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Information Processing in Robotics

Exercise Sheet 6

Topic: Support vector machines

Exercise 1: Implementation of a Support Vector Machine

In this exercise, we will implement a support vector machine classificator. Skeleton code is provided in Python but any language is suitable as soon as a quadratic programming solver[1] is available.

(a) What is the input of the training of support vector machines?

(b) What are the support vectors? After training, what information is needed for classification? Deduce the signature of a `train` service a SVM could provide.

(c) We have a solver able to optimize quadratic functions under linear equality or inequality constraints:

$$\begin{cases} \text{minimize} & \frac{1}{2}\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x} + \boldsymbol{q}^T\boldsymbol{x} \\ \text{subject to} & \boldsymbol{G}\boldsymbol{x} \leq \boldsymbol{h} \\ & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \end{cases}$$

where $\boldsymbol{x}$ is the vector of unknowns.

Give the expressions of:

- matrix $\boldsymbol{P}$,
- vector $\boldsymbol{q}$,
- matrix $\boldsymbol{G}$,
- vector $\boldsymbol{h}$,
- matrix $\boldsymbol{A}$,
- vector $\boldsymbol{b}$.

(d) Implement the handler that trains a SVM.

[1] We use `cvxopt` in Python for our example available as a ubuntu package: `$ sudo apt-get install python-cvxopt`

(e) What is the expression of the prediction value for a new point? Implement a service doing prediction.

```
float64[] x_vector
---
float64[] t_vector
```

Exercise 2: Experimenting with SVMs and kernel

We want a SVM classifier to discriminate points that are inside a disk centered on $(0, 0)$ with radius 1 from points that are outside this disk.

(a) Is it feasible with support vector machines, and if so with which mechanism?

(b) In the video shown in class, the points were projected on a 2D parabola. Write a function ϕ to change from the 2D point space to the new 3D feature space; write the associated kernel function k.

(c) In this feature space what will be the boundary? Give its mathematical expression.

(d) If the first class is not a disk anymore but an ellipsis centered on $(0, 0)$ with length 4 and width 2, can we use the same kernel? and why?

(e) Propose a new kernel that can help in this case.

(f) Using the node you implemented (or the `svmtrain` and `svmclassify` matlab functions) try to test different kernel: generate some points and try to do classification with:

- a constant kernel,
- a linear kernel,
- a Gaussian kernel,
- the two kernels from this exercise.