

# A discrete approach for decomposing noisy digital contours into arcs and segments

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# Arcs and segments decomposition

## Motivation

Arcs and segments are the most appearing primitives in images

- ▶ Detection of shapes
  - ▶ medical imaging, technical images, manual drawings
- ▶ Automatic character recognition
  - ▶ sketch, scanned documents

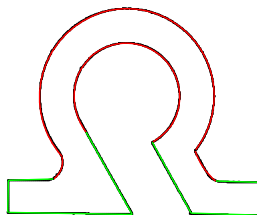
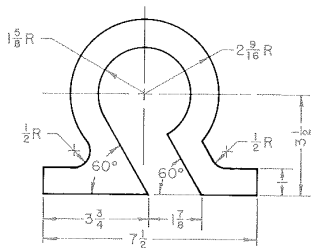


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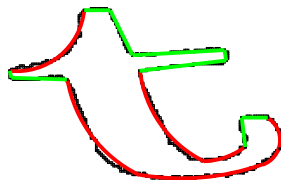
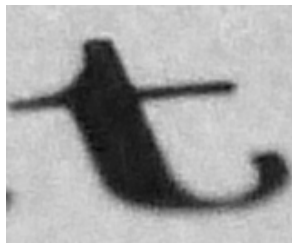


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## Tools

- ▶ Adaptive tangential cover [Ngo16]
- ▶ Dominant point detection [Ngo15]
- ▶ Tangent space representation [Nguyen11a]

# Overview

- 1 MOTIVATION
- 2 ADAPTIVE TANGENTIAL COVER
- 3 DOMINANT POINT DETECTION
- 4 TANGENT SPACE REPRESENTATION
- 5 DECOMPOSITION ALGORITHM
- 6 CONCLUSION & PERSPECTIVES

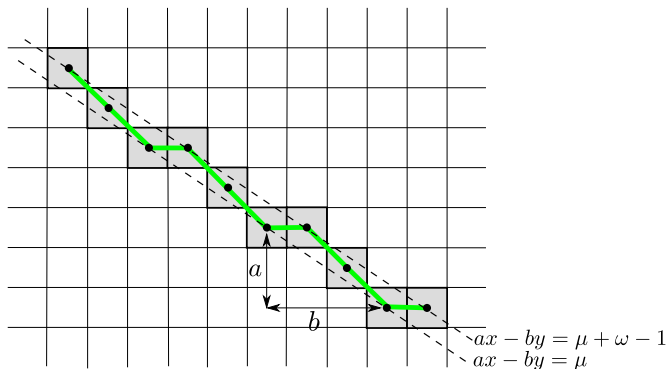
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# Discrete line and segment

## Definition

A **discrete line**  $\mathcal{D}(a, b, \mu, \omega)$  is the set of integer points  $(x, y)$  verifying  $\mu \leq ax - by < \mu + \omega$  where  $a, b, \mu, \omega \in \mathbb{Z}$  and  $\gcd(a, b) = 1$ .

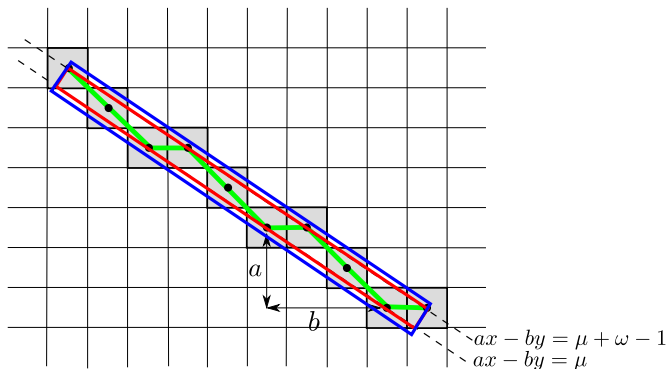




# Discrete line and segment

## Definition

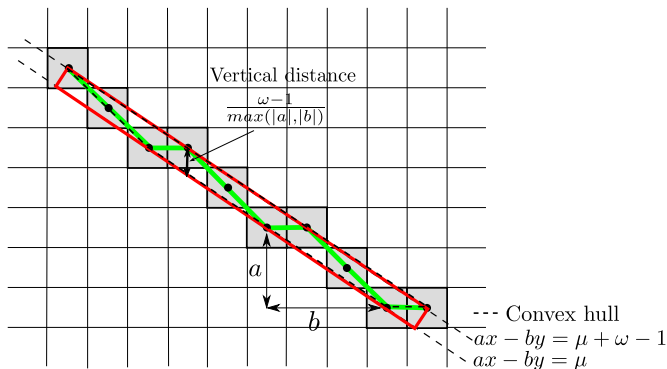
A **discrete segment** is a finite set  $\mathcal{S}_f$  of integer points bounded by the discrete line  $\mathcal{D}(a, b, \mu, \omega)$ .



# Discrete line and segment

## Definition

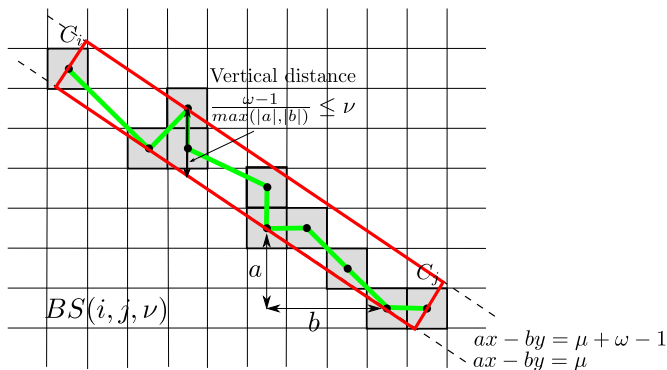
A discrete segment  $\mathcal{S}_f$  is **optimal** if its vertical (or horizontal) distance is equal to the vertical (or horizontal) thickness of its convex hull.



# Blurred segment

## Definition

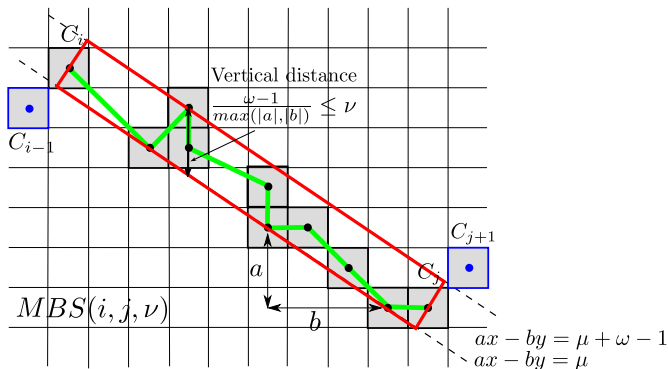
A sequence integer points  $S_f$  is a **blurred segment of width  $\nu$**  if its optimal bounding discrete segment  $\mathcal{D}(a, b, \mu, \omega)$  has the vertical or horizontal distance less than or equal to  $\nu$ .



# Blurred segment

## Definition

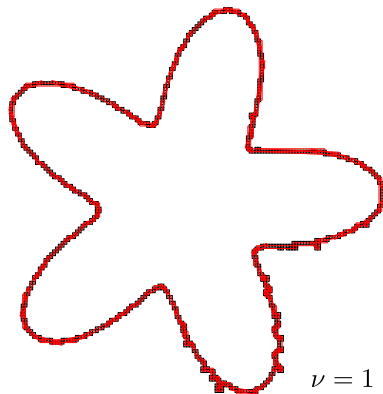
A blurred segment of width  $\nu$   $BS(i, j, \nu)$  is **maximal**, and noted  $MBS(i, j, \nu)$ , iff  $\neg BS(i, j + 1, \nu)$  and  $\neg BS(i - 1, j, \nu)$ .



# Maximal blurred segment decomposition

## Definition

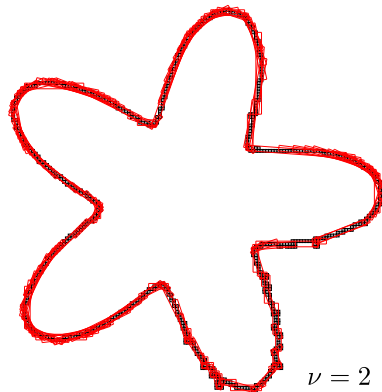
For any discrete curve  $C$ , its decomposition into maximal blurred segments of width  $\nu$  is called a **width  $\nu$  tangential cover** of  $C$ .



# Maximal blurred segment decomposition

## Definition

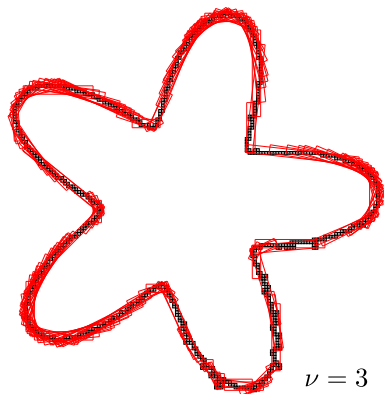
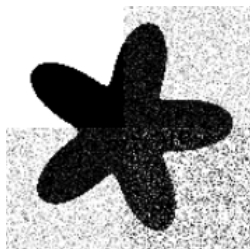
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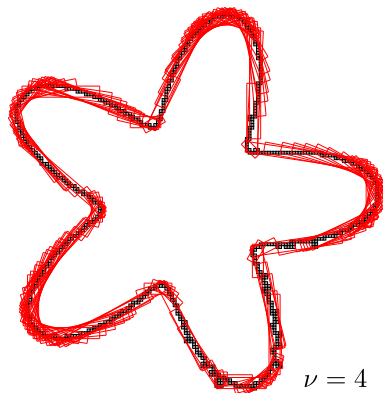
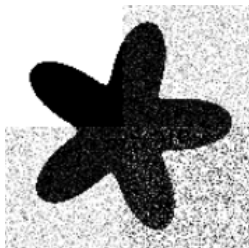
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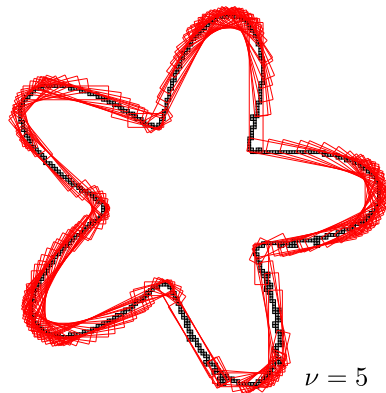
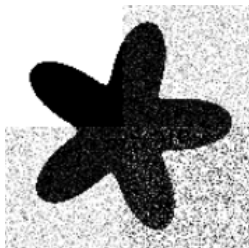




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## Solution

Tangential cover of different widths: **Adaptive Tangential Cover**

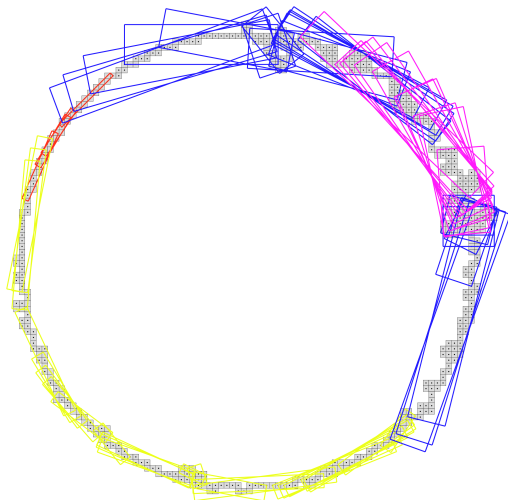
- ▶ appropriated widths based on a local noise estimation
  - ▶ **meaningful thickness detection** [Kerautret12]
- ▶ parameter-free computation

# Construction of adaptive tangential cover

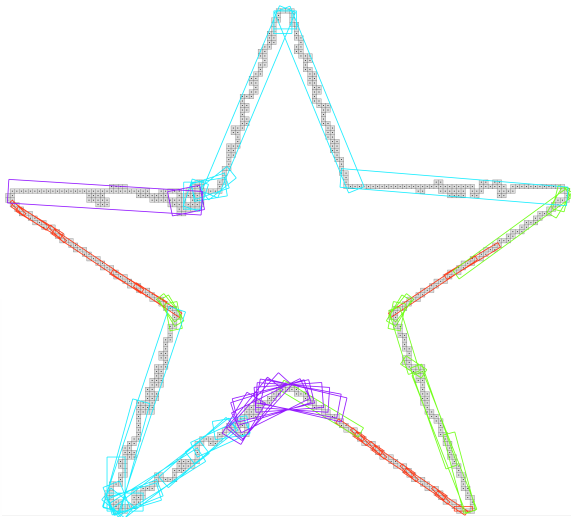
## Principles [Ngo16]

- ▶ Input:
  - ▶ A discrete curve  $C$  of  $n$  points
  - ▶ Vector of meaningful thickness  $\eta$  associated to each point of  $C$
- ▶ Output:
  - ▶ An ATC of  $C$  associated to the meaningful thickness vector  $\eta$
- ▶ The method for computing ATC is divided into two steps:
  - ▶ Labeling the points from the meaningful thickness values
    - ▶ Maximum meaningful thickness of MBS passing the point
  - ▶ Building the ATC with MBS of widths from the obtained labels
    - ▶ MBS of width being the label of at least one point in the MBS

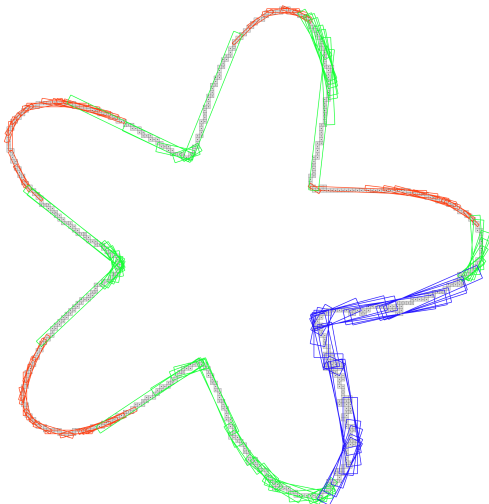
# Examples of Adaptive Tangential Cover



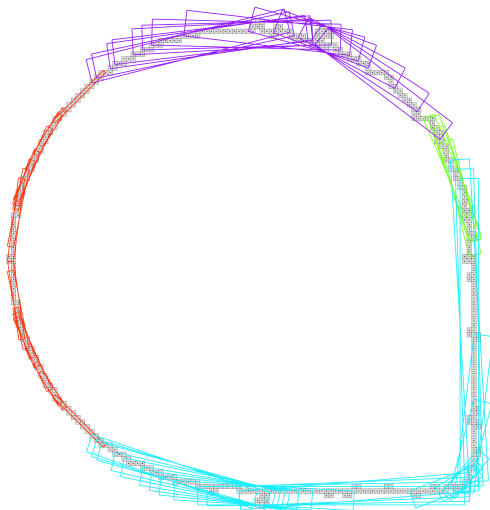
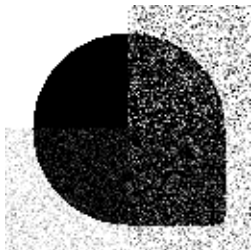
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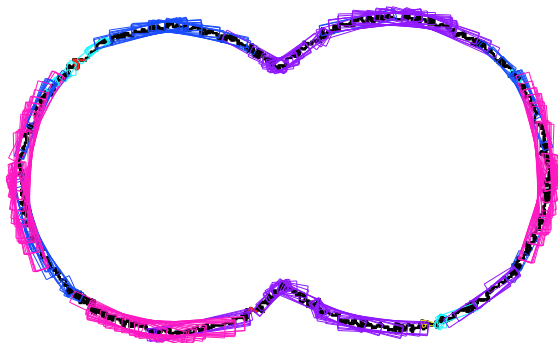
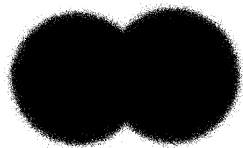
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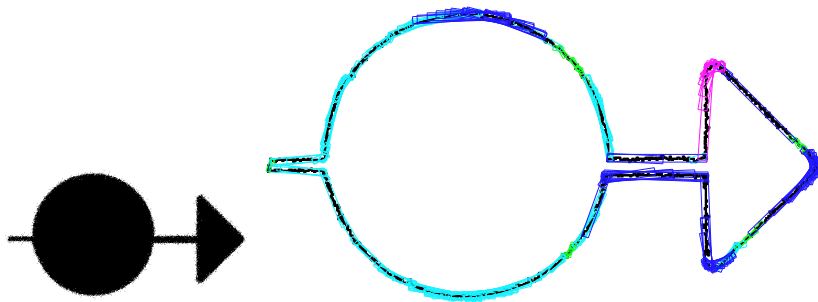


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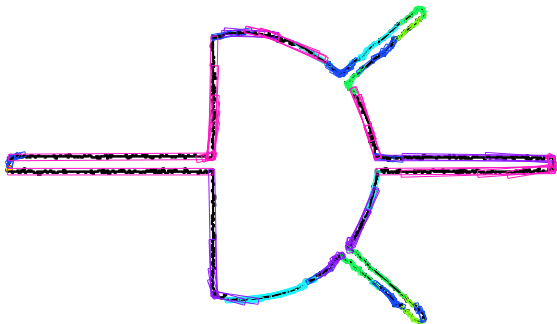
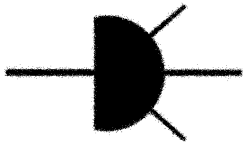




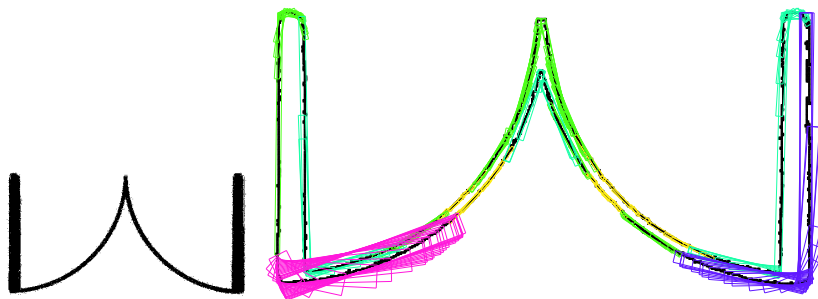
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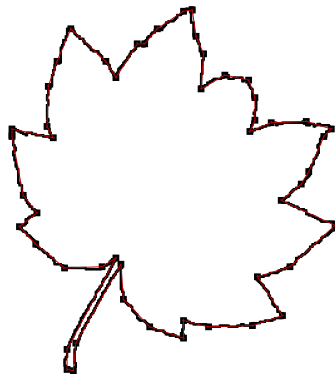
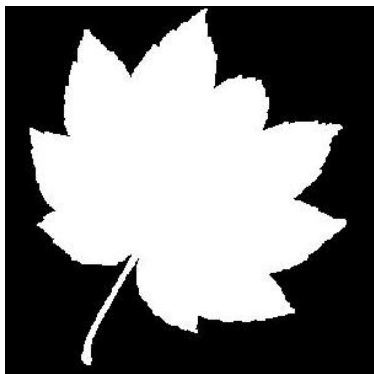
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# Dominant point

## Definition

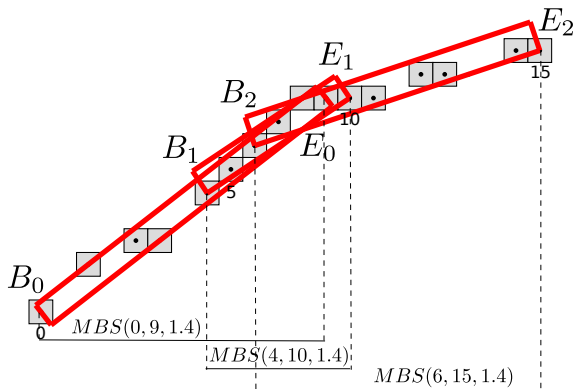
A **dominant point** (corner point) on a curve is a point of local maximum curvature.



# Dominant point detection

## Proposition [Nguyen11b]

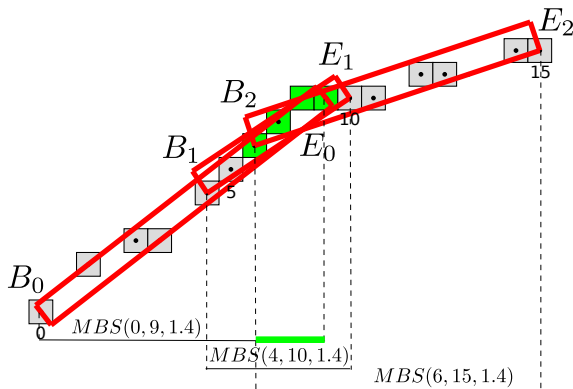
Dominant points of the curve is located in the **common zones** of successive maximal blurred segments.



# Dominant point detection

## Proposition [Nguyen11b]

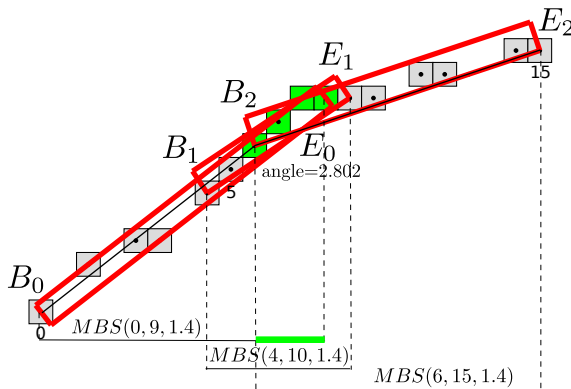
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# Dominant point detection

## Proposition [Ngo15]

Dominant point is detected as the point with **minimum angle measure** estimated with extremities of MBS composing the common zone.



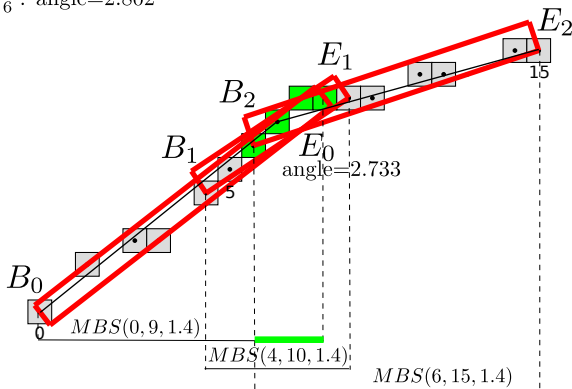


# Dominant point detection

## Proposition [Ngo15]

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$P_6$  : angle=2.802



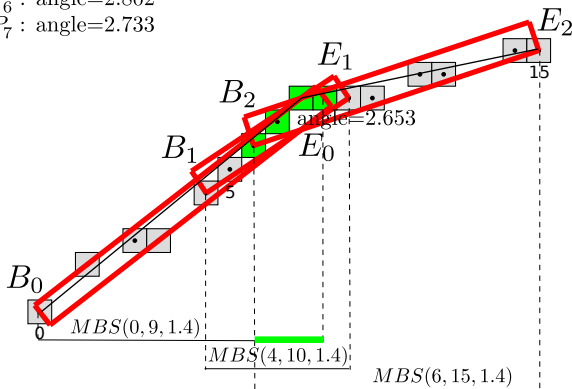
# Dominant point detection

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$P_6$  : angle=2.802

$P_7$  : angle=2.733



# Dominant point detection

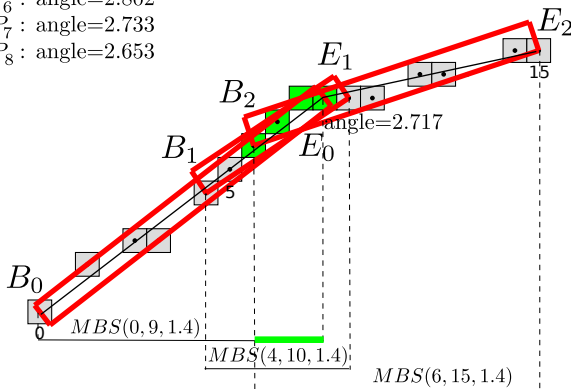
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$P_8$  : angle=2.653



# Dominant point detection

## Proposition [Ngo15]

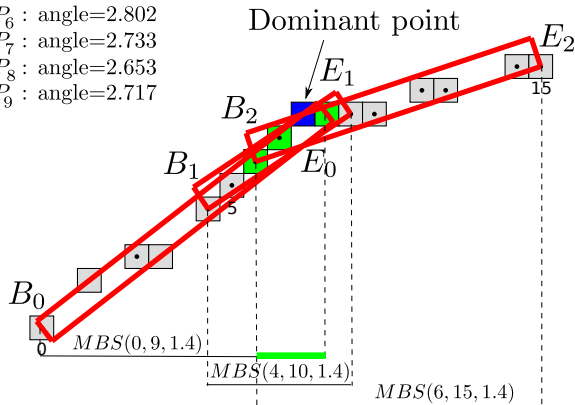
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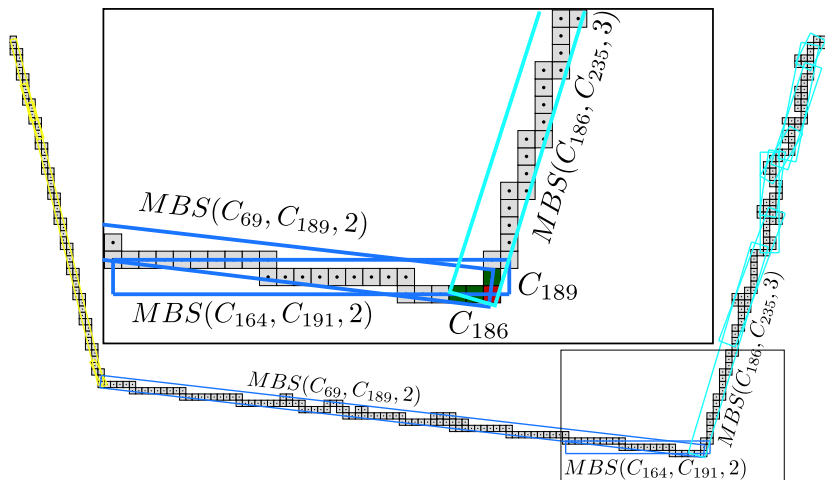
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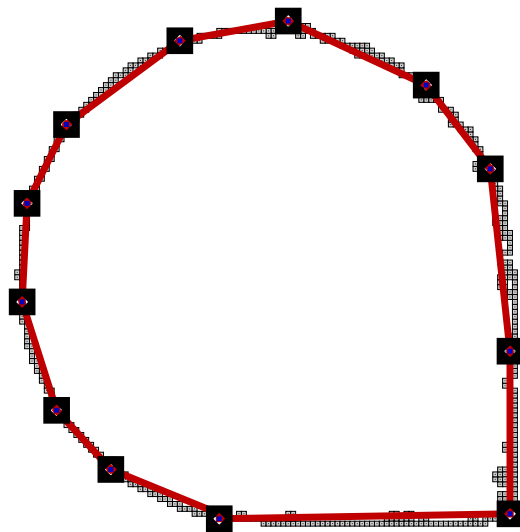
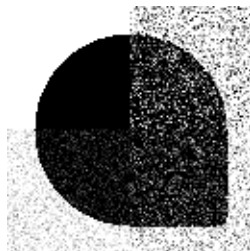
$P_9$  : angle=2.717



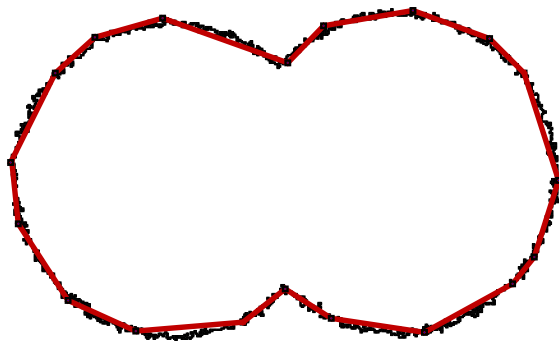
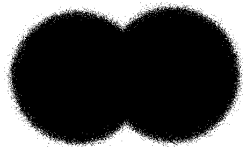
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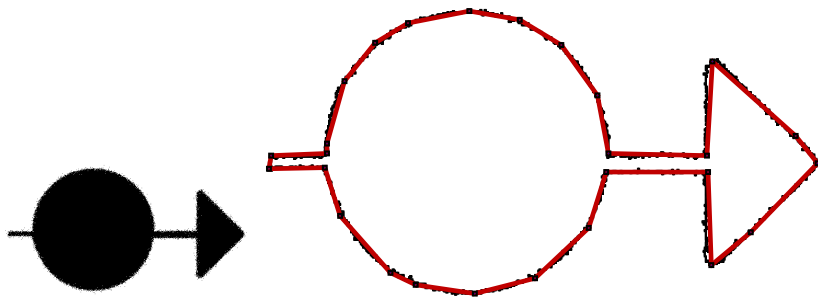
# Dominant point detection results



# Dominant point detection results

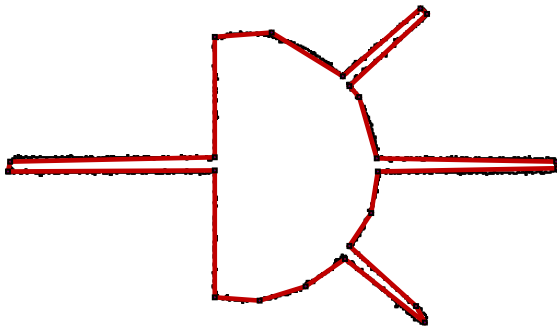
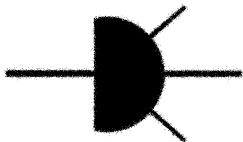


# Dominant point detection results

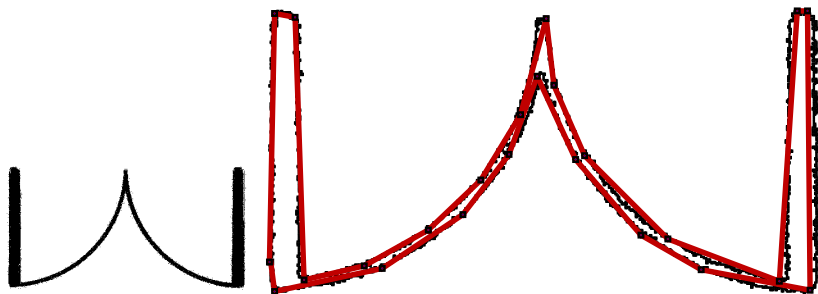




# Dominant point detection results



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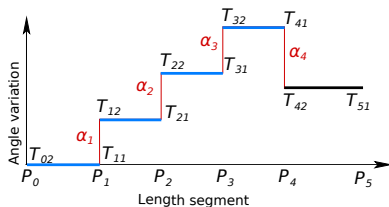
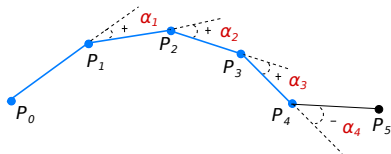
# Tangent space representation

## Definition

Let  $P = \{P_i\}_{i=0}^m$  be a polygon,  $l_i = |\overrightarrow{P_i P_{i+1}}|$  and  $\alpha_i = \angle(\overrightarrow{P_{i-1} P_i}, \overrightarrow{P_i P_{i+1}})$  s.t.  $\alpha_i > 0$  if  $P_{i+1}$  is on the right side of  $\overrightarrow{P_{i-1} P_i}$  and  $\alpha_i < 0$  otherwise.

A **tangent space representation**  $T(P)$  of  $P$  is a step function which is constituted of segments  $T_{i2}T_{(i+1)1}$  and  $T_{(i+1)1}T_{(i+1)2}$  for  $0 \leq i < m$  with

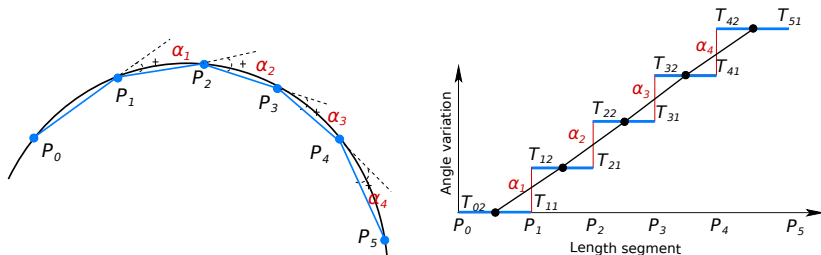
- ▶  $T_{02} = (0, 0)$ ,
- ▶  $T_{i1} = (T_{(i-1)2}.x + l_{i-1}, T_{(i-1)2}.y)$  for  $1 \leq i \leq m$ ,
- ▶  $T_{i2} = (T_{i1}.x, T_{i1}.y + \alpha_i)$ ,  $1 \leq i \leq (m - 1)$ .



# Tangent space representation

## Proposition [Nguyen11a]

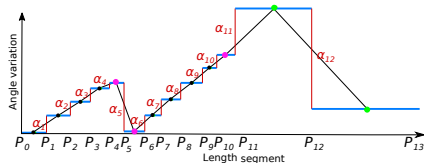
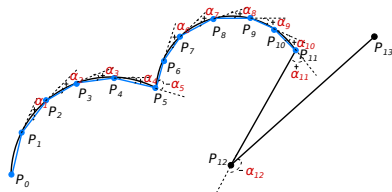
Let  $P = \{P_i\}_{i=0}^m$  be a polygon,  $l_i = |\overrightarrow{P_i P_{i+1}}|$ ,  $\alpha_i = \angle(\overrightarrow{P_{i-1} P_i}, \overrightarrow{P_i P_{i+1}})$  s.t.  $\alpha_i \leq \alpha \leq \frac{\pi}{4}$  for  $0 \leq i < n$ ,  $T(P)$  the tangent space representation of  $P$  and  $T(P)$  constitutes of segments  $T_{i2} T_{(i+1)1}$ ,  $T_{(i+1)1} T_{(i+1)2}$  for  $0 \leq i < m$ ,  $M = \{M_i\}_{i=0}^{m-1}$  the midpoint set of  $\{T_{i2} T_{(i+1)1}\}_{i=0}^{m-1}$ .  $P$  is a polygon whose vertices are on a real arc only if  $M_i$  belongs to a small width strip bounded by two real parallel lines, namely *quasi collinear* points.



# Tangent space representation

In the tangent space representation, the midpoints can be classified as

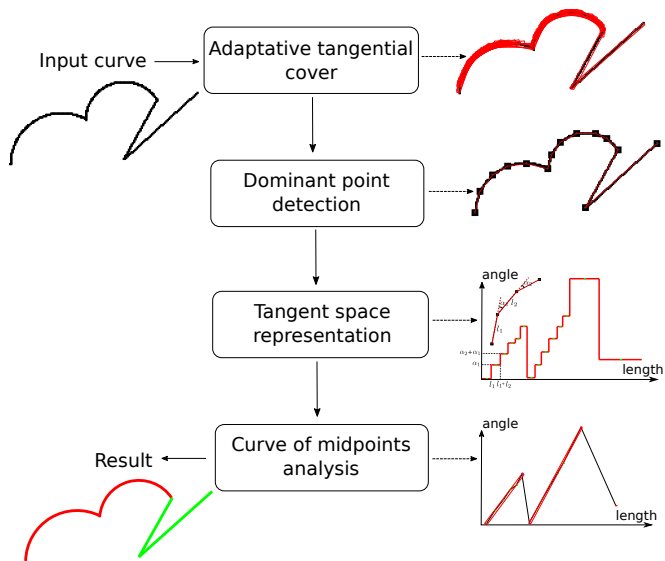
- ▶ **isolated point** if either  $(|M_i.y - M_{i-1}.y| > \alpha)$  or  $(|M_i.y - M_{i+1}.y| > \alpha) \implies$  a junction between two primitives
- ▶ **fully isolated point** if  $(|M_i.y - M_{i-1}.y| > \alpha)$  and  $(|M_i.y - M_{i+1}.y| > \alpha) \implies$  a segment
- ▶ **arc point** otherwise  $\implies$  an arc chord



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# Algorithm of arcs and segments decomposition





# Algorithm of arcs and segments decomposition

**Input:**  $C = (C_i)_{0 \leq i \leq n-1}$  discrete curve of  $n$  points  
 $\nu, \alpha$  test of collinear and admissible angle in tangent space

**Output:** *ARCs* and *SEGs* sets of arcs and segments of  $C$

**Begin**

$ARCs \leftarrow \emptyset, SEGs \leftarrow \emptyset$

Detect the dominant point  $D$  from ATC of  $C$

Transform  $D$  into the tangent space  $T(D)$

Construct the midpoint curve  $\{M_i\}_{i=0}^{m-1}$  of  $T(D)$

**for**  $i \leftarrow 1$  to  $m - 2$  **do**

$C_{b_i} C_{e_i}$  the part of  $C$  corresponds to  $M_i$

**if**  $(|M_i.y - M_{i-1}.y| > \alpha) \& (|M_i.y - M_{i+1}.y| > \alpha)$  **then**

$SEGs \leftarrow SEGs \cup \{\text{find a segment from } C_{b_i} C_{e_i}\}$

$MBS_\nu \leftarrow \emptyset$

**else**

...

**end if**

**end for**

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**Begin**

...

**for**  $i \leftarrow 1$  to  $m - 2$  **do**

$C_{b_i} C_{e_i}$  the part of  $C$  corresponds to  $M_i$

...

**if**  $MBS_\nu \leftarrow MBS \cup \{M_i\}$  is a MBS of width  $\nu$  **then**

$MBS_\nu \leftarrow MBS_\nu \cup \{M_i\}$

$pARC \leftarrow pARC \cup \{C_{b_i} C_{e_i}\}$

**else**

$ARCs \leftarrow ARCs \cup \{\text{find an arc from } pARC\}$

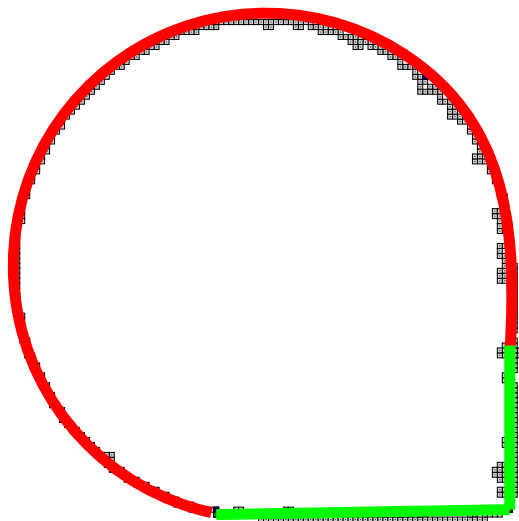
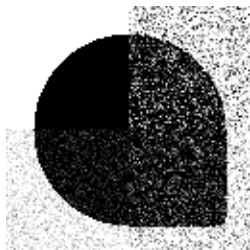
$pARC \leftarrow \emptyset$

**end if**

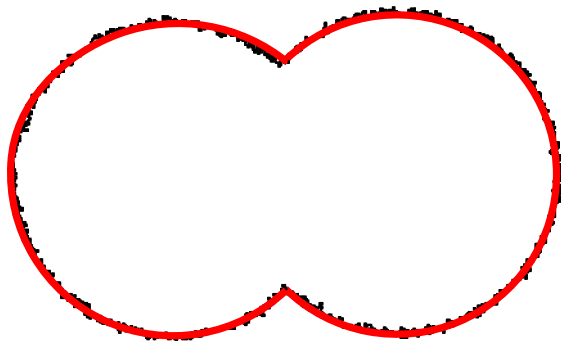
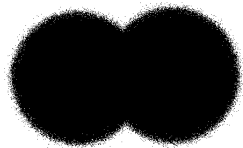
**end for**

**End**

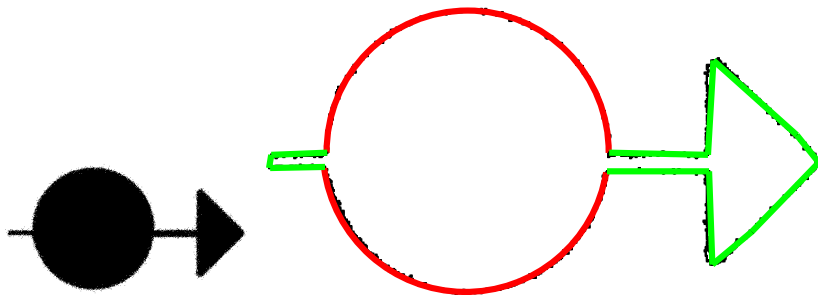
# Experimental results



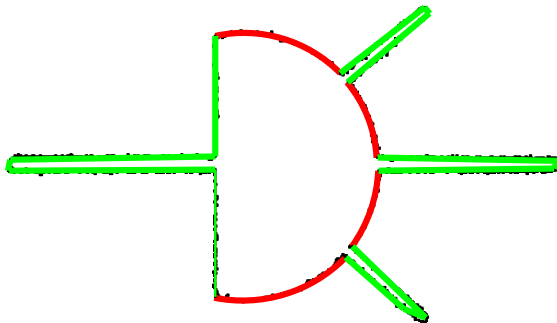
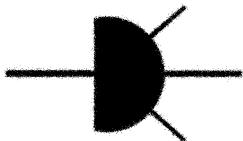
# Experimental results



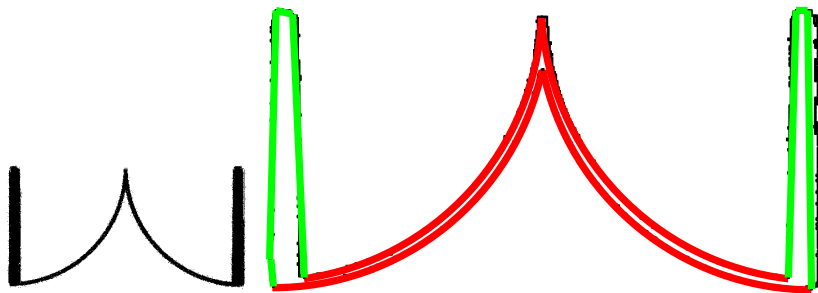
# Experimental results



# Experimental results



# Experimental results



# Online demonstration

An online demonstration based on the DGtal and ImaGene library at

[http://ipol-geometry.loria.fr/~phuc/ipol\\_demo/ATC\\_ArcSegDecom\\_IPOLDemo](http://ipol-geometry.loria.fr/~phuc/ipol_demo/ATC_ArcSegDecom_IPOLDemo)

## Arcs and Segments Decomposition of Digital Contours: Online Demonstration

[article](#) [demo](#) [archive](#)

Please cite the reference article if you publish results obtained with this online demo.

This demonstration applies the Adaptive Tangential Cover algorithm for arcs and segments decomposition of noisy digital curves.

### Select Data

Click on an image to use it as the algorithm input.



[image credits](#)

### Upload Data

Upload your own image files to use as the algorithm input.

input image

Images larger than 16777216 pixels will be resized. Upload size is limited to 16MB per image file and 10MB for the whole upload set. TIFF, JPEG, PNG, GIF, PNM (and other standard formats) are supported. The uploaded will be publicly archived unless you switch to private mode on the result page. Only upload suitable images. See the copyright and legal conditions for details.



# Overview

- 1 MOTIVATION
- 2 ADAPTIVE TANGENTIAL COVER
- 3 DOMINANT POINT DETECTION
- 4 TANGENT SPACE REPRESENTATION
- 5 DECOMPOSITION ALGORITHM
- 6 CONCLUSION & PERSPECTIVES**

# Conclusion

- ▶ Curve decomposition into arcs and segments with
  - ▶ Adaptive tangential cover
  - ▶ Dominant point detection
  - ▶ Tangent space representation
- ▶ Perspectives
  - ▶ Extension to other primitives
  - ▶ Reduction of the number of parameters
  - ▶ Integration of topology into the decomposition

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Thank you for your attention!