Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Dr. Francis Colas
Institute of Robotics and Intelligent Systems
Autonomous Systems Lab

ETH Zürich
CLA E 26
Tannenstraße 3
8092 Zürich Switzerland

fcolas@mavt.ethz.ch
www.asl.ethz.ch

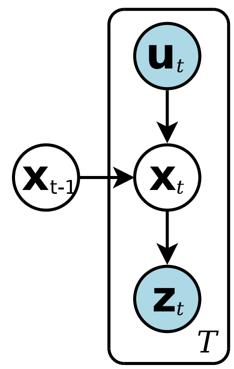
Information Processing in Robotics

Exercise Sheet 3

Topic: Online estimation: application to localization and mapping

Exercise 1: Kalman filter

In this exercise, we will investigate in more details the equations of the Kalman filter. To do that, we will rely on some relations for Gaussian distributions.
Assuming:



- $p(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda})$,
- $p(\boldsymbol{y} \mid \boldsymbol{x}) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}, \boldsymbol{L})$,

we have:

- $p(\boldsymbol{y}) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{A}\boldsymbol{\mu} + \boldsymbol{b}, \boldsymbol{L} + \boldsymbol{A}\boldsymbol{\Lambda}\boldsymbol{A}^T)$,
- $p(\boldsymbol{x} \mid \boldsymbol{y}) = \mathcal{N}\left(\boldsymbol{x}|(\boldsymbol{\Lambda}^{-1} + \boldsymbol{A}^T\boldsymbol{L}^{-1}\boldsymbol{A})^{-1}\left\{\boldsymbol{A}^T\boldsymbol{L}^{-1}(\boldsymbol{y} - \boldsymbol{b}) + \boldsymbol{\Lambda}^{-1}\boldsymbol{\mu}\right\}, (\boldsymbol{\Lambda}^{-1} + \boldsymbol{A}^T\boldsymbol{L}^{-1}\boldsymbol{A})^{-1}\right)$.

(a) Write the expression of the prediction inference: $P(\boldsymbol{x}_t|\boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{0:t})$. What are the mean and covariance?

(b) Write the expression of the update inference: $P(\boldsymbol{x}_t|\boldsymbol{z}_{0:t}, \boldsymbol{u}_{0:t})$. What are the mean and covariance?

(c) Compare your expressions with the Kalman filter algorithm. (Hint: use the following matrix identities: $(A + BD^{-1}C)^{-1} = A^{-1} - A^{-1}B(D + CA^{-1}B)^{-1}CA^{-1}$ and $(P^{-1} + B^TR^{-1}B)^{-1}B^TR^{-1} = PB^T(BPB^T + R)^{-1}$.)

(d) Implement a service that computes the mean and covariance of a state given an observation. The signature can be:

```
float64[] command
float64[] observation
```

```
---
float64[] mean
float64[] covariance
```

(e) What should be changed in order to have an EKF?

Exercise 2: SLAM

Using the `office_room` package, you can experience what SLAM is about.

(a) Untar the package besides your other packages.

(b) Copy `worlds/office_slam.jpg` into the gazebo textures directory (`gazebo/gazebo/share/gazebo/`

(c) Launch `office_slam.launch`.

(d) Launch `rviz` (`$ rosrun rviz rviz`).

(e) Move the robot slowly and observe the update of the map.

(f) Try to have a complete map.