Total variation minimization for spectrum interpolation in quasiperiodic noise removal

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June 2015

1 Motivation

Algorithms for quasiperiodic noise removal are proposed in [7, 8, 9]. They are based on a notch filter which smooths out the noise contribution by removing spikes in the spectrum which are not likely to be observed in natural images. These spikes are detected as statistical outliers, either from the power law fitting the distribution of the Fourier coefficients against their frequency [8, 9], or from an a-contrario model [7].

In some situations, ringing artifacts can be observed, especially when the contribution of periodic noise is superimposed to the contribution of actual structures of the image. Basic interpolation methods are described in [2] for strain maps in experimental mechanics with metrological performance concerns. However, concerning natural images, spectrum interpolation with total variation (TV) minimization is known to reduce ringing artifacts. Following [3] (see also [5, 10]), spectrum interpolation through total variation minimization is detailed here. Recent references on this topic can be found in, e.g., [1, 4].

2 Constrained total variation minimization

Let u be the image of interest impaired by quasiperiodic noise, and Ω be the set of frequencies where the noise contribution has been located by the algorithm described in [8, 9] or [7]. The problem is to find $\widetilde{u}$ such that the spectra of $\widetilde{u}$ and u coincide on the complement of Ω (noted Ω^C) and such that the spectrum of $\widetilde{u}$ restricted to Ω permits to minimize the total variation of $\widetilde{u}$.

In this note, the regularized total variation of any image v is given by:

$$\mathrm{TV}_\varepsilon(v) = \int ||\nabla v||_\varepsilon \tag{1}$$

where $||\nabla v||_\varepsilon = \sqrt{\varepsilon^2 + ||\nabla v||^2}$, $||\cdot||$ being the Euclidean norm in $\mathbb{R}^2$ and ∇ the gradient. With a small value of ε, $\mathrm{TV}_\varepsilon(v)$ is differentiable at points where the gradient of v vanishes.

The problem of interest is thus to retrieve $\widetilde{u}$, solution of the following constrained minimization problem:

$$\begin{cases} \widetilde{u} = \mathrm{argmin}_v \ \mathrm{TV}_\varepsilon(v) \\ \text{under the constraint that: } \mathcal{F}(v)|_{\Omega^C} = \mathcal{F}(u)|_{\Omega^C} \end{cases} \tag{2}$$

where $\mathcal{F}(\cdot)$ denotes the Fourier transform of any image.

3 Algorithm

3.1 Projected gradient descent

As in [3, 5, 10], $\widetilde{u}$ is estimated with a projected gradient descent. Here, the algorithm writes:

1. Initialization with any v_0 such that $\mathcal{F}(v_0)|_{\Omega^C} = \mathcal{F}(u)|_{\Omega^C}$.
2. Iterate for $n \geq 0$:

$$v_{n+1} = v_n - \alpha_n P_\Omega \left(\nabla \mathrm{TV}_\varepsilon(v_n) \right) \tag{3}$$

with a step size α_n, until some stopping criterion is satisfied.

Here, the (linear) projection P_Ω is such that

$$\begin{cases} \mathcal{F}(P_\Omega(\cdot))|_\Omega = \mathcal{F}(\cdot)|_\Omega \\ \mathcal{F}(P_\Omega(\cdot))|_{\Omega^C} = 0 \end{cases} \tag{4}$$

The gradient of the regularized total variation is given by [6]:

$$\nabla \mathrm{TV}_\varepsilon(v) = -\mathrm{div} \left(\frac{\nabla v}{||\nabla v||_\varepsilon} \right) \tag{5}$$

where div is the divergence of any vector field.

3.2 Implementation

In the proposed implementation:

- v_0 is the raw output of [7] or [9], and may show ringing;
- The derivatives are estimated using forward and backward difference schemes;

- The stopping criterion is:

$$\overline{\mathrm{TV}_\varepsilon}(v_{n+1}) > \overline{\mathrm{TV}_\varepsilon}(v_n) - 10^{-4} \tag{6}$$

  where $\overline{\mathrm{TV}_\varepsilon}(v) = \frac{1}{NM}\sum_{i,j} ||\nabla v(i,j)||_\varepsilon$ for an $N \times M$ 8-bit image v;

- A simple backtracking line search is used for setting the step size α_n.

