Exercise 1: Kalman filter

In this exercise, we will investigate in more details the equations of the Kalman filter. To do that, we will rely on some relations for Gaussian distributions.

Assuming:

- \( p(x) = \mathcal{N}(x|\mu, \Lambda) \),
- \( p(y|x) = \mathcal{N}(y|Ax + b, L) \),

we have:

- \( p(y) = \mathcal{N}(y|A\mu + b, L + AA^T) \),
- \( p(x|y) = \mathcal{N} \left( x | (\Lambda^{-1} + A^T L^{-1} A)^{-1} \left\{ A^T L^{-1} (y - b) + \Lambda^{-1} \mu \right\} , (\Lambda^{-1} + A^T L^{-1} A)^{-1} \right) \).

(a) Write the expression of the prediction inference: \( P(x_{t}|z_{0:t-1}, u_{0:t}) \). What are the mean and covariance?

(b) Write the expression of the update inference: \( P(x_{t}|z_{0:t}, u_{0:t}) \). What are the mean and covariance?

(c) Compare your expressions with the Kalman filter algorithm. (Hint: use the following matrix identities: \((A + BD^{-1}C)^{-1} = A^{-1} - A^{-1}B(D + CA^{-1}B)^{-1}CA^{-1}\) and \((P^{-1} + B^T R^{-1} B)^{-1} B^T R^{-1} = PB^T (BPB^T + R)^{-1}\).)

(d) Implement a service that computes the mean and covariance of a state given an observation. The signature can be:

```c
float64[] command
float64[] observation
```
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float64[] mean
float64[] covariance

(e) What should be changed in order to have an EKF?

**Exercise 2: SLAM**

Using the office_room package, you can experience what SLAM is about.

(a) Untar the package besides your other packages.

(b) Copy worlds/office_slam.jpg into the gazebo textures directory (gazebo/gazebo/share/gazebo/Media/materials/textures).

(c) Launch office_slam.launch.

(d) Launch rviz ($ rosrun rviz rviz).

(e) Move the robot slowly and observe the update of the map.

(f) Try to have a complete map.