

# Structure discrète des courbes bruités

Couverture tantentielle adaptative et applications en analyse d'image

Phuc Ngo

Collaboration avec :

Hayat Nasser

Isabelle Debled

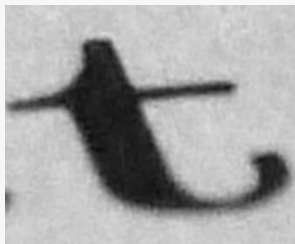
Bertrand Kerautret



19 Mai 2016

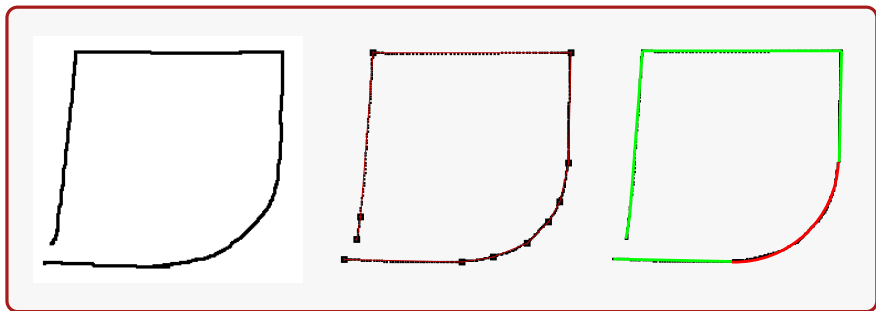
# Motivation

- ▶ Estimateurs géométriques
- ▶ Description de formes
- ▶ Approximation polygonale
- ▶ Extraction de caractéristiques



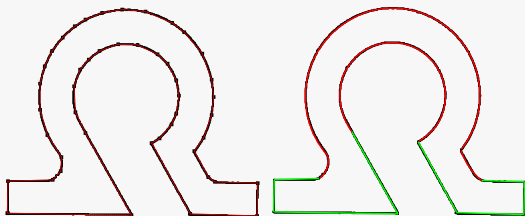
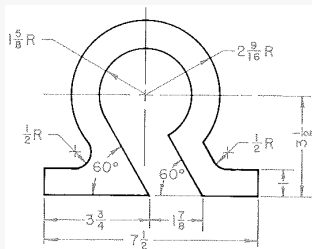
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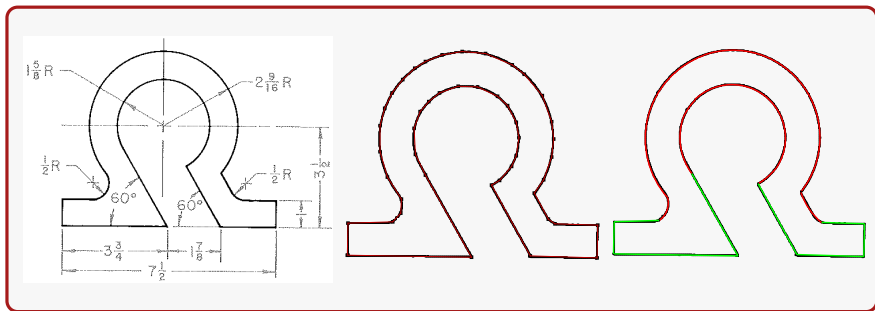
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# Motivation

- ▶ Estimateurs géométriques
- ▶ Description de formes
- ▶ Approximation polygonale
- ▶ Extraction de caractéristiques



⇒ Présence de **bruit** dans l'image par l'acquisition !

# Plan de la présentation

- 1 Notions de base
- 2 Couverture tangentielle adaptative
- 3 Application : Détection de points dominants
- 4 Application : Décomposition de courbe en arcs et segments
- 5 Conclusion & perspectives

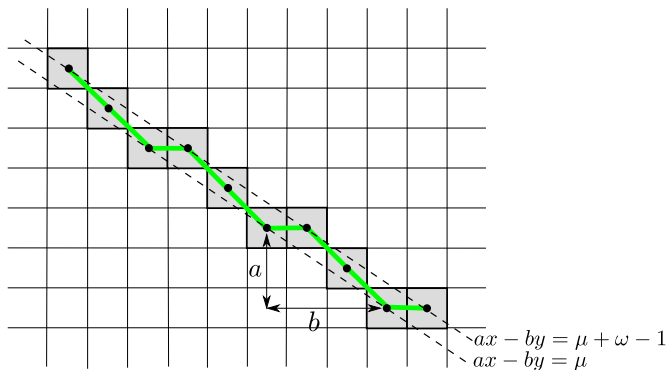
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# Discrete line and segment

## Definition

A **discrete line**  $\mathcal{D}(a, b, \mu, \omega)$  is the set of integer points  $(x, y)$  verifying  $\mu \leq ax - by < \mu + \omega$  where  $a, b, \mu, \omega \in \mathbb{Z}$  and  $\gcd(a, b) = 1$ .

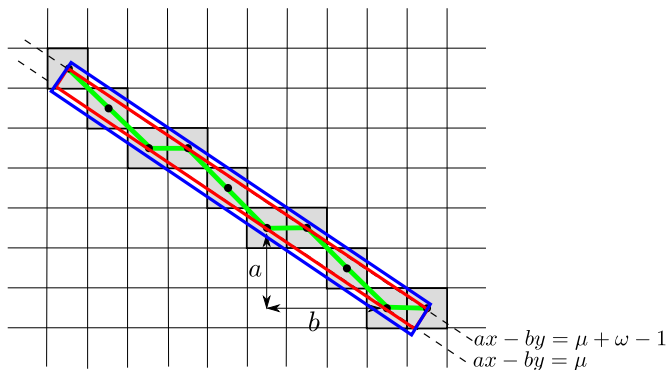




# Discrete line and segment

## Definition

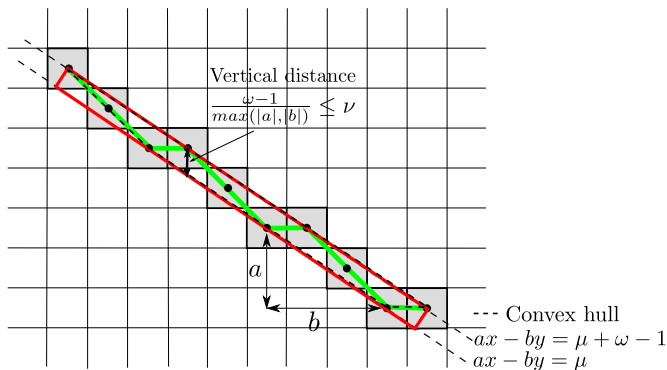
A **discrete segment** is a finite set  $\mathcal{S}_f$  of integer points bounded by the discrete line  $\mathcal{D}(a, b, \mu, \omega)$ .



# Discrete line and segment

## Definition

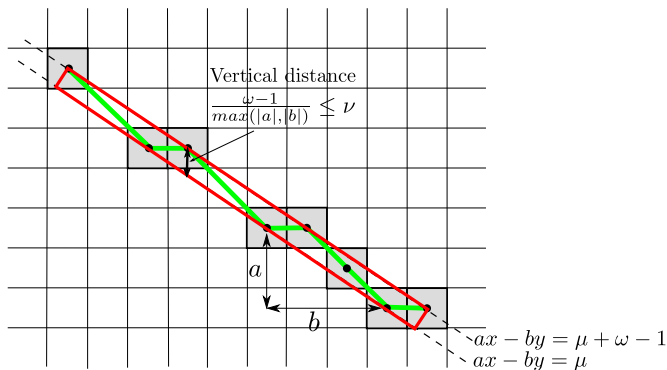
A discrete segment  $\mathcal{S}_f$  is **optimal** if its vertical (or horizontal) distance is equal to the vertical (or horizontal) thickness of its convex hull.



