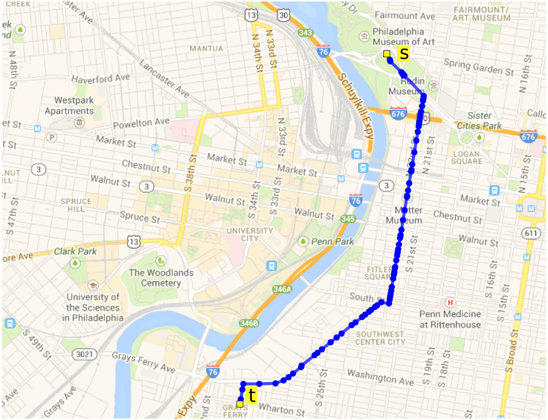
Example

After a visit to the
Philadelphia Museum of Art

Rocky wants to walk home,
on Warthon street.

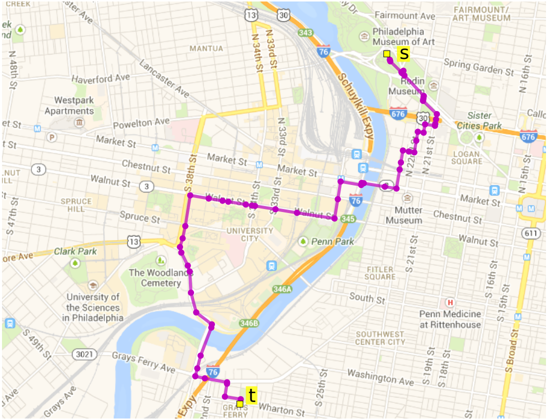
Example

the **shortest path**



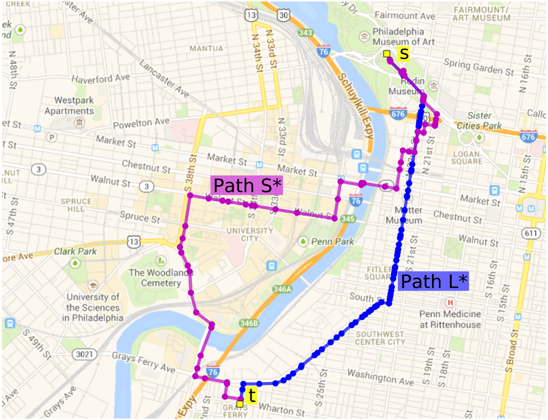
Example

the **safest path**



Example

the **shortest path** or the **safest path** ?



Example

trade-offs

Path	Length ℓ (m)	Risk r (10^{-3})
1	3955	2.32
2	4027	2.02
3	4060	2.01
4	4922	1.71
5	5988	1.70

Road network

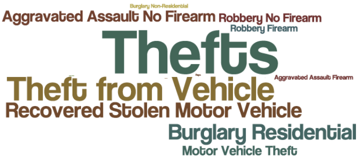
Exported from OpenStreetMap

- **Nodes** Intersections
- **Edges** Street segments
- **Weight $\ell(e)$** Physical length

Risk model

Built from publicly available civic datasets

```
DISPATCH_DATE_TIME,TEXT_GENERAL_CODE,POINT_X,POINT_Y
2010-05-05 11:34:00,Robbery No Firearm,-75.17699712,40.0528852
2010-03-31 12:47:00,Thefts,-75.22697891,39.99847574
2007-08-13 07:40:00,Theft from Vehicle,-75.24469105,39.96263542
...
```



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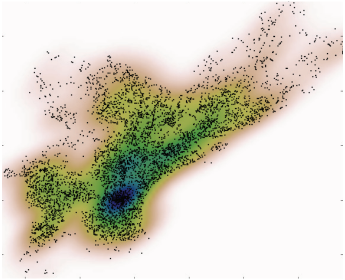
1. Obtain a spatial density with Kernel Density Estimation



Risk model

Built from publicly available civic datasets

1. Obtain a spatial density with Kernel Density Estimation
2. Evaluate along street segments and normalize

Weight $r(e)$ relative crime probability

Definitions

Given a graph $G = (V, E)$, a path P from s to t.

