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Information Processing in Robotics
Solution Sheet 1
Topic: Introduction to Learning and Probabilistic Reasoning

Exercise 2: Bayes' rule

(a) Product rule writes: $P(A,B) = P(A|B)P(B)$ and, by symmetry: $P(A,B) = P(B|A)P(A)$. Therefore:

$$\begin{aligned} P(A|B)P(B) &= P(B|A)P(A) \\ P(A|B) &= \frac{P(B|A)P(A)}{P(B)} \end{aligned}$$

Furthermore:

$$\begin{aligned} P(S|K) &= \frac{P(S,K)}{P(K)} \\ P(S|K) &= \frac{\sum_F P(S,F,K)}{\sum_{S,F} P(S,F,K)} \end{aligned}$$

(b) For a given b:

- $P(A)$ is a probability distribution over variable A: it has to be normalized.
- $P(b|A)$ is a probability value for each cases of A; therefore it is a function over the domain of A but not a probability distribution (it doesn't need to be normalized).
- $P(b)$ is a scalar: the marginal probability of value b for B.
- $P(A|b)$ is a probability distribution over variable A (thus it is normalized).

(c) Using sum and product rules we have:

$$P(b) = \sum_A P(b|A)P(A)$$

(d) To compute the posterior we can:

$Z \leftarrow 0$
for all $a \in A$ **do**
  $post(a) \leftarrow prior(a) * likelihood(a)$
  $Z \leftarrow Z + post(a)$
end for
$Z \leftarrow 0$
for all $a \in A$ **do**
  $post(a) \leftarrow post(a)/Z$
end for

(e) see source code.