# Blurred segment

## Definition

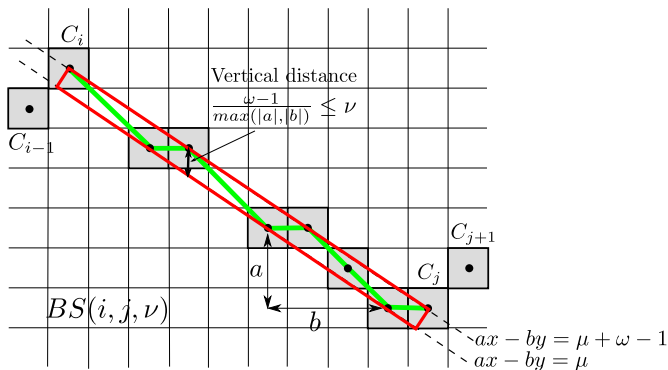
A sequence integer points  $\mathcal{S}_f$  is a **blurred segment of width  $\nu$**  if its optimal bounding discrete segment  $\mathcal{D}(a, b, \mu, \omega)$  has the vertical or horizontal distance less than or equal to  $\nu$ .



# Blurred segment

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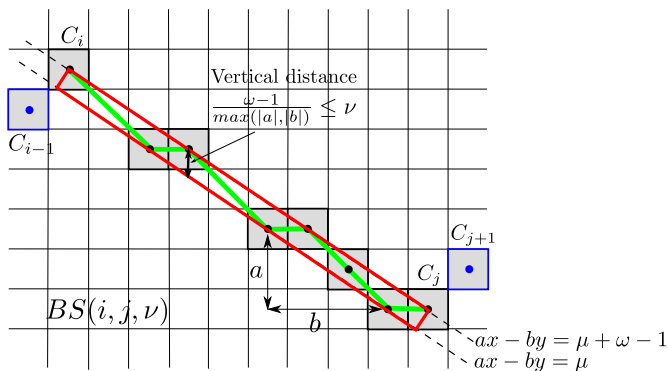
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# Blurred segment

## Definition

A blurred segment of width  $\nu$   $BS(i, j, \nu)$  is **maximal**, and noted  $MBS(i, j, \nu)$ , iff  $\neg BS(i, j + 1, \nu)$  and  $\neg BS(i - 1, j, \nu)$ .



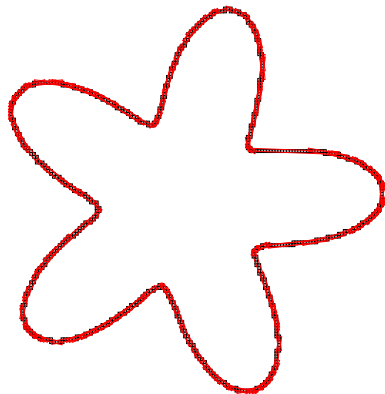
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# Maximal blurred segment decomposition

## Definition

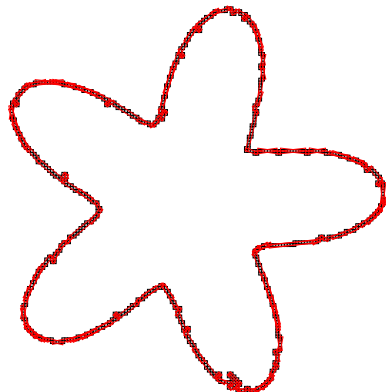
For any discrete curve  $C$ , its decomposition into maximal segments is called a **tangential cover** of  $C$ .



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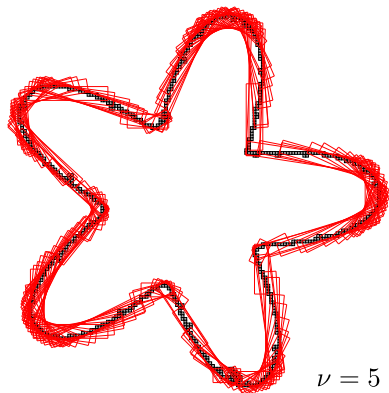




# Maximal blurred segment decomposition

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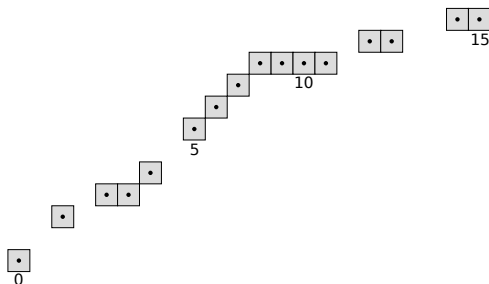
For any discrete curve  $C$ , its decomposition into maximal blurred segments of width  $\nu$  is called a **width  $\nu$  tangential cover** of  $C$ .

 $\nu = 5$

# Maximal blurred segment decomposition

## Algorithm

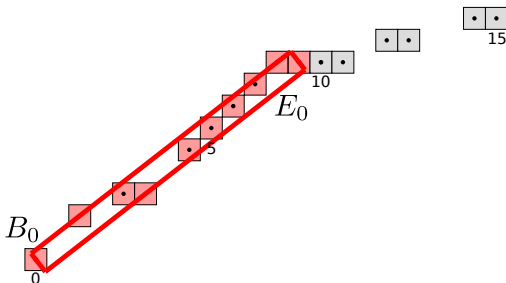
- ▶ Input: A discrete curve  $C$  and a width  $\nu$
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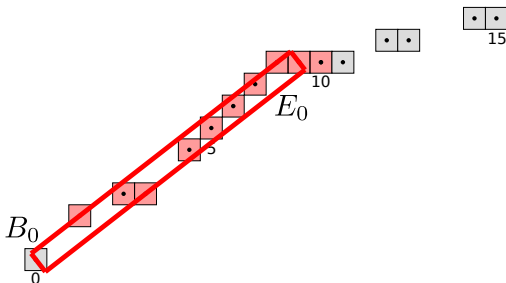
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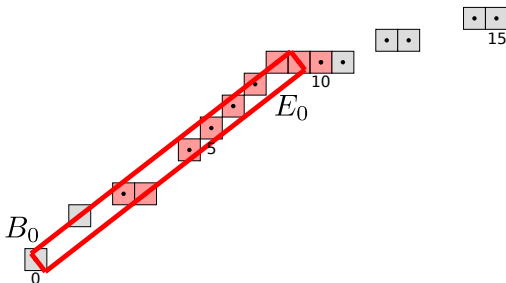
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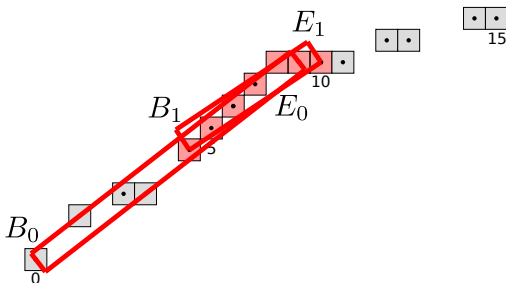
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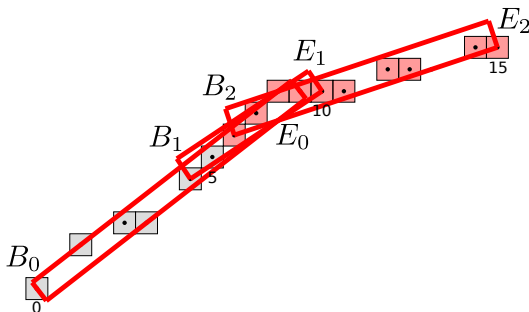
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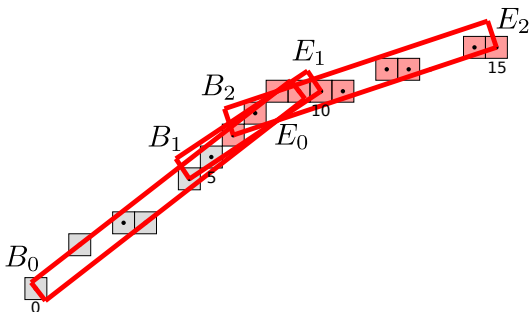
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⇒ Algorithm of decomposition is in quasi-linear time [1]

