Iterative Closest Point Algorithm

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Introduction

Several problems:

- find object into a scene,
- estimate motion of sensor,
- compare two scenes,
- merge observations into a map,
- ...

Finding objects

Input:

- description of an object (mesh),
- scene (point clouds);

Objective:

- detect object,
- estimate position,
- estimate some parameters.

Sensor Motion

Input:

- sensor data (image or point cloud) at time t_1,
- sensor data at time t_2;

Objectives:

- estimate motion of the sensor,
- associate part of the data between frames,
- estimate some sensor parameters.

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Scene Comparison

Input:

- sensor data from first point of view (time, space or sensor),
- sensor data from second point of view;

Objectives:

- find the differences,
- find the similarities.

Mapping

Input:

- some map,
- sensor data;

Objectives:

- integrate new data into map,
- find location of new data,
- finding overlap between map and data.

Introduction

Several problems:

- find object into a scene,
- estimate motion of sensor,
- compare two scenes,
- merge observations into a map,
- ...

Solution:

- ICP: Iterative Closest Point algorithm.

Iterative Closest Point

Definition:

- find transformation between 2 set of points;

Input:

- reference point cloud,
- data point cloud;

Output:

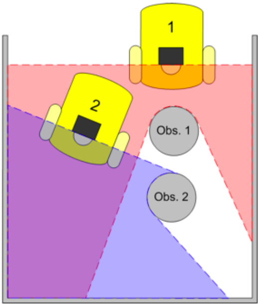
- transformation between reference and data:
  - 3 degrees of freedom in 2D,
  - 6 degrees of freedom in 3D.

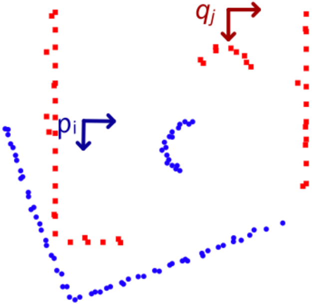
Algorithm

Outline:

- find corresponding points in both clouds,
- compute transformation minimzing error,
- move data point according to the transformation,
- loop...

Algorithm



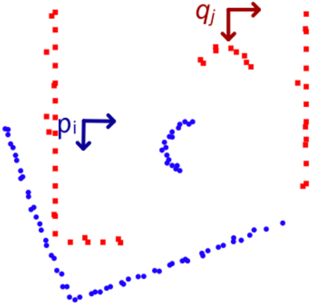


Images courtesy François Pomerleau

Algorithm

1. Preprocessing
2. Matching
3. Weighting
4. Rejection
5. Error
6. Minimization

(Steps defined in Rusinkiewicz 01)



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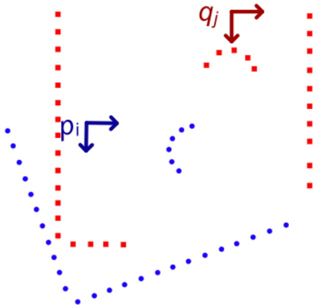

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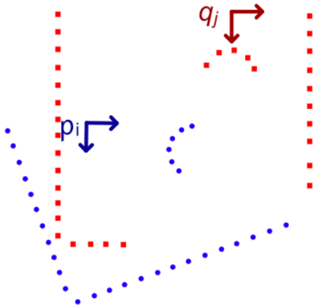

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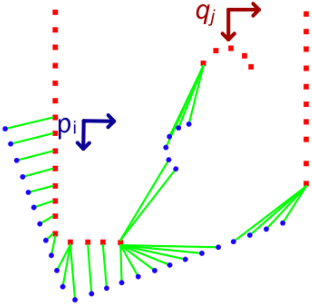

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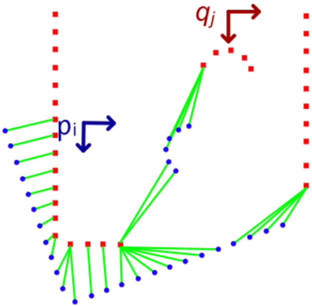

