Dynamics of Abstract Turing Machines

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Turing machines have been well studided in the context of Computability theory, looking at computations starting from a very specific state and a given input. We intend to study here Turing machines from the point of view of dynamical systems, that is consider Turing machines starting from any input, and any possible state. This model is very interesting from this point of view, and intimately linked with the dynamics of piecewise affine maps and area-preserving diffeomorphisms [4, 3]

While most properties of the model are of course intrisically undecidable, it turns out that many dynamical properties, and more generally properties about its long term behaviour, are computable [2, 1].

Many of these properties can be obtained from the trace of the Turing machine, which consists in the sequences of states and symbols the machine visits. In fact, it is often sufficient to know where the machine is coming from instead of its exact state. This means many results may be formulated and proved for a larger class of dynamical systems, called abstract Turing machines, which share many properties with Turing machines, but for which the set of states is not visible (and may even be infinite albeit compact)

The goal of the internship is to investigate this abstract model, and to formulate efficient criteria to decide whether a given abstract Turing machine corresponds to a concrete Turing machine.

Knowledge of Computability Theory, in particular of Turing machines, is appreciated. Knowledge of Symbolic Dynamics, or basics of Topology, will be useful but is not required.

References

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