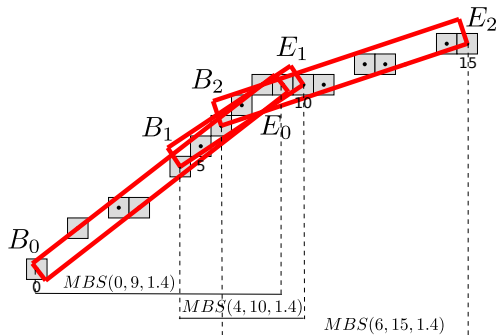


# Maximal blurred segment decomposition

## Property

Let  $MBS_{\nu}(C) = \{MBS(B_0, E_0, \nu), \dots, MBS(B_{m-1}, E_{m-1}, \nu)\}$  be the maximal blurred segment decomposition of width  $\nu$  of  $C$ , we have:

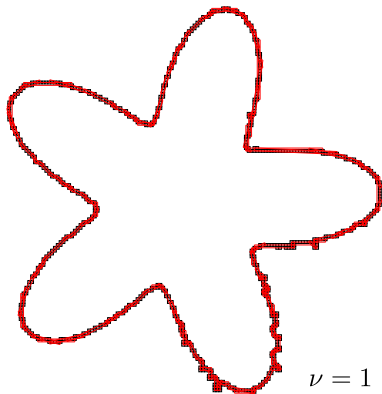
$$B_0 < B_1 < \dots < B_{m-1} \text{ and } E_0 < E_1 < \dots < E_{m-1}$$



# Maximal blurred segment decomposition

## Issues

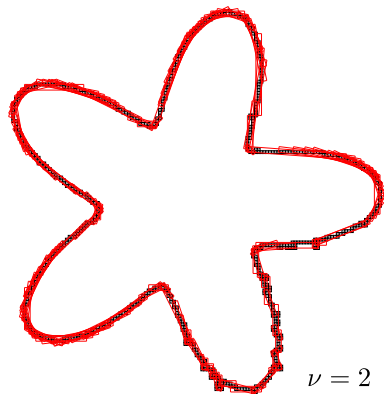
- ▶ Width value  $\nu$  is manually adjusted to deal with noise
- ▶ Mono-width  $\nu$  is not adapted to local noise along the contour



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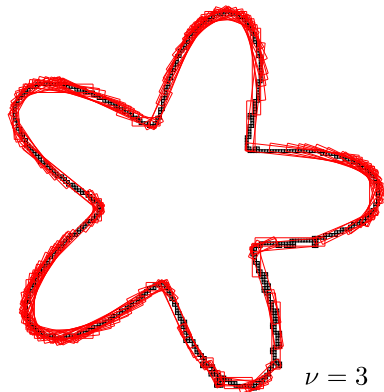
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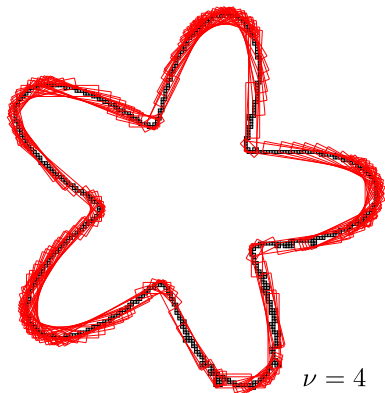
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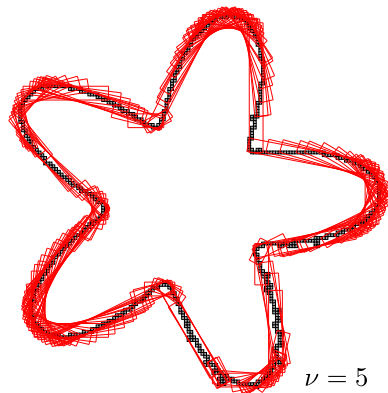
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## Solution

Tangential cover of different widths: **Adaptive Tangential Cover**

- ▶ appropriated widths based on a local noise estimation
  - ▶ **meaningful thickness detection**
- ▶ parameter-free computation

# Meaningful thickness detection

- ▶ Based on the asymptotic properties of the discrete length of maximal segments of perfect shape discretization
- ▶ Extend these properties with MBS at each point of the contour
- ▶ Compare to determine significant width of MBS of points

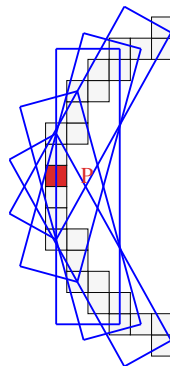
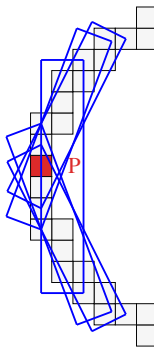
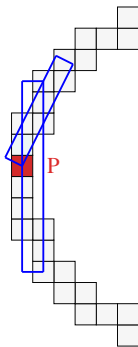
## Theorem

- ▶  $X$  simple connected shape in  $R^2$  with the boundary  $\delta X$  with a piecewise boundary  $C^3$
- ▶  $U$  an open connected neighborhood of  $p \in \delta X$ ,
- ▶  $(L_j^h)$  the digital lengths of the maximal segments covering  $p$  along the boundary of  $Dig_h(X)$ , where  $h$  is the grid size
  - ▶ if  $U$  is strictly convex/concave, then  $\Omega(1/h^{1/3}) \leq (L_j^h) \leq O(1/h^{1/2})$
  - ▶ if  $U$  has null curvature everywhere, then  $\Omega(1/h) \leq (L_j^h) \leq O(1/h)$



