Which Pattern for a Low Pattern-induced Bias?

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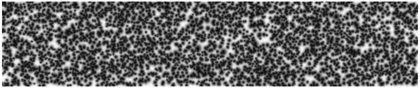
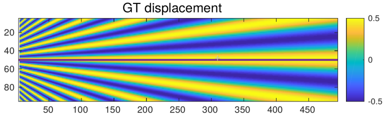
Biases in displacement estimation through DIC





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→ Ground-truth (GT) displacement along horizontal axis

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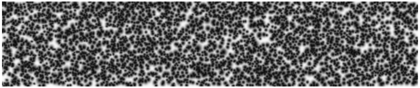

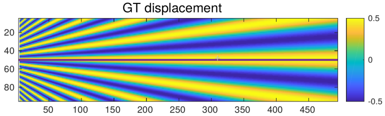

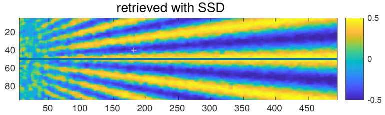

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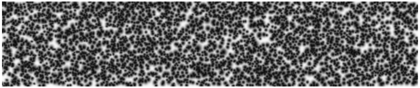

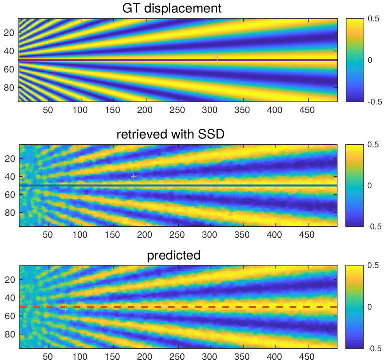

Input: two speckle images of the surface of a specimen, before and after deformation (synthetic images, **no noise**)

→ Ground-truth (GT) displacement along horizontal axis

→ typical result of DIC

→ our predictive formula *pattern-induced bias* (PIB) term introduced in [Fayad Seidl Reu ExpMech2020]

Reminder: digital image correlation (DIC)

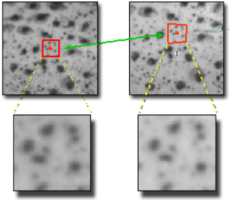

Image credit: Correlated Solutions

For any pixel $\mathbf{x}$ in the image domain, minimize sum of squared difference w.r.t. displacement ϕ over $\Omega_{\mathbf{x}}$:

$$\sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} \left(\mathcal{I}(\mathbf{x}_i) - \widetilde{\mathcal{I}}'(\mathbf{x}_i + \phi(\mathbf{x}_i)) \right)^2$$

where:

- images known at pixels: $\mathcal{I}$ reference state, $\mathcal{I}'$ deformed state
- $\widetilde{\mathcal{I}}'$ continuous interpolation of $\mathcal{I}'$ (known at pixel coordinates)
- $\Omega_{\mathbf{x}}$ subset of M pixels $(\mathbf{x}_1, \ldots, \mathbf{x}_M)$.
- ϕ expected to approximate the unknown displacement $\mathbf{u}$ over $\Omega_{\mathbf{x}}$.

$\rightarrow$ for any $\mathbf{x}$, displacement $=$ value of ϕ at the **center of the subset** $\Omega_{\mathbf{x}}$

Reminder: displacement estimation is an ill-posed problem

$$\sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} \left(\mathcal{I}(\mathbf{x}_i) - \widetilde{\mathcal{I}}'(\mathbf{x}_i + \phi(\mathbf{x}_i)) \right)^2$$

small displacement ϕ (realistic here):

$$\forall \mathbf{x}_i \in \Omega_{\mathbf{x}},\ \widetilde{\mathcal{I}}'(\mathbf{x}_i + \phi(\mathbf{x}_i)) = \widetilde{\mathcal{I}}'(\mathbf{x}_i) + \underbrace{< \phi(\mathbf{x}_i), \nabla \widetilde{\mathcal{I}}'(\mathbf{x}_i) >}_{\substack{\textit{no information for} \\ \textit{displacement orthogonal} \\ \textit{to image gradient}}} + \mathcal{O}(|\phi(\mathbf{x}_i)|)$$