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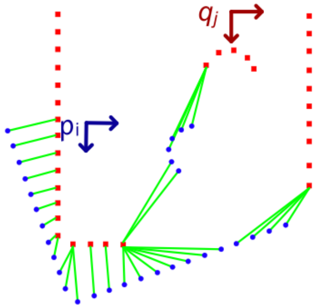

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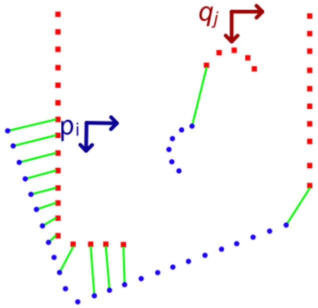

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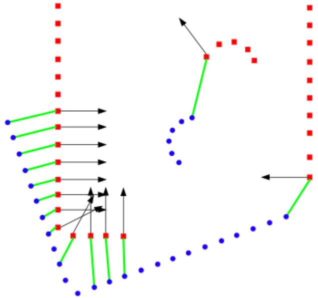

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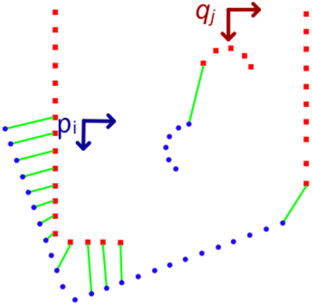

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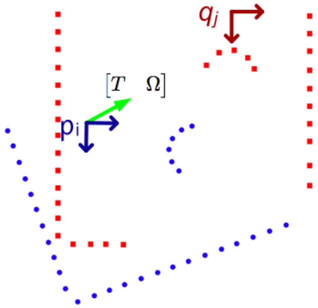

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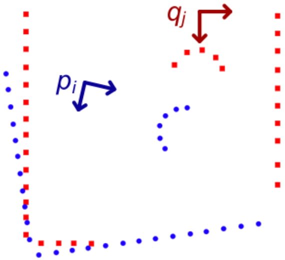

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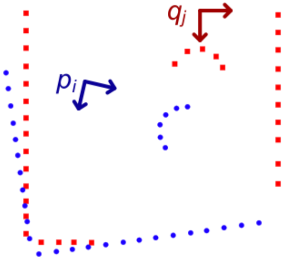

Algorithm

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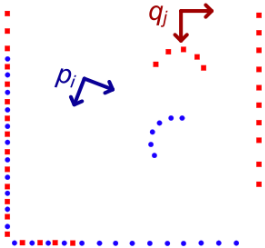
End conditions



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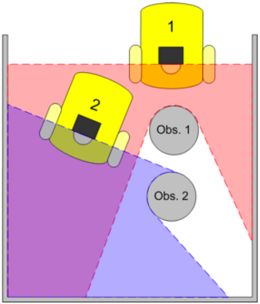
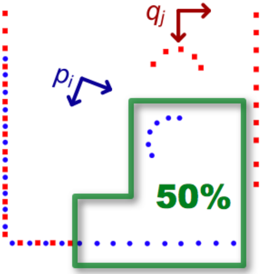
Algorithm





ICP

Algorithm:

1. preprocessing
2. matching
3. weighting
4. rejection
5. error
6. minimization
7. loop to 2. unless convergence.

ICP

Algorithm:

1. preprocessing:
   - clean data,
2. matching
3. weighting
4. rejection
5. error
6. minimization
7. loop to 2. unless convergence.

ICP

Algorithm:

1. preprocessing
2. matching:
   - associate points from reference to data,
   - use neighbor search,
   - can use features,
3. weighting
4. rejection
5. error
6. minimization
7. loop to 2. unless convergence.

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ICP

Algorithm:

1. preprocessing
2. matching
3. weighting:
   - change importance of some pairs,
4. rejection
5. error
6. minimization
7. loop to 2. unless convergence.