Length of P $\ell(P) = \sum_{e \in P} \ell(e)$

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Risk of P $r_t(P) = 1 - \prod_{e \in P}(1 - r(e))$, or (*total risk*)

$r_m(P) = \max_{e \in P} r(e)$. (*max risk*)

Definitions

P* dominates *P′ if

$$\ell(P) \leq \ell(P') \text{ and } r(P) < r(P'), \text{ or}$$
$$\ell(P) < \ell(P') \text{ and } r(P) \leq r(P').$$

SafePaths

Problem (SafePaths)

If $\mathscr{P}$ is the set of all possible paths from s to t, our goal is to select a ***small*** *subset of these paths $\mathscr{S} \subseteq \mathscr{P}$ such that for every path $P \in \mathscr{S}$, P is neither dominated by nor equivalent to any other path $P' \in \mathscr{S}$.*

Total-Paths

Solving SafePaths with length and total risk:

$$\ell(P) = \sum_{e \in P} \ell(e)$$

$$r_t(P) = 1 - \prod_{e \in P} (1 - r(e))$$

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sum-sum biobjective minimization problem

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sum-sum biobjective minimization problem

- `Sum-Recursive`

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sum-max biobjective minimization problem

- `Max-Exhaustive`
- `Max-Recursive`

Speed-ups

- Approximation via early stopping

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- Ellipse pruning

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- Approximation via early stopping (*subset of paths*)
- Ellipse pruning (*same set of paths*)

Setting

- Sample (s, t) pairs

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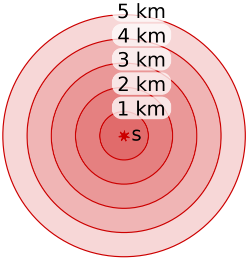

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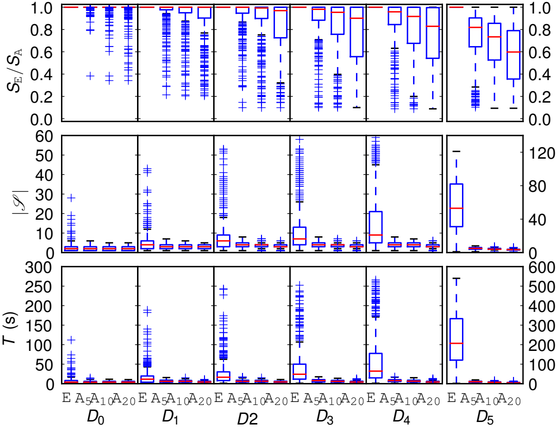
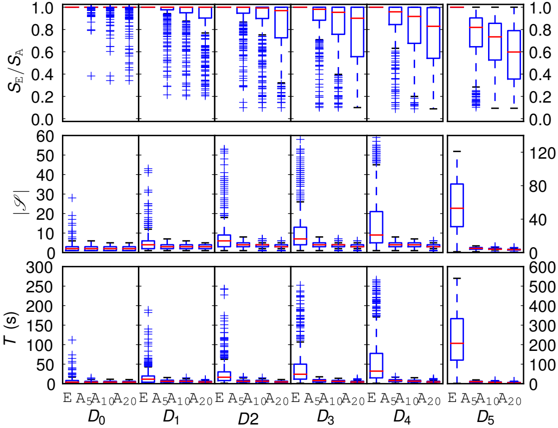
- Sample (s, t) pairs
- Focus on short distances
- Run algorithms with/without approximation and pruning
- Compare solution quality and speed

Results



- Fairly accurate approximations

Results



- Fairly accurate approximations
- Significant speed-ups

Conclusions

- Built a risk model from civic datasets
- Formalized a problem of safe urban navigation
  Two variants: total-risk and max-risk
- Proposed practical algorithmic solutions