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$\rightarrow$ *aperture problem*: noise-free case is an ill-posed problem

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$\rightarrow$ regularization required

ϕ sought as a linear combination of N basis functions $(\phi_j)_{1 \leq j \leq N}$

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Actual optimization problem: minimize SSD w.r.t. $(\lambda_j)_{1 \leq j \leq N}$

$$\mathrm{SSD}(\lambda_1, \ldots, \lambda_N) = \sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} \left(\mathcal{I}(\mathbf{x}_i) - \widetilde{\mathcal{I}}' \left(\mathbf{x}_i + \textstyle\sum_{j=1}^{N} \lambda_j \phi_j(\mathbf{x}_i) \right) \right)^2$$

With a polynomial basis, displacement at $\mathbf{x}$ is $\begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix}$

Potential bias sources

$$\mathrm{SSD}(\lambda_1, \ldots, \lambda_N) = \sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} \left(\mathcal{I}(\mathbf{x}_i) - \widetilde{\mathcal{I}'} \left(\mathbf{x}_i + \sum_{j=1}^{N} \lambda_j \phi_j(\mathbf{x}_i) \right) \right)^2$$

Bias = difference between unknown $\mathbf{u}$ and retrieved ϕ

Caused by:

- interpolation $\widetilde{\mathcal{I}'}$
- $\mathbf{u}$ approximated by a linear combination of the functions (ϕ_j)
- aperture problem
- (sensor noise)

$\rightarrow$ several papers in the literature analyze these bias sources

$\rightarrow$ we discuss a unifying predictive formula

Post-optimal analysis (noise-free case)

The solution $\mathbf{\Lambda} = (\lambda_1, \ldots, \lambda_N)^T$ is a stationary point of SSD

Theorem (after derivation of SSD w.r.t. λ_i + some calculations...)

If $\mathbf{\Lambda}$ minimizes the SSD criterion, then:

$$\mathbf{\Lambda} = ((L^{\mathbf{u}})^T L^{\mathbf{u}})^{-1} (L^{\mathbf{u}})^T \mathbf{G}^{\mathbf{u}} + ((L^{\mathbf{u}})^T L^{\mathbf{u}})^{-1} (L^{\mathbf{u}})^T \mathbf{D}\mathcal{I}' + \mathcal{O}(|\boldsymbol{\delta}|^2) + \mathcal{O}\left(|\mathbf{D}\mathcal{I}'| \cdot |\mathbf{D}\nabla\mathcal{I}'|\right) + \mathcal{O}\left(|\mathbf{D}\mathcal{I}'| \cdot |\boldsymbol{\delta}|\right)$$

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- $\forall i \in \{1, \ldots, M\}$, $\mathbf{G}^{\mathbf{u}}_i = \langle \nabla\mathcal{I}'(\mathbf{x}_i + \mathbf{u}(\mathbf{x}_i)), \mathbf{u}(\mathbf{x}_i) \rangle$

$\rightarrow$ only information colinear with image gradient is integrated

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- $\forall i \in \{1, \ldots, M\}$, $j \in \{1, \ldots, N\}$, $L^{\mathbf{u}}_{i,j} = \langle \nabla\mathcal{I}'\left(\mathbf{x}_i + \mathbf{u}(\mathbf{x}_i)\right), \phi_j(\mathbf{x}_i) \rangle$

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$\rightarrow$ only ϕ_j components colinear with image gradient are integrated
- $\mathbf{D}\mathcal{I}' = \left(\mathcal{I}' - \widetilde{\mathcal{I}}'\right)(\mathbf{x}_i + \phi(\mathbf{x}_i))$
$\rightarrow$ interpolation error

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- $\mathbf{D}\mathcal{I}' = \left(\mathcal{I}' - \widetilde{\mathcal{I}}'\right)(\mathbf{x}_i + \phi(\mathbf{x}_i))$
  $\rightarrow$ interpolation error
- $\forall i \in \{1, \ldots, M\}$, $\boldsymbol{\delta}_i = \mathbf{u}(\mathbf{x}_i) - \phi(\mathbf{x}_i)$
  $\rightarrow$ estimation error

Discussion: what are the bias sources?