The Matlab implementation of the TV minimization for [9] and [7] can be found on the following webpages, respectively:
`http://www.loria.fr/%7Esur/software/ARPENOS/`
`http://www.loria.fr/%7Esur/software/ACARPENOS/`

4 Illustrative example

Figure 1 shows an image affected by periodic noise. It is possible to see in Figure 2 that notch filtering gives ringing artifacts: the reason is that the contribution of periodic noise to the spectrum is superimposed to the contribution of some line segments of the picture (see the discussion in [9]). Constrained TV minimization gives a ringing-free output. The reader is kindly asked to zoom in in the pdf file.

Figure 3 compares a detail of the output of notch filtering before and after TV minimization. The spectrum is interpolated in such a way that the straight line caused by the masts is appropriately reconstructed.

The difference between the raw output of notch filtering and the output of the constrained TV minimization is shown in Figure 4. A significant amount of ringing has been removed. As noted in [5], ringing is less visible in textured regions than in uniform regions.

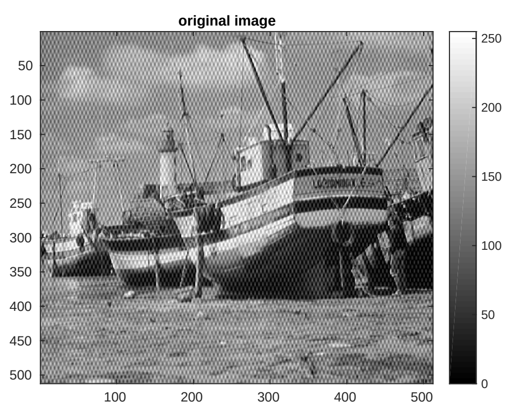

Figure 1: An image impaired by periodic noise.