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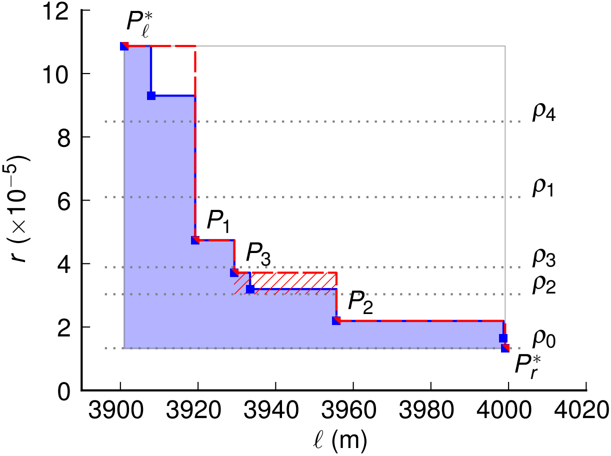
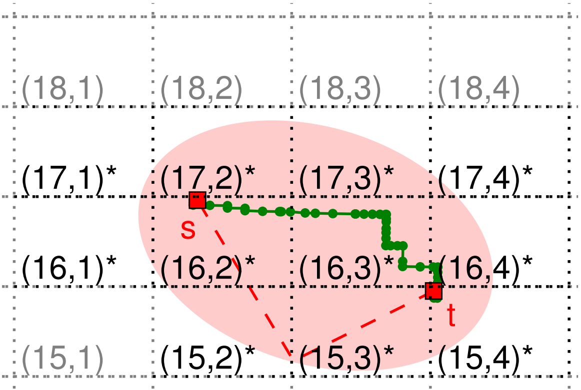
Thanks!


12
10
8
6
4
2
0
$r\ (\times 10^{-5})$
3900
3920
3940
3960
3980
4000
4020
ℓ (m)
P_ℓ^*
P_1
P_3
P_2
P_r^*
ρ_4
ρ_1
ρ_3
ρ_2
ρ_0


(18,1)
(18,2)
(18,3)
(18,4)
(17,1)*
(17,2)*
(17,3)*
(17,4)*
s
(16,1)*
(16,2)*
(16,3)*
(16,4)*
t
(15,1)
(15,2)*
(15,3)*
(15,4)*

$\texttt{Sum-Recursive}(G, \ell(), r())$

$P_\ell^* \leftarrow \texttt{Dijkstra}(G, \ell())$

$P_r^* \leftarrow \texttt{Dijkstra}(G, r())$

$\mathscr{S} \leftarrow \{P_\ell^*, P_r^*\}$

$\texttt{sum-rec}(P_\ell^*, P_r^*, \mathscr{S})$

return $\mathscr{S}$

Routine `sum-rec`$(P_u, P_l, \mathscr{S})$

$\lambda \leftarrow (r(P_u) - r(P_l))/(\ell(P_u) - \ell(P_l))$
$\forall e \in E : f(e) = r(e) - \lambda \ell(e)$
$P_i \leftarrow$ `Dijkstra`$(G, f())$
if $P_i \neq P_u$ and $P_i \neq P_l$ **then**
    $\mathscr{S} \leftarrow \mathscr{S} \cup \{P_i\}$
    `sum-rec`$(P_u, P_i, \mathscr{S})$
    `sum-rec`$(P_i, P_l, \mathscr{S})$

Routine $\texttt{max-rec}(P_u, P_l, \mathscr{S})$

$\rho \leftarrow (r(P_u) + t(P_l))/2$

if $\exists e \in E, t(P_l) < r(e) < \rho$ **then**

  $P_i \leftarrow \texttt{Dijkstra}(G_{r(e)<\rho}, \ell())$

  if $P_i \neq P_l$ **then**

    $\mathscr{S} \leftarrow \mathscr{S} \cup \{P_i\}$

    $\texttt{max-rec}(P_i, P_l, \mathscr{S})$

  $\texttt{max-rec}(P_u, P_i, \mathscr{S})$

	Total-Paths		Max-Paths	
	Chicago	Philadelphia	Chicago	Philadelphia
Total	2400	2400	2400	2400
$\lvert\mathscr{S}\rvert = 1$	111	407	263	562
$\lvert\mathscr{S}\rvert = 2$	182	148	317	253
Ident. $\mathtt{A}_5$	520	476	398	424
Ident. $\mathtt{A}_{10}$	420	407	340	351
Ident. $\mathtt{A}_{20}$	337	327	14	49