Another formulation, with $\mathbf{u} = \sum_k \lambda_k^{\mathbf{u}} \phi_k + \epsilon$
($\epsilon \neq 0$ *with undermatched shape function* [Schreier Sutton ExpMech2002])

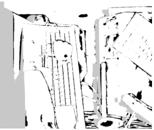
$$\boldsymbol{\Lambda} = \boldsymbol{\Lambda}^{\mathbf{u}} + \underbrace{((L^{\mathbf{u}})^T L^{\mathbf{u}})^{-1}(L^{\mathbf{u}})^T \mathbf{E}}_{\text{approximation error}} + \underbrace{((L^{\mathbf{u}})^T L^{\mathbf{u}})^{-1}(L^{\mathbf{u}})^T \mathbf{D}\mathcal{I}'}_{\text{interpolation error}}$$

with $\forall i \in \{1, \dots, M\}$, $\mathbf{E}_i = \langle \nabla\mathcal{I}'(\mathbf{x}_i + \mathbf{u}(\mathbf{x}_i)), \epsilon(\mathbf{x}_i)\rangle$

1. **Interpolation error**
   $\rightarrow$ impossible to bound rigorously without additional information, but small under mild assumptions
   - well-sampled images (Nyquist conditions satisfied): no interpolation error with Fourier interpolation.
     Problem: images are usually **not** well sampled
   - linear or cubic interpolation: error depends on derivatives of $\mathcal{I}'$
2. **Approximation error** : *(focus of the current presentation)*
   $\rightarrow$ plays a role since $\epsilon \neq 0$
   basis: polynomial functions $(\phi_j)_{1 \leq j \leq N}$

Special case: 1D displacement

In stereoscopy:
disparity estimation over rectified images

Popular approach: displacement ϕ constant inside $\Omega_{\mathbf{x}}$
(corresponds to 0-order basis functions)

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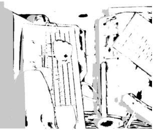

Popular approach: displacement ϕ constant inside $\Omega_{\mathbf{x}}$
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Displacement retrieved in $\Omega_{\mathbf{x}}$ by minimizing DIC:

$$\phi = \frac{\sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} \left(\mathcal{I}'_x(\mathbf{x}_i + u(\mathbf{x}_i))\right)^2 u(\mathbf{x}_i)}{\sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} \left(\mathcal{I}'_x(\mathbf{x}_i + u(\mathbf{x}_i))\right)^2}$$

[Sabater Morel Almansa SIAM J. on Imaging Sciences 2011]

$\rightarrow$ it is a special case of our predictive formula

Conclusion: approximation error depends on the image gradient distribution
$\rightarrow$ in stereoscopy: *adhesion* or *fattening* effect

A paradox: link with Savitzky-Golay convolution

[Schreier Sutton ExpMech2002]

Claim:

The solution ϕ of DIC also minimizes $\displaystyle\sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} |\mathbf{u}(\mathbf{x}_i) - \phi(\mathbf{x}_i)|^2$

Consequence in the case of a polynomial basis $(\phi_j)_{1 \le j \le N}$:

$$\mathbf{\Lambda} = SG * \mathbf{u}$$

where SG is a (bank of) Savitzky-Golay filter (tabulated values) which depends on

d: degree of the polynomials ϕ_j

m: size of the square window $\Omega_{\mathbf{x}}$

Paradox: distribution of image gradient *does not* affect displacement estimation?