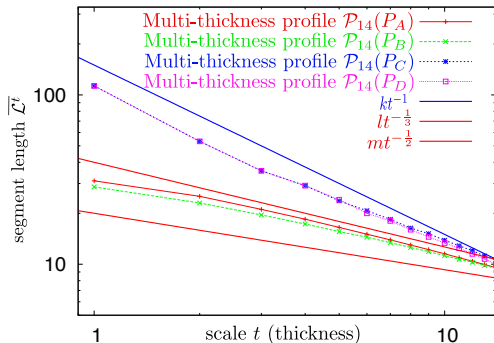
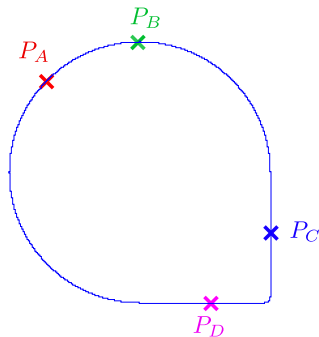
# Meaningful thickness detection

- ▶ Based on the asymptotic properties of the discrete length of maximal segments of perfect shape discretization
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- ▶ Compare to determine significant width of MBS of points



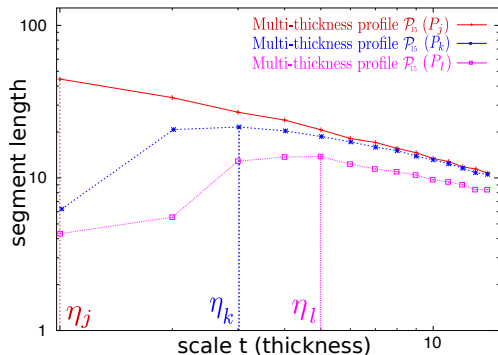
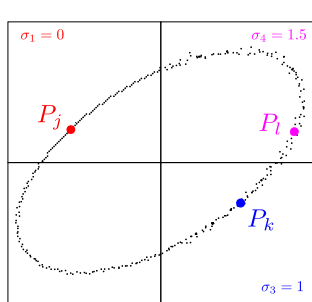
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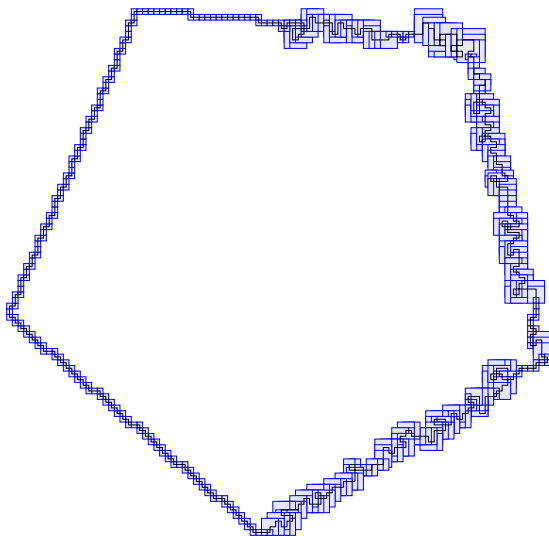


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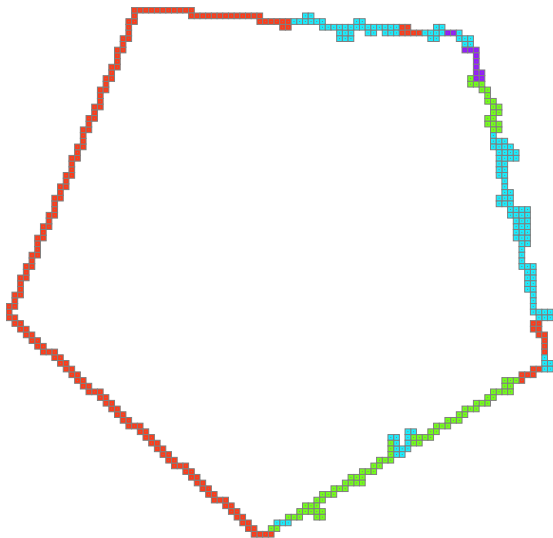
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# Meaningful thickness detection



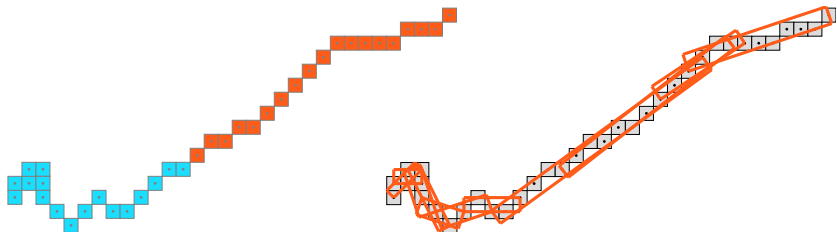
# Meaningful thickness detection



# Adaptive tangential cover

## Definition

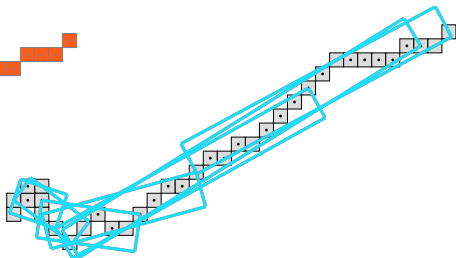
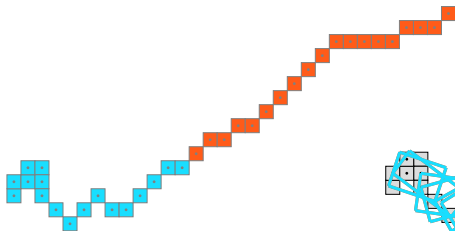
Let  $MBS_i = MBS(B_i, E_i, \cdot)$ ,  $MBS_j = MBS(B_j, E_j, \cdot)$  be two MBS. We say  $MBS_j$  is **included** in  $MBS_i$ , note as  $MBS_j \subseteq MBS_i$ , if  $B_i \leq B_j$  and  $E_i \geq E_j$ .



# Adaptive tangential cover

## Definition

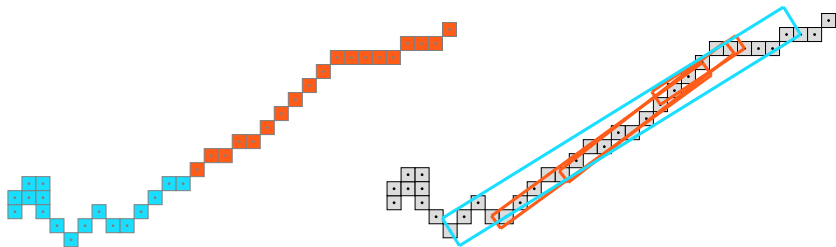
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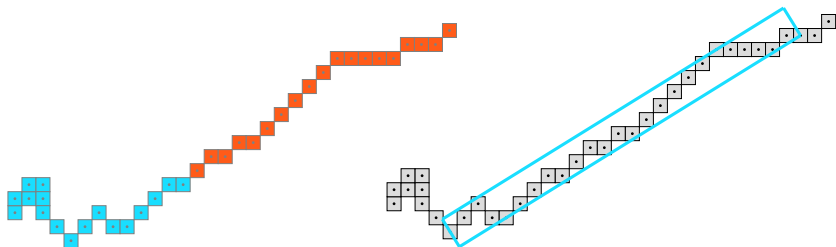




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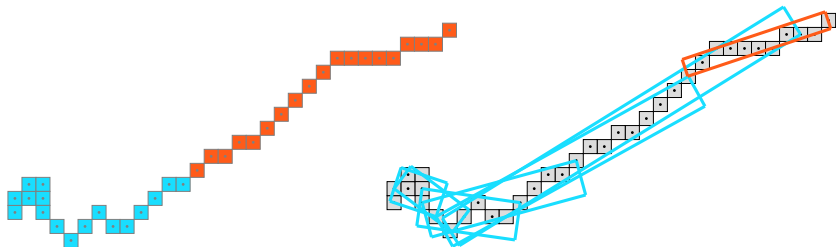
Let  $MBS(C)$  be a set of MBS of a discrete curve  $C$ . We say  $MBS_i \in MBS(C)$  is **largest** if for all  $MBS_j \in MBS(C)$  with  $i \neq j$ ,  $MBS_j \not\subseteq MBS_i$ .