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ICP

Algorithm:

1. preprocessing
2. matching
3. weighting
4. rejection:
   - discard some pairs,
5. error
6. minimization
7. loop to 2. unless convergence.

ICP

Algorithm:

1. preprocessing
2. matching
3. weighting
4. rejection
5. error:
    - compute error for each pair,
6. minimization
7. loop to 2. unless convergence.

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ICP

Algorithm:

1. preprocessing
2. matching
3. weighting
4. rejection
5. error
6. minimization:
   - find best transform;
7. loop to 2. unless convergence.

Summary

ICP:

- iterative algorithm,
- estimate matches,
- minimize error;

Features:

- similar to EM scheme (like Baum-Welch),
- local optimization,
- sensitive to overlap/outliers.

Matching

Objective:

- find point in reference to associate to each point in data,
- based on position,
- also normal vectors, colors...

Means:

- additional information as additional dimensions of point
- nearest neighbor search.

Done a lot:

- needs to be efficient.

Matching

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Done a lot:

- needs to be efficient.

Nearest Neighbor Search

Two main approaches:

- look at all points: *linear search*,
- look only where you want: *space partitioning*.

Features:

- exact or appoximate,
- complexity with respect to dimension.

Linear search

Exhaustive:

- compute distance for every point,
- keep current maximum;

Complexity:

- linear in number of points,
- depends on dimensionality only for distance function.

k-d tree

k-dimensional tree:

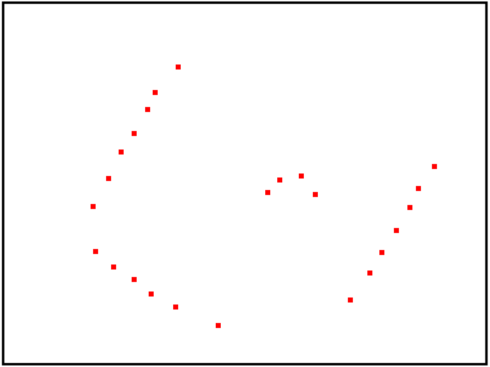
- split in 2 on middle or median point,
- according to the widest dimension,
- until a given bucket size.

Search:

- tree search,
- go down the tree to bucket,
- linear search in bucket.

k-d tree

Build the tree for reference:



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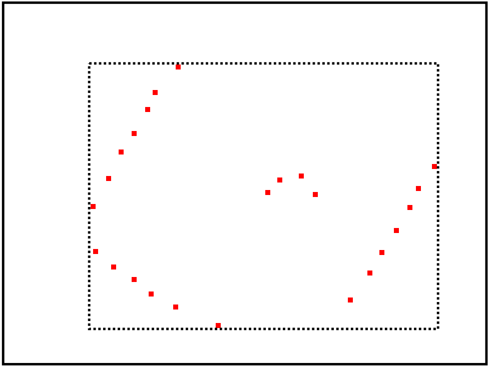

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k-d tree

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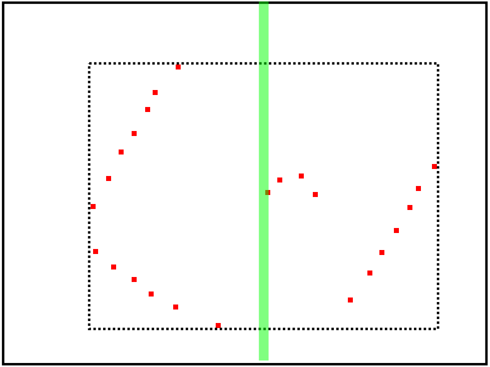

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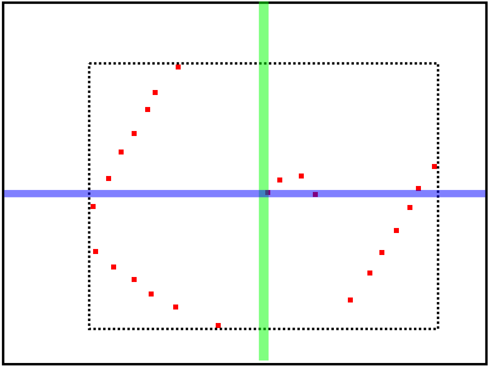