Solving the paradox (1)

$$
\begin{aligned}
\mathrm{SSD}(\lambda_1,\ldots,\lambda_N) &= \sum_{\mathbf{x}_i\in\Omega_{\mathbf{x}}} \left(\mathcal{I}(\mathbf{x}_i) - \widetilde{\mathcal{I}'}\left(\mathbf{x}_i + \textstyle\sum_{j=1}^{N}\lambda_j \boldsymbol{\phi}_j(\mathbf{x}_i)\right)\right)^2 \\
&= \sum_{\mathbf{x}_i\in\Omega_{\mathbf{x}}} \left(\mathcal{I}'(\mathbf{x}_i + \mathbf{u}(\mathbf{x}_i)) - \widetilde{\mathcal{I}'}\left(\mathbf{x}_i + \boldsymbol{\phi}(\mathbf{x}_i)\right)\right)^2 \\
&\simeq \sum_{\mathbf{x}_i\in\Omega_{\mathbf{x}}} (< \nabla\mathcal{I}'(\mathbf{x}_i + \mathbf{u}(\mathbf{x}_i)), \mathbf{u}(\mathbf{x}_i) - \boldsymbol{\phi}(\mathbf{x}_i) >)^2 \\
&= \sum_{\mathbf{x}_i\in\Omega_{\mathbf{x}}} g_i^2 \cos^2(\theta_i)|\mathbf{u}(\mathbf{x}_i) - \boldsymbol{\phi}(\mathbf{x}_i)|^2
\end{aligned}
$$

where

- $g_i = |\nabla\mathcal{I}'(\mathbf{x}_i + \mathbf{u}(\mathbf{x}_i))|$
  $\rightarrow$ small variations of g_i would give small PIB
- θ_i: angle between image gradient and $\mathbf{u}(\mathbf{x}_i) - \boldsymbol{\phi}(\mathbf{x}_i)$
  $\rightarrow$ can be considered as random since $\mathbf{u} - \boldsymbol{\phi}$ is small

Note: [Lehoucq Reu Turner ExpMech 2017] gives a similar equation

Solving the paradox (2)

Assumption: speckle images are *spatially correlated* random fields

Theorem (with Bernstein's weak law of large numbers)

$$\sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} g_i^2 \cos^2(\theta_i) |\mathbf{u}(\mathbf{x}_i) - \phi(\mathbf{x}_i)|^2 \xrightarrow[m \to +\infty]{\mathrm{p}} K \sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} |\mathbf{u}(\mathbf{x}_i) - \phi(\mathbf{x}_i)|^2$$

Consequence: minimizing DIC amounts to minimizing $\sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} |\mathbf{u}(\mathbf{x}_i) - \phi(\mathbf{x}_i)|^2$ if m is "large enough" (w.r.t. correlation range of speckle)

$\rightarrow$ holds because speckle images are random fields

$\rightarrow$ PIB gives spurious fluctuations around the output of SG filter

$\rightarrow$ this explains why we need fine speckle patterns: in this case small m can be used (small SG bias) and still give low PIB.

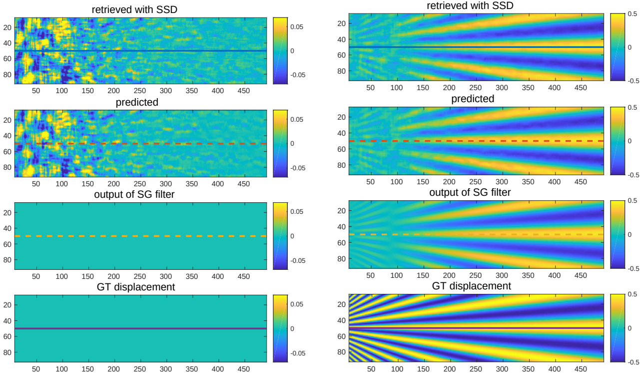
Illustrative numerical experiment (1)

First-order shape functions: $\phi(\mathbf{x}) = \begin{pmatrix} \lambda_1 + \lambda_3 x + \lambda_5 y \\ \lambda_2 + \lambda_4 x + \lambda_6 y \end{pmatrix}$

$\Omega_{\mathbf{x}}$: 13×13 window centered at any pixel $\mathbf{x}$ of the domain

Cubic interpolation to compute $\widetilde{\mathcal{I}}'(\mathbf{x} + \phi(\mathbf{x}))$

