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Information Processing in Robotics

Exercise Sheet 2

Topic: Graphical models and Hidden Markov Models

Exercise 1: Bayesian network and independence

Assuming we have the following Bayesian network:

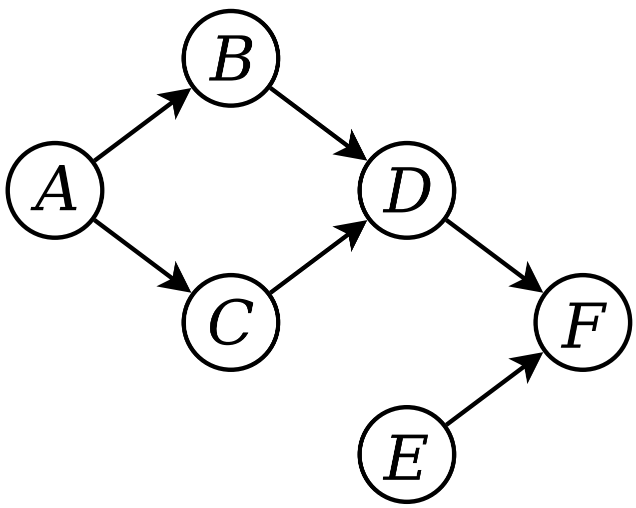

Figure 1: Example Bayesian network

(a) Write the joint probability distribution corresponding to Fig. 1.

(b) Which of the folowing is true, and why?

- $B \perp\!\!\!\perp C \mid A$,
- $B \perp\!\!\!\perp C \mid D$,
- $B \perp\!\!\!\perp C \mid F$,
- $B \perp\!\!\!\perp C \mid E$,
- $A \perp\!\!\!\perp D \mid B$,

- $A \perp\!\!\!\perp D \mid B, C$,
- $A \perp\!\!\!\perp E$,
- $A \perp\!\!\!\perp E \mid D$,
- $A \perp\!\!\!\perp E \mid F$,
- $A \perp\!\!\!\perp E \mid D, F$.

Exercise 2: Hidden Markov Models

We've seen several different uses of HMM in the class each with its own inference. Among those, we presented the forward algorithm that can be summarized by:

$$\begin{aligned} \alpha(S^t) &= P(O^0, \ldots, O^t, S^t), \\ \alpha(S^{t+1}) &= P(O^{t+1}|S^{t+1}) \sum_{S^t} P(S^{t+1}|S^t)\alpha(S^t). \end{aligned}$$

(a) Restate the update of α based on the parameters $\theta = (\pi, A, B)$ (consider α to be a vector).

(b) Write the forward algorithm completely.

(c) Write a ROS node proposing a service that takes an observation and returns the current $\alpha(S^t)$ values. The signature could be:

```
int32 observation
---
float64[] alpha
```

Exercise 3: Wifi localization

Assuming a building with several wifi access points, it is possible to know where we are just according to the identity of the access point we can observe.
The environment is show in Fig. 2, where A, B, ... are the rooms and 1, 2, and 3 are the wifi spots.

(a) Give an expression of π assuming that we have no a priori knowledge on the robots location.

(b) Give an expression of A assuming that the robot can go to any connected room or corridor with equal probability and will stay in the same location with probability 50%.

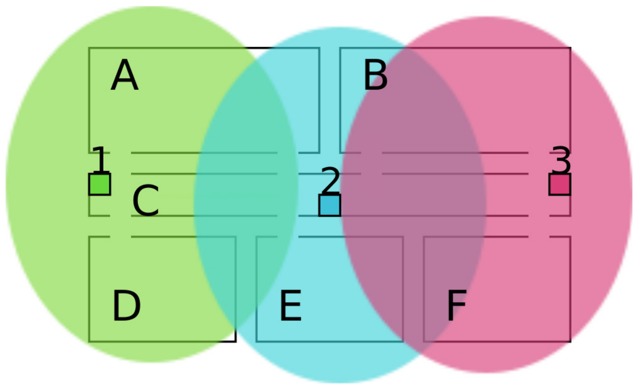

Figure 2: Environment; letters are room and colors/numbers are access points.

(c) Give an expression of B (guess reasonable values when there is overlap).

(d) Using the service written in the previous exercice, compute the probability distribution over the rooms after having successively observed: 1, 1, 2, 2, 3, 3, 3, 2, 2, 2, 2, 2.

(e) What happen to the α values if you continue having observations? and if you continue observing 2?