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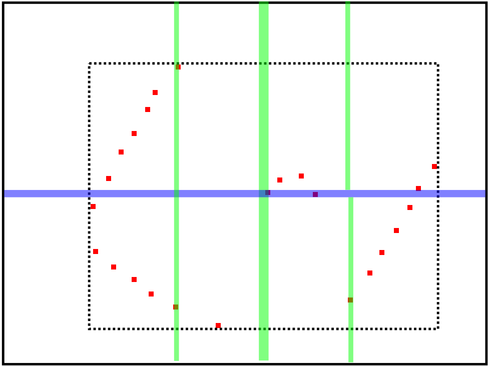

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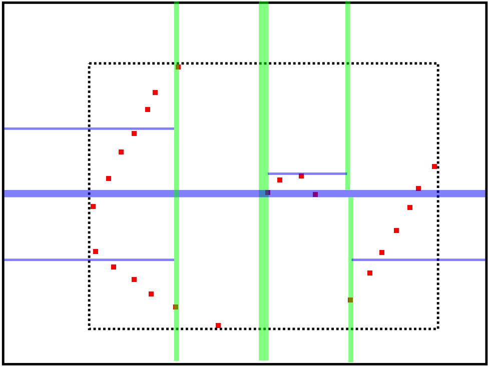

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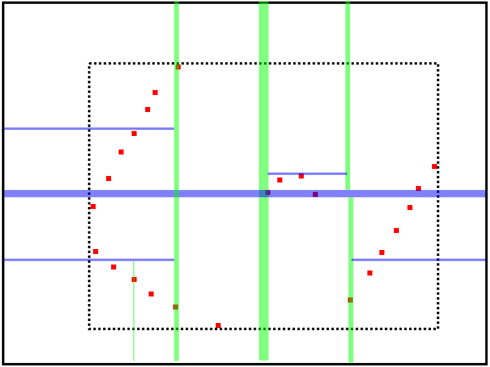
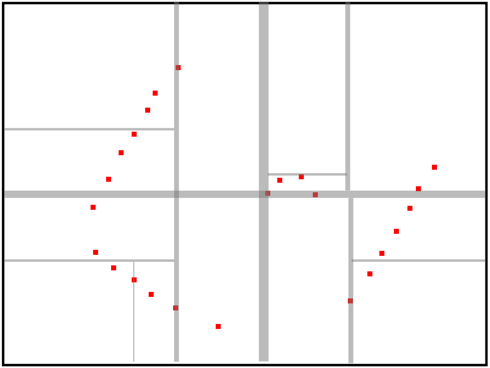


Nearest Neighbor Search in a *k*-d Tree

Looking for the neighbors of data points:



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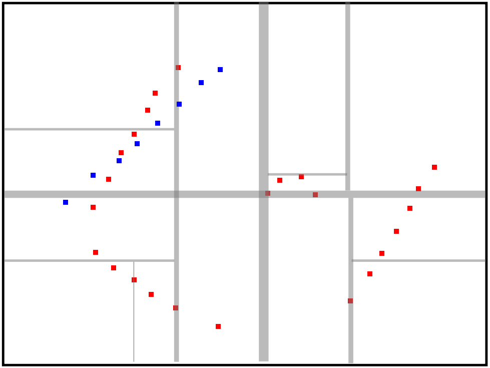

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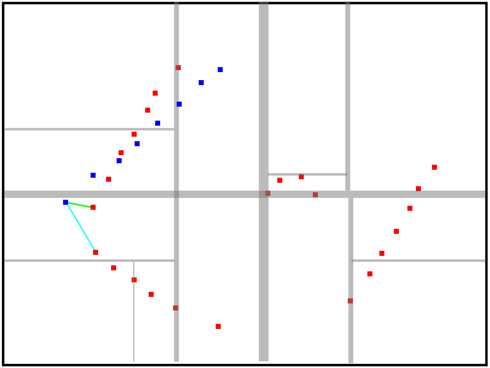