Predictive formula: $\boldsymbol{\Lambda} = ((L^{\mathbf{u}})^T L^{\mathbf{u}})^{-1} (L^{\mathbf{u}})^T \mathbf{G}^{\mathbf{u}} = \boldsymbol{\Lambda}^{\mathbf{u}} + ((L^{\mathbf{u}})^T L^{\mathbf{u}})^{-1} (L^{\mathbf{u}})^T \mathbf{E}$

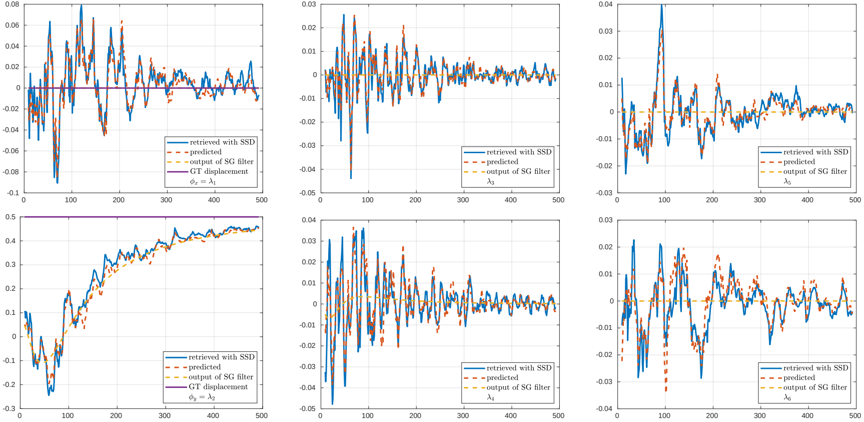


x-displacement / λ_1 $\quad$ y-displacement / λ_2

$\rightarrow$ strong variations of $\mathbf{u} = (u_x, u_y)$ give strong variations of (λ_1, λ_2)

Illustrative numerical experiment (2)

Cross-section plots:



$\rightarrow$ pattern-induced bias may give large spurious fluctuations

Is it possible to get rid of PIB?

In stereoscopy (**1D constant displacement** along x-direction)

[Blanchet et al. J. of Mathematical Imaging and Vision 2011]

minimizing the following weighted SSD:

$$\sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} \frac{1}{|\mathcal{I}'_x(\mathbf{x}_i + \phi)|^2} \left(\mathcal{I}(\mathbf{x}_i) - \widetilde{\mathcal{I}}'(\mathbf{x}_i + \phi)\right)^2$$

gives the displacement (constant inside $\Omega_{\mathbf{x}}$):

$$\phi = \frac{1}{m} \sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} u(\mathbf{x}_i)$$

$\rightarrow$ convolution equation, independent from image gradient

What about the 2D-case in photomechanics?

Remark: [Fayad Seidl Reu ExpMech2020] after [Yuan et al. OLEN2014] weighted DIC to penalize pixels far from the center of $\Omega_{\mathbf{x}}$

We introduce:

$$\widetilde{SSD}(\lambda_1, \ldots, \lambda_N) = \sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} \frac{\left(\mathcal{I}(\mathbf{x}_i) - \widetilde{\mathcal{I}}'(\mathbf{x}_i + \phi(\mathbf{x}_i))\right)^2}{\max(|\nabla \mathcal{I}'(\mathbf{x}_i + \phi(\mathbf{x}_i))|^2, \kappa)}$$

It is also possible to show that ϕ minimizing $\widetilde{\text{SSD}}$ also minimizes $\sum_{\mathbf{x}_i \in \Omega_{\mathbf{x}}} |\mathbf{u}(\mathbf{x}_i) - \phi(\mathbf{x}_i)|^2$ (with m "large enough" in 2D)

However, 2D displacement is much more difficult than 1D displacement:

- in 1D: aperture problem only if $\mathcal{I}'_x = 0$.
- in 2D: information orthogonal to the image gradient lost *at any point* of $\Omega_{\mathbf{x}}$.

