Complex root isolation

Given the polynomial $p(x) = a_0 + \cdots + a_n x^N$, a classical problem is to compute the solutions of the equation p(x) = 0. Unfortunately, the fastest theoretical algorithm to solve this problem [1] is not efficient in practice. Recently, several efforts were made to design an algorithm efficient in practice, using notably algorithms based on subdivision approaches [2].

The target of this internship is to explore a new subdivision shape, that is better suited for polynomial evaluation (see Figure below). In particular, this shape will allows us to use the Fast Fourrier Transform algorithm [3,chap.13], which is fast both in theory and in practice.



References

[1] Pan VY. Univariate Polynomials: Nearly Optimal Algorithms for Numerical Factorization and Root-finding. Journal of Symbolic Computation. 2002;33:701–733.

[2] Becker R, Sagraloff M, Sharma V, et al. A near-optimal subdivision algorithm for complex root isolation based on the Pellet test and Newton iteration. Journal of Symbolic Computation. 2018;86:51–96.

[3] Gathen J von zur, Gerhard J. Modern Computer Algebra. 3rd ed. Cambridge University Press; 2013.