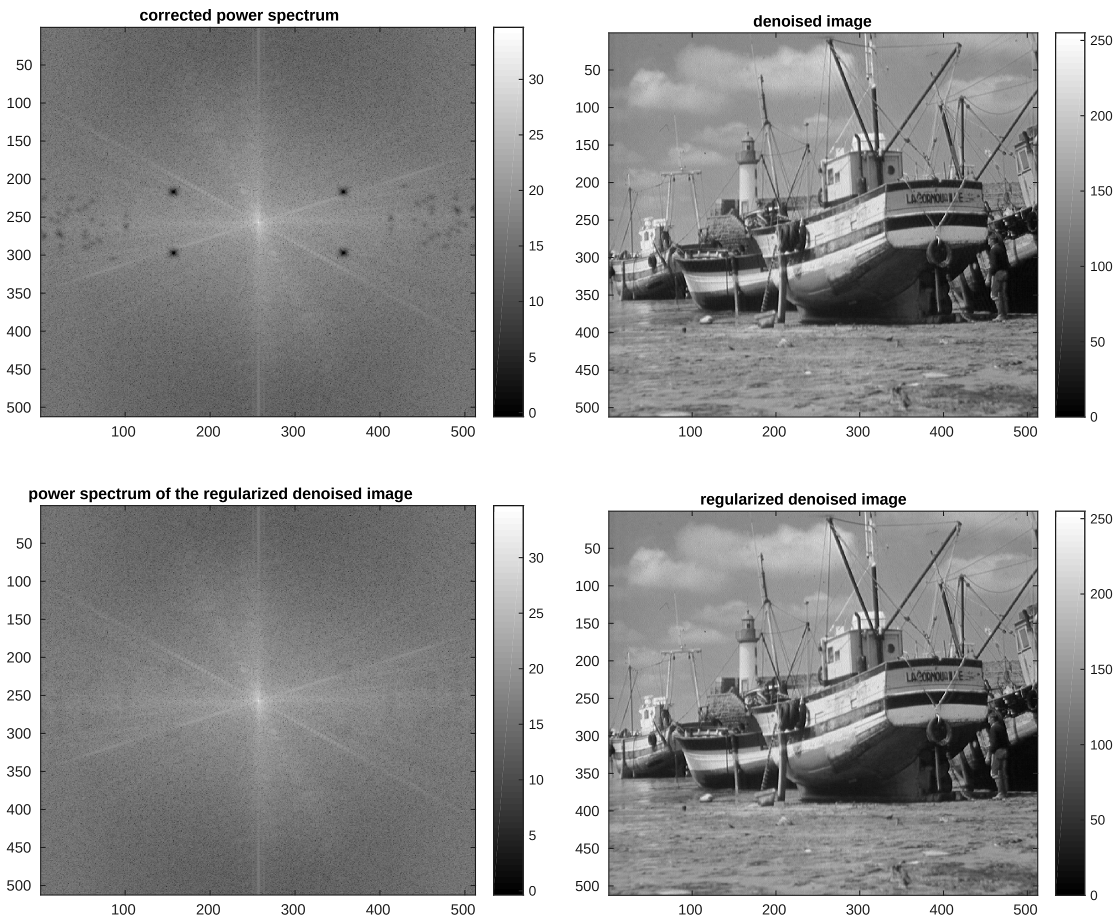

Figure 2: Top: image affected by a periodic noise. Middle: power spectrum after notch filtering (left) and denoised image (right). Bottom: power spectrum after constrained TV minimization (left) and corresponding image (right). Low frequencies are in the middle of the spectrum image.

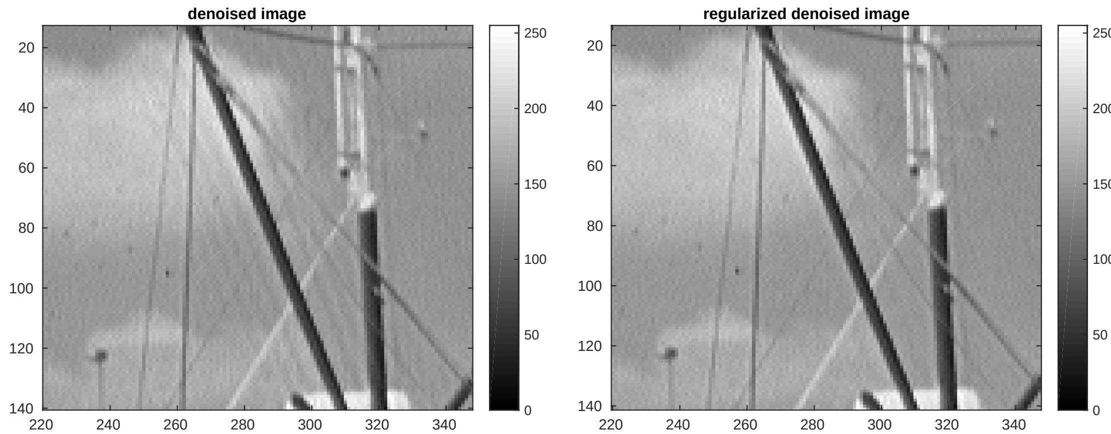

Figure 3: Close-up of the output of notch filtering (left) and after TV minimization (right).

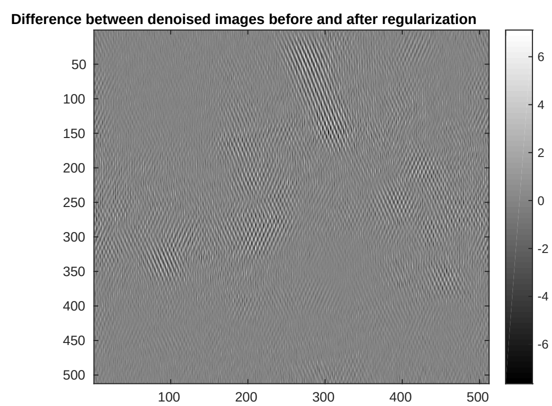

Figure 4: Difference between the output of notch filtering and TV minimization.

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