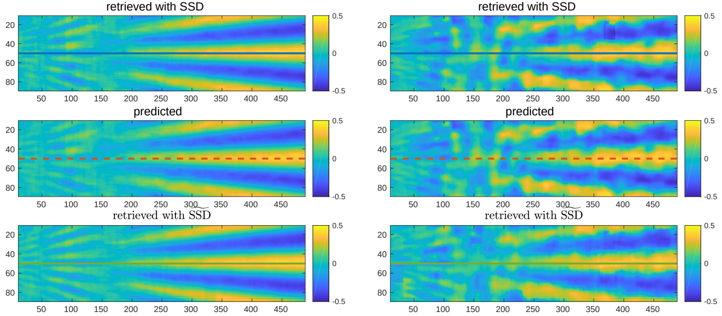
Illustrative numerical experiment

First order basis function

19×19 subset

0-order basis function

19×19 subset



1D-$\widetilde{\text{SSD}}$

2D-$\widetilde{\text{SSD}}$

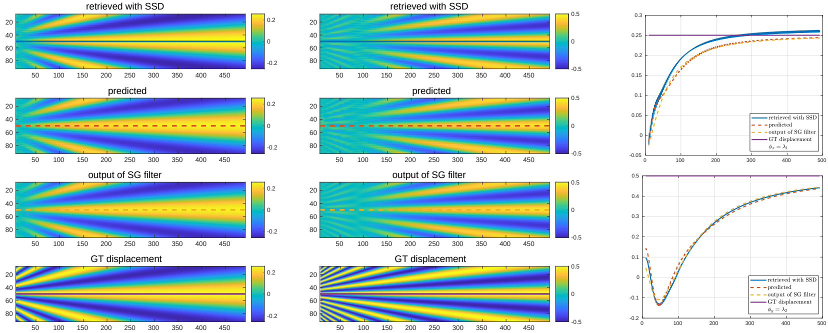
$\rightarrow$ 1D fattening-free weighted SSD from stereoscopy indeed gives consistent results

$\rightarrow$ 2D extension is not satisfactory

Using a periodic pattern instead of speckle. . .

$\Omega_{\mathbf{x}}$: 13×13 size
First-order basis functions
GT disp$_x \neq \mathbf{0}$

deformed "checkerboard" - period: 6 pixels
product of sine waves + 12-bit quantization



x-displacement y-displacement cross-sections

$\rightarrow$ lower pattern-induced bias than with speckle
(also observed in [Fayad Seidl Reu ExpMech2020])
$\rightarrow$ predictive formula not as accurate (aliasing effect?)

Conclusion: pattern-induced bias (PIB)

$\rightarrow$ predictive formula to express ϕ as a function of **the true u**

$\rightarrow$ PIB may be large, in addition to bias induced by SG filtering

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Which pattern for a low PIB?

Some open questions...

- small gradient norm (g_i) variations for an optimized speckle?
- periodic pattern ? (processed by Fourier instead of DIC?)
- how to remove or reduce PIB with a weighted SSD?
- how to predict PIB in a real experiment? (non-smooth images, unknown **u**, Poisson-Gaussian noise...)

Conclusion: pattern-induced bias (PIB)

$\rightarrow$ predictive formula to express ϕ as a function of **the true u**

$\rightarrow$ PIB may be large, in addition to bias induced by SG filtering

Which pattern for a low PIB?

Some open questions...

- small gradient norm (g_i) variations for an optimized speckle?
- periodic pattern ? (processed by Fourier instead of DIC?)
- how to remove or reduce PIB with a weighted SSD?
- how to predict PIB in a real experiment? (non-smooth images, unknown **u**, Poisson-Gaussian noise...)

Details in:

F. Sur, B. Blaysat, M. Grédiac, *On biases in displacement estimation for image registration, with a focus on photomechanics*
Journal of Mathematical Imaging and Vision, 2021

Extended version (noisy images + additional numerical experiments):
https://hal.archives-ouvertes.fr/hal-02862808/

Software code: members.loria.fr/FSur/software/PIB/