# Adaptive tangential cover

## Definition

An **adaptive tangential cover (ATC)** associated to the meaningful thickness vector  $\eta$  of  $C$  is defined as the set of the largest MBS of  $\{MBS_j = MBS(B_j, E_j, v_k) \in MBS(C) \mid v_k = \max\{\eta_t \mid t \in \llbracket B_j, E_j \rrbracket\}\}$ .



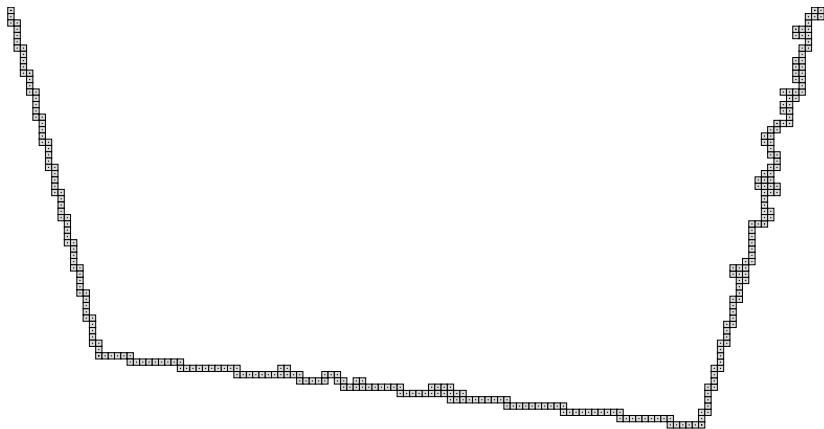
# Construction of adaptive tangential cover

## Principes

- ▶ Input:
  - ▶ A discrete curve  $C$  of  $n$  points
  - ▶ Vector of meaningful thickness  $\nu$  associated to each point of  $C$
- ▶ Output:
  - ▶ An ATC of  $C$  associated to the meaningful thickness vector  $\nu$
- ▶ The method for computing ATC is divided into two steps:
  - ▶ Labeling the points from the meaningful thickness values
    - ▶ Maximum meaningful thickness of MBS passing the point
  - ▶ Building the ATC with MBS of widths from the obtained labels
    - ▶ MBS of width being the label of at least one point in the MBS

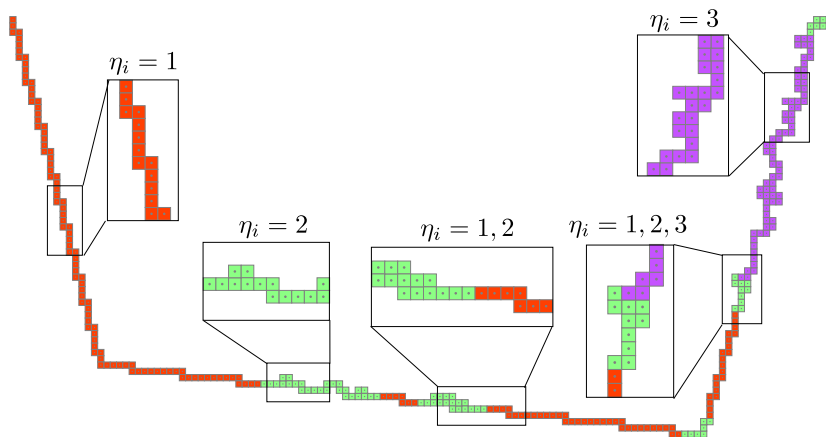
# Construction of adaptive tangential cover

## ► Input curve



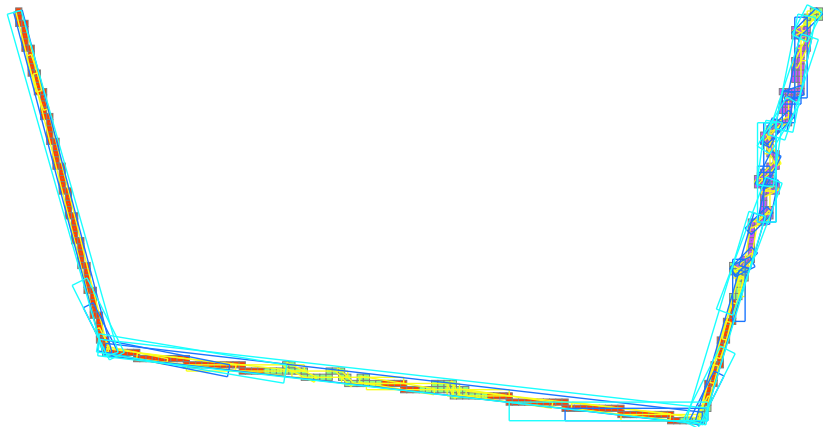
# Construction of adaptive tangential cover

- ▶ Input curve
- ▶ Meaningful thickness vector  $\nu$



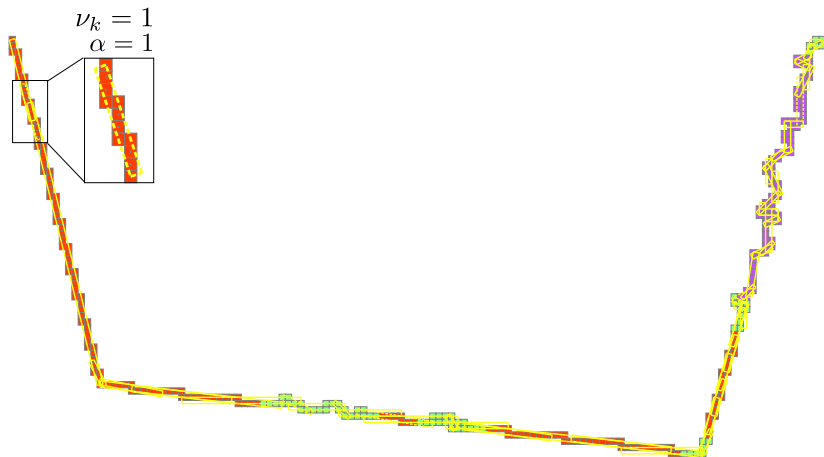
# Construction of adaptive tangential cover

- ▶ Input curve
- ▶ Meaningful thickness vector  $\nu$
- ▶ Tangent covers of ...



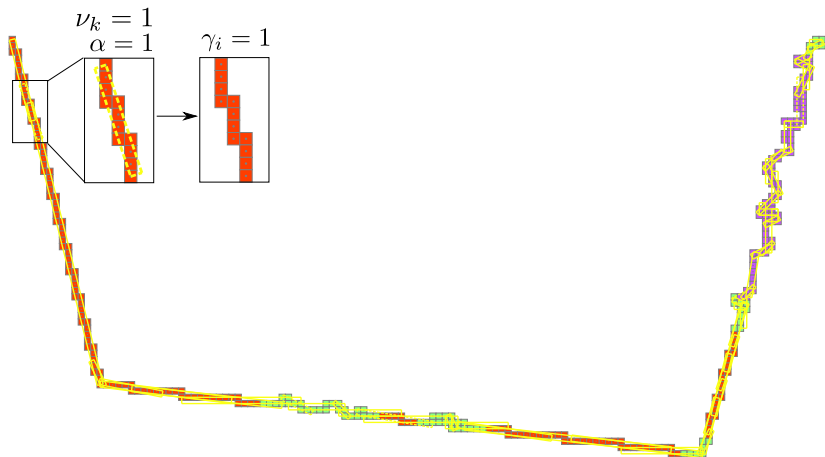
# Construction of adaptive tangential cover

- ▶ Step 1: Labeling points from meaningful thickness values
  - ▶  $\alpha$  max and  $\gamma$  label and  $\gamma_i = \nu_i$  at initialization



# Construction of adaptive tangential cover

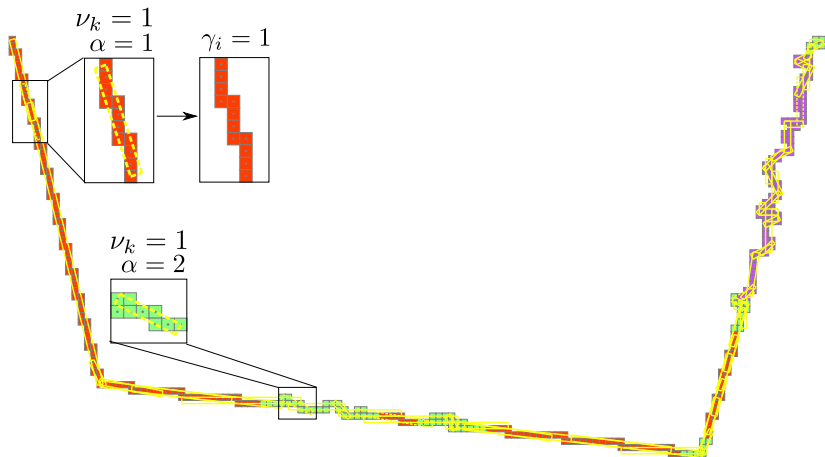
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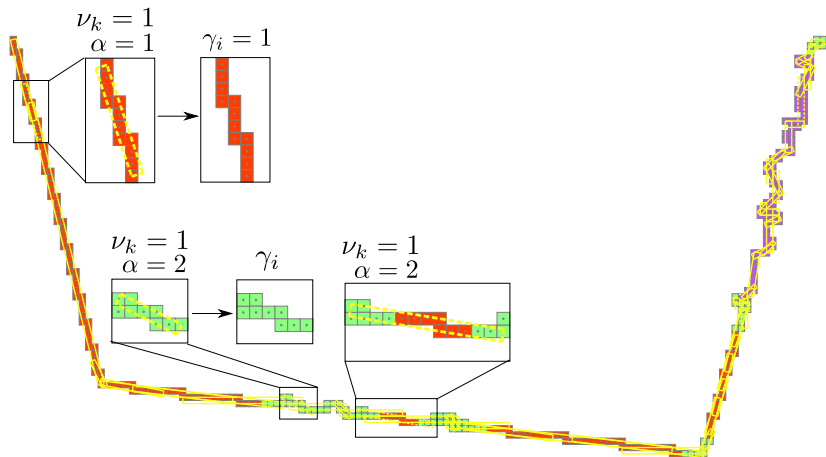
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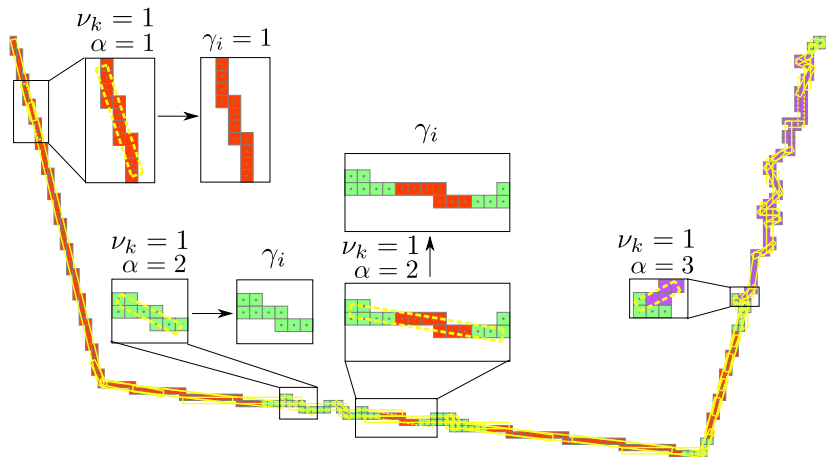
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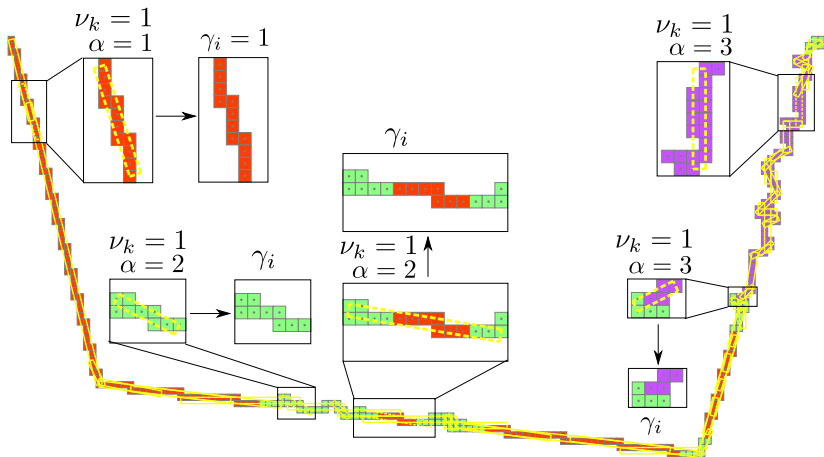
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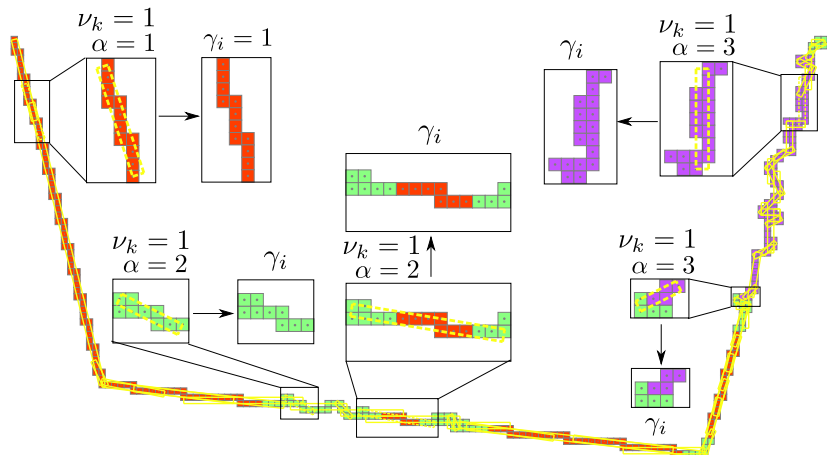
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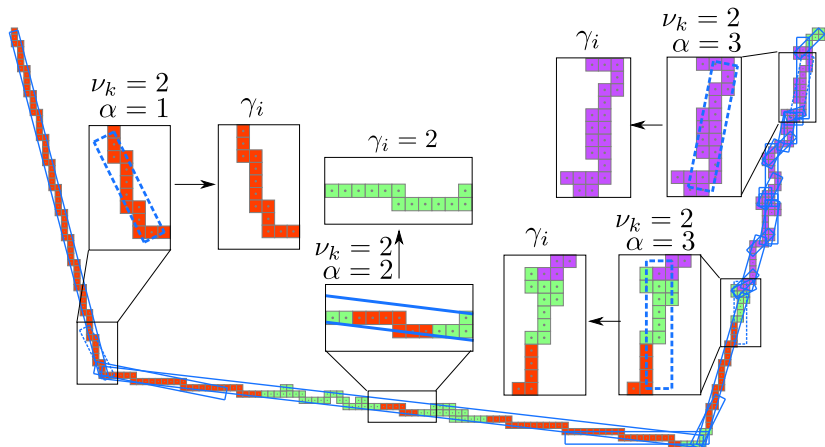
# Construction of adaptive tangential cover

- ▶ Step 1: Labeling points from meaningful thickness values
  - ▶  $\alpha$  max and  $\gamma$  label and  $\gamma_i = \nu_i$  at initialization



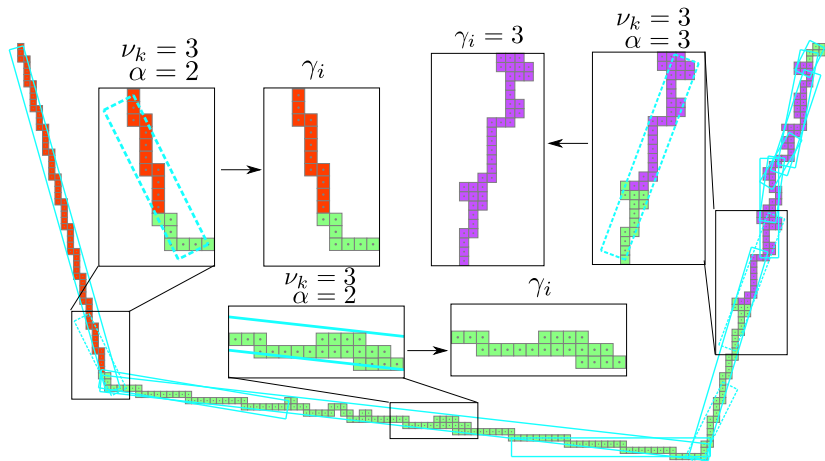
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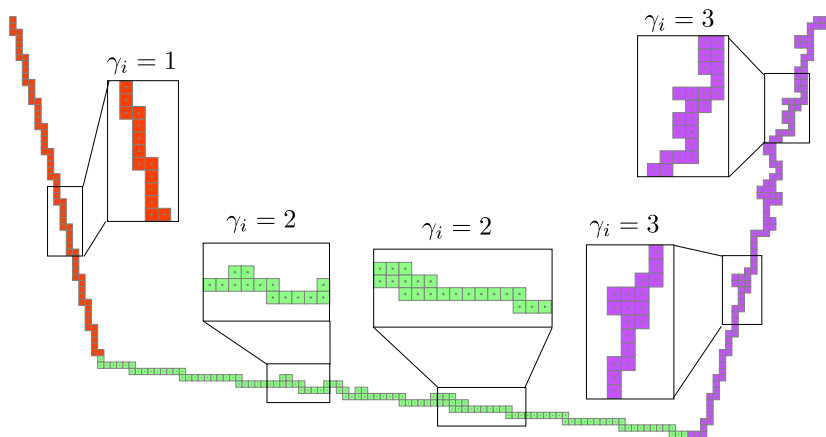
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- ▶ Step 1: Labeling points from meaningful thickness values
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# Construction of adaptive tangential cover

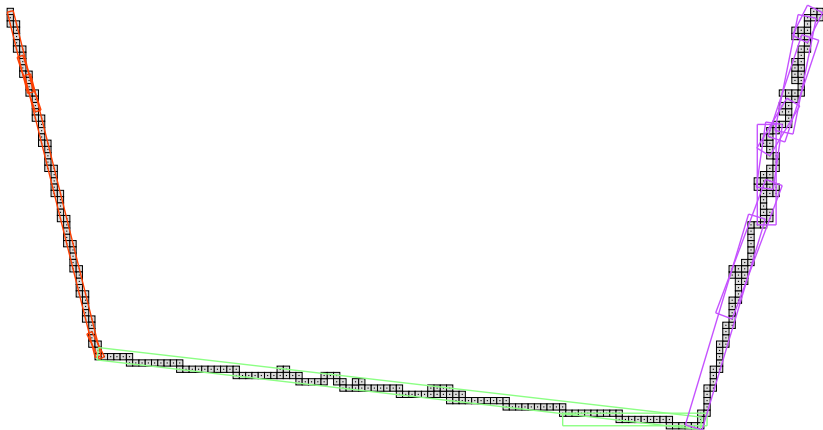
- ▶ Step 1: Labeling points from meaningful thickness values
  - ▶  $\alpha$  max and  $\gamma$  label and  $\gamma_i = \nu_i$  at initialization





# Construction of adaptive tangential cover

- ▶ Step 1: Labeling points from meaningful thickness values
  - ▶  $\alpha$  max and  $\gamma$  label and  $\gamma_i = \nu_i$  at initialization
- ▶ Step 2: Determining the MBS of the ATC



# Algorithm of adaptive tangential cover construction

**Input:**  $C = (C_i)_{0 \leq i \leq n-1}$  discrete curve of  $n$  points  
 $\eta = (\eta_i)_{0 \leq i \leq n-1}$  vector of MT associated to  $C$   
 $\nu = \{\nu_k \mid \nu_k \in \eta\}$  ordered set of MT value of  $\eta$   
 $MBS(C) = \{MBS_{\nu_k}(C)\}_{k=0}^{m-1}$  sets of MBS of  $C$  for  $\nu_k \in \nu$

**Output:**  $ATC(C)$  adaptive tangential cover of  $C$

**Begin**

$ATC(C) = \emptyset$ ;  $\gamma_i = \eta_i$  for  $i \in \llbracket 0, n-1 \rrbracket$

**for**  $\nu_k \in \nu$  **do**

**for**  $MBS(B_i, E_i, \nu_k) \in MBS_{\nu_k}(C)$  **do**

$\alpha = \max\{\eta_i \mid i \in \llbracket B_i, E_i \rrbracket\}$

**if**  $\alpha = \nu_k$  **then**

$\gamma_i = \nu_k$  for  $i \in \llbracket B_i, E_i \rrbracket$

**end if**

**end for**

**end for**

...

# Algorithm of adaptive tangential cover construction

**Input:**  $C = (C_i)_{0 \leq i \leq n-1}$  discrete curve of  $n$  points  
 $\eta = (\eta_i)_{0 \leq i \leq n-1}$  vector of MT associated to  $C$   
 $\nu = \{\nu_k \mid \nu_k \in \eta\}$  ordered set of MT value of  $\eta$   
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**Output:**  $ATC(C)$  adaptive tangential cover of  $C$

**Begin**

...

**for**  $\nu_k \in \nu$  **do**

**for**  $MBS(B_i, E_i, \nu_k) \in MBS_{\nu_k}(C)$  **do**

$\alpha = \max\{\eta_i \mid i \in \llbracket B_i, E_i \rrbracket\}$

**if**  $\exists \gamma_i$ , for  $i \in \llbracket B_i, E_i \rrbracket$ , such that  $\gamma_i = \nu_k$  **then**

$ATC(C) = ATC(C) \cup \{MBS(B_i, E_i, \nu_k)\}$

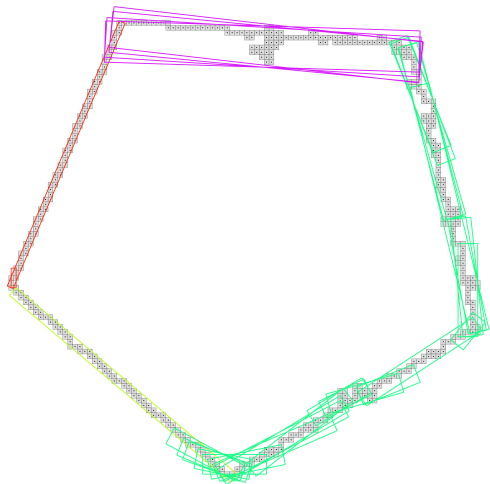
**end if**

**end for**

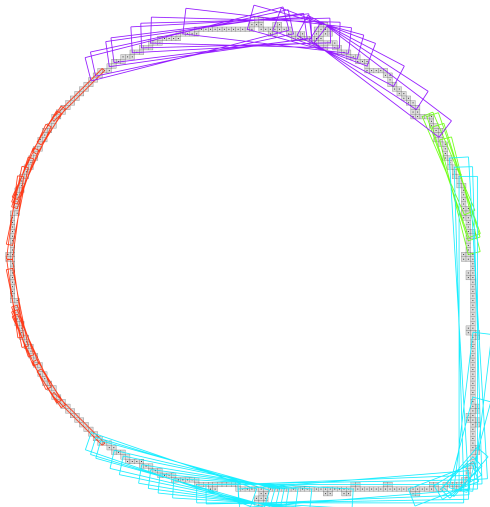
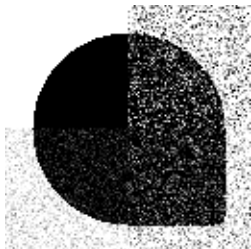
**end for**

**End**

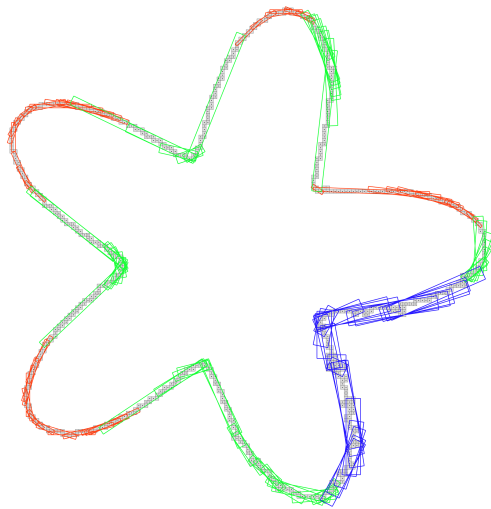
# Examples of Adaptive Tangential Cover



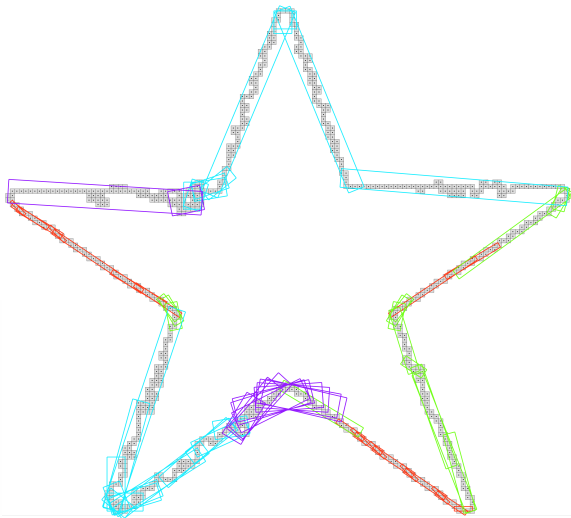
# Examples of Adaptive Tangential Cover



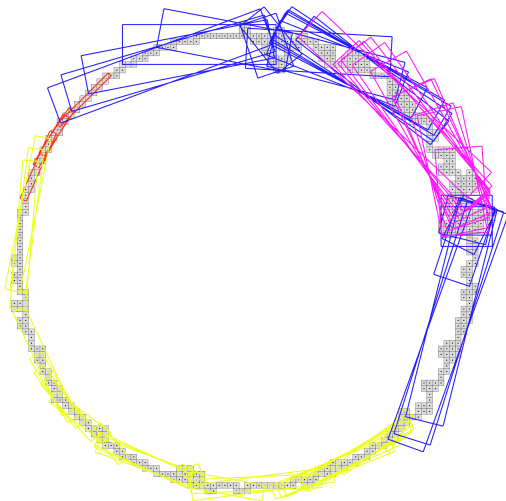
# Examples of Adaptive Tangential Cover



# Examples of Adaptive Tangential Cover

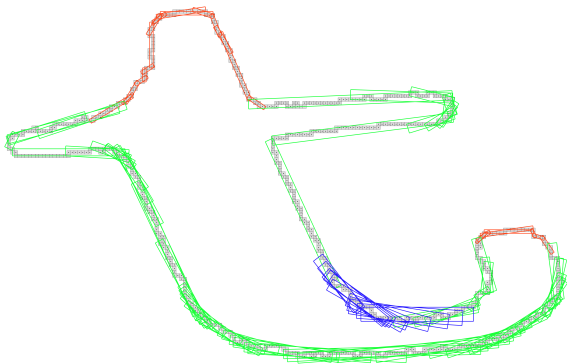
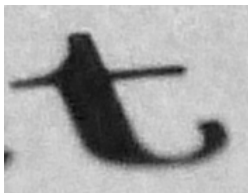


# Examples of Adaptive Tangential Cover





# Examples of Adaptive Tangential Cover



# Online demonstration

An online demonstration based on the DGtal and ImaGene library at [http://ipol-geometry.loria.fr/~kerautre/ipol\\_demo/ATC\\_IPOLDemo](http://ipol-geometry.loria.fr/~kerautre/ipol_demo/ATC_IPOLDemo)

## Adaptive Tangential Cover for Noisy Digital Contours: Online Demonstration

[article](#) [demo](#) [archive](#)

Please cite the reference article if you publish results obtained with this online demo.

This demonstration applies the Adaptive Tangential Cover algorithm with the application to curve simplification.

### Select Data

Click on an image to use it as the algorithm input.



[image credits](#)

### Upload Data

Upload your own image files to use as the algorithm input.

input image

Images larger than 16777216 pixels will be resized. Upload size is limited to 16MB per image file and 10MB for the whole upload set. TIFF, JPEG, PNG, GIF, PNM (and other standard formats) are supported. The uploaded will be publicly archived unless you switch to private mode on the result page. Only upload suitable images. See the copyