Iterative Closest Point Algorithm

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Introduction

Several problems:

- find object into a scene,
- estimate motion of sensor,
- compare two scenes,
- merge observations into a map,
- ...

...
Finding objects

Input:
- description of an object (mesh),
- scene (point clouds);

Objective:
- detect object,
- estimate position,
- estimate some parameters.
Sensor Motion

Input:
- sensor data (image or point cloud) at time $t_1$,
- sensor data at time $t_2$;

Objectives:
- estimate motion of the sensor,
- associate part of the data between frames,
- estimate some sensor parameters.
Scene Comparison

Input:
- sensor data from first point of view (time, space or sensor),
- sensor data from second point of view;

Objectives:
- find the differences,
- find the similarities.
Mapping

Input:
- some map,
- sensor data;

Objectives:
- integrate new data into map,
- find location of new data,
- finding overlap between map and data.
Introduction

Several problems:

- find object into a scene,
- estimate motion of sensor,
- compare two scenes,
- merge observations into a map,
- ...

Solution:

- ICP: Iterative Closest Point algorithm.
Iterative Closest Point

Definition:
- find transformation between 2 set of points;

Input:
- reference point cloud,
- data point cloud;

Output:
- transformation between reference and data:
  - 3 degrees of freedom in 2D,
  - 6 degrees of freedom in 3D.
Algorithm

Outline:

- find corresponding points in both clouds,
- compute transformation minimizing error,
- move data point according to the transformation,
- loop...
Images courtesy François Pomerleau
1. Preprocessing
2. Matching
3. Weighting
4. Rejection
5. Error
6. Minimization

(Steps defined in Rusinkiewicz 01)
Algorithm

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ICP

Algorithm:
1. preprocessing
2. matching
3. weighting
4. rejection
5. error
6. minimization
7. loop to 2. unless convergence.
ICP

Algorithm:
1. preprocessing:
   ▶ clean data,
2. matching
3. weighting
4. rejection
5. error
6. minimization
7. loop to 2. unless convergence.
ICP

Algorithm:

1. preprocessing
2. matching:
   ▶ associate points from reference to data,
   ▶ use neighbor search,
   ▶ can use features,
3. weighting
4. rejection
5. error
6. minimization
7. loop to 2. unless convergence.
ICP

Algorithm:

1. preprocessing
2. matching
3. weighting:
   ▶ change importance of some pairs,
4. rejection
5. error
6. minimization
7. loop to 2. unless convergence.
ICP

Algorithm:
1. preprocessing
2. matching
3. weighting
4. rejection:
   ▶ discard some pairs,
5. error
6. minimization
7. loop to 2. unless convergence.
ICP

Algorithm:
1. preprocessing
2. matching
3. weighting
4. rejection
5. error:
   ▶ compute error for each pair,
6. minimization
7. loop to 2. unless convergence.
ICP

Algorithm:
1. preprocessing
2. matching
3. weighting
4. rejection
5. error
6. minimization:
   ▶ find best transform;
7. loop to 2. unless convergence.
Summary

ICP:
► iterative algorithm,
► estimate matches,
► minimize error;

Features:
► similar to EM scheme (like Baum-Welch),
► local optimization,
► sensitive to overlap/outliers.
Matching

Objective:
- find point in reference to associate to each point in data,
- based on position,
- also normal vectors, colors...

Means:
- additional information as additional dimensions of point
- nearest neighbor search.

Done a lot:
- needs to be efficient.
Matching

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Nearest Neighbor Search

Two main approaches:
- look at all points: *linear search*,
- look only where you want: *space partitioning*.

Features:
- exact or approximate,
- complexity with respect to dimension.
Linear search

Exhaustive:
- compute distance for every point,
- keep current maximum;

Complexity:
- linear in number of points,
- depends on dimensionality only for distance function.
**$k$-d tree**

$k$-dimensional tree:
- split in 2 on middle or median point,
- according to the widest dimension,
- until a given bucket size.

Search:
- tree search,
- go down the tree to bucket,
- linear search in bucket.
$k$-d tree

Build the tree for reference:
Build the tree for reference:
$k$-d tree

Build the tree for reference:
**k-d tree**

Build the tree for reference:
**$k$-d tree**

**Build the tree for reference:**
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Build the tree for reference:
Nearest Neighbor Search in a $k$-d Tree

Looking for the neighbors of data points:
Nearest Neighbor Search in a k-d Tree

Looking for the neighbors of data points:
Nearest Neighbor Search in a \( k \)-d Tree

Looking for the neighbors of data points:

![Diagram of nearest neighbor search in a \( k \)-d Tree]
Nearest Neighbor Search in a $k$-d Tree

Looking for the neighbors of data points:
Nearest Neighbor Search in a $k$-d Tree

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Search:
- go down the tree,
- find closest point in the leaf,
- check when getting back up;

Complexity:
- normally $O(\log(N))$,
- worst case is linear,
- high dimension: worst case.

Building a $k$-d tree:
- complex: $O(N \log^2(N))$,
- efficient in low dimension ($\leq 20$),
- worthwhile for repetitive queries.
Summary

Nearest neighbor search:
- linear search,
- space partitioning;

Linear search:
- exhaustive,
- depends on the number of points,
- good for really high dimension or low number of points;

Space partitioning:
- $k$-d tree,
- several variants,
- more complex to build,
- faster for many points in not too high dimension.
Error Minimization

Several error functions:

- **point-to-point:**

\[
E(R, t) = \sum_{i=1}^{N} ||Rp_i + t - m_i||^2
\]

where \(m_i\) is the reference point corresponding to data point \(p_i\).

- **point-to-plane:**

\[
E(R, t) = \sum_{i=1}^{N} ||Rp_i + t - m'_i||^2
\]

where \(m'_i\) is projection of \(p_i\) on the closest surface.

- **scaled:**

\[
E(R, t, S) = \sum_{i=1}^{N} ||RSp_i + t - m_i||^2
\]
Error Minimization

Getting the best transformation:

\[
R, t = \arg \min_{R, t} E(R, t)
\]

Iterative process:

\[
R_k, t_k = \arg \min_{R, t} \sum_i = 1^N \| Rp_i^k + t - m_i^k \|^2
\]

where \( p_i^k = R_{k-1} p_i^{k-1} + t_{k-1} \).

Solution:

- point-to-point: closed-form solutions with SVD, quaternions...
- point-to-plane: no closed-form solution; linearization or Levenberg-Marquardt.
Summary

ICP:
- matching,
- error minimization,

Matching:
- linear nearest-neighbor search,
- \( k \)-d tree NNS;

Error minimization:
- point-to-point: standard with closed-form solution,
- point-to-plane: better but with approximate techniques.