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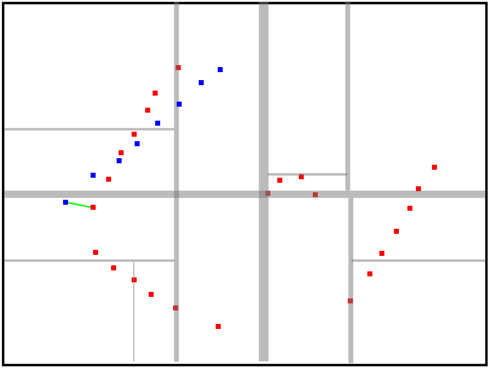

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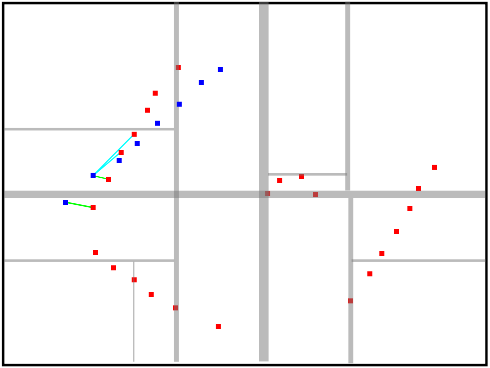

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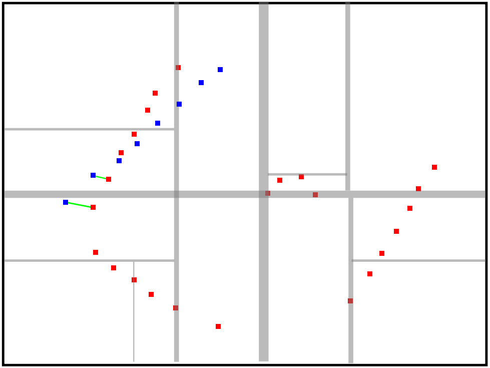

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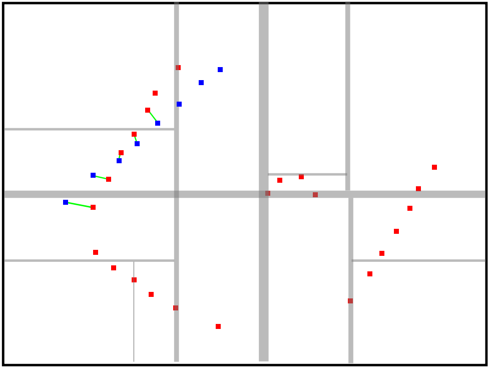

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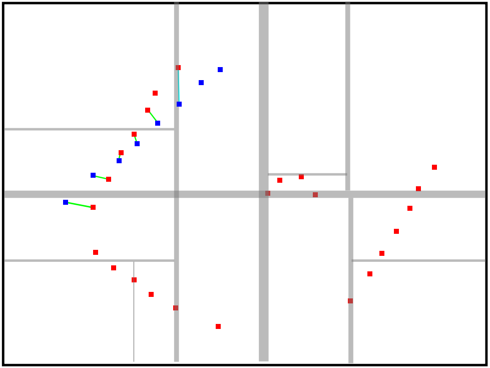

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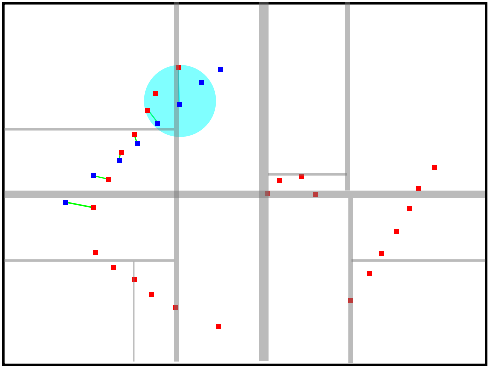

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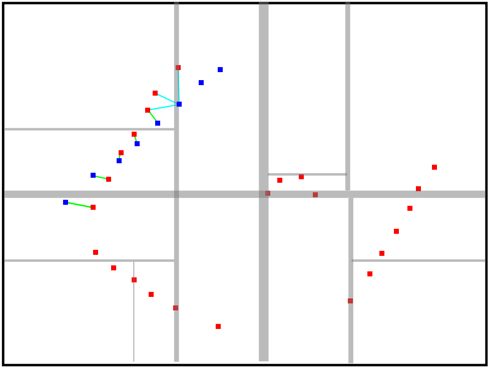

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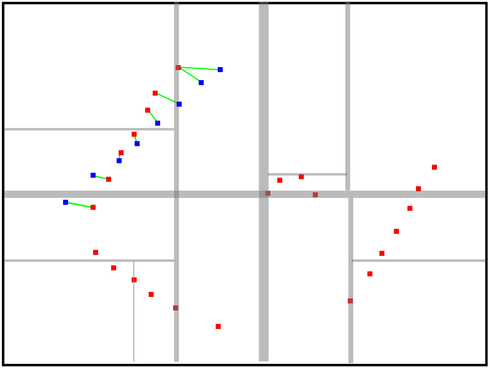

Nearest Neighbor Search in a k-d Tree

Search:

- go down the tree,
- find closest point in the leaf,
- check when getting back up;

Complexity:

- normally $O(\log(N))$,
- worst case is linear,
- high dimension: worst case.

Building a k-d tree:

- complex: $O(N \log^2(N))$,
- efficient in low dimension (≤ 20),
- worthwhile for repetitive queries.

Summary

Nearest neighbor search:

- linear search,
- space partitioning;

Linear search:

- exhaustive,
- depends on the number of points,
- good for really high dimension or low number of points;

Space partitioning:

- *k*-d tree,
- several variants,
- more complex to build,
- faster for many points in not too high dimension.

Error Minimization

Several error functions:

- point-to-point:

$$E(\mathbf{R},\mathbf{t}) = \sum_i = 1^N \|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{m}_i\|^2$$

  where $\mathbf{m}_i$ is the reference point corresponding to data point $\mathbf{p}_i$.
- point-to-plane:

$$E(\mathbf{R},\mathbf{t}) = \sum_i = 1^N \|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{m}'_i\|^2$$

  where $\mathbf{m}'_i$ is projection of $\mathbf{p}_i$ on the closest surface
- scaled:

$$E(\mathbf{R},\mathbf{t},\mathbf{S}) = \sum_i = 1^N \|\mathbf{R}\mathbf{S}\mathbf{p}_i + \mathbf{t} - \mathbf{m}_i\|^2$$

Error Minimization

Getting the best transformation:

$$\mathbf{R}, \mathbf{t} = \operatorname*{arg\,min}_{\mathbf{R},\mathbf{t}} E(\mathbf{R}, \mathbf{t})$$

Iterative process:

$$\mathbf{R}_k, \mathbf{t}_k = \operatorname*{arg\,min}_{\mathbf{R},\mathbf{t}} \sum_i = 1^N \|\mathbf{R}\mathbf{p}_i^k + \mathbf{t} - \mathbf{m}_i^k\|^2$$

where $\mathbf{p}_i^k = \mathbf{R}_{k-1}\mathbf{p}_i^{k-1} + \mathbf{t}_{k-1}$.
Solution:

- point-to-point: closed-form solutions with SVD, quaternions...
- point-to-plane: no closed-form solution; linearization or Levenberg-Marquardt.

Summary

ICP:

- matching,
- error minimization,

Matching:

- linear nearest-neighbor search,
- k-d tree NNS;

Error minimization:

- point-to-point: standard with closed-form solution,
- point-to-plane: better but with approximate techniques.