Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

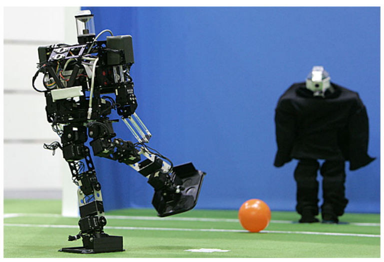
Dr. Francis Colas
Institute of Robotics and Intelligent Systems
Autonomous Systems Lab

ETH Zürich
CLA E 26
Tannenstraße 3
8092 Zürich Switzerland

fcolas@mavt.ethz.ch
www.asl.ethz.ch

Information Processing in Robotics
Exercise Sheet 4
Topic: Regression

Exercise 1: Constant Regression



We want to build a goal keeper robot. We have a vision system (noisy) that can detect the position of the ball and we want to estimate the trajectory of the ball to be able to catch it.
The vision system gives a set of positions $(x, y)_t$ on the ground for each timestep t.

(a) What is linear in the linear regression?

(b) What do we expect the trajectory to look like? Which kind of basis function should we choose?

(c) With only the constant function $\phi(t) = 1$ (polynomial fit of order 0), write the expression of the sum of squared errors E as a function of the coefficient w of ϕ. Deduce an algebraic expression for the optimal $\hat{w} = argmin_w E(w)$.

Exercice 2: Affine Regression

Assuming the robots do not put spin on the ball, we consider the trajectory to follow a straight line on the (x, y) plane. We will therefore do a first order polynomial regression. In this exercise we will compute the parameters a and b of the fit $y = ax + b$ with an alternative method. Let $\bar{x}$ and $\bar{y}$ be the respective means of the x and y coordinates of the points.

(a) Write the expression for the sum of squared errors.

(b) Derive it with respect to both a and b and pose the system of equations to be solved.

(c) Deduce a property of the point $(\bar{x}, \bar{y})$ with respect to the fit and give an expression of b as a function of a, $\bar{x}$ and $\bar{y}$. (Hint: use the derivative with respect to b.)

(d) Substitute the expression of b in both equations and deduce an expression of a. (Hint: consider $x_t - \bar{x}$ and $y_x - \bar{y}$.)

(e) The goal is on the x axis between the points $(-1, 0)$ and $(1, 0)$. After a shot from an opponent robot, the ball has been seen by the camera in points: $(-0.25, 5)$, $(0, 3.9)$, $(0.26, 3)$, and $(0.5, 2)$. Compute a, b, and deduce the position at which the ball should cross the goal line. Should the robot move?

(f) The expressions above are not symmetric in x and y. With the same method compute α and β as the coefficients of the equation $x = \alpha y + \beta$. Compute where the ball should cross the goal line.

(g) Bring the equation of the second fit under the form $y = a'x + b'$ and compare the sum of square error for both solutions. Explain qualitatively why there is a difference between both results.

Exercise 3: Polynomial Regression

Now robots have progressed and can shoot the ball in the air. We need to take into account the altitude z of the ball, which is also estimated by cameras. The coordinates of the ball from frame 1 to 4 are now: $(-0.25, 5, -0.09)$, $(0, 3.9, 0.55)$, $(0.26, 3, 0.73)$, and $(0.5, 2, 1.05)$. 0 is the ground level and the height of the goal is 1: anything higher is out. (Yes the first point is reported below the ground: never completely trust a vision algorithm.)

(a) Plot the altitude z as a function of y. Do you expect the ball to enter the goal?

(b) Write the Φ matrix for the polynomial regression of the altitude against time for orders 1, 2, and 3 then compute the coefficients of each polynome.
(Hint: In matlab, you can use `pinv(Phi)` to get the pseudo inverse of a matrix or you can use `regress(Z', Phi)`. You can check your results by comparison with `polyfit(T, Z, n)`.)

(c) Compute the expected altitude value according to each fit for time $t = 6$. Does the ball enter the goal? Plot the three polynomial fits and the data up to time $t = 6$. Which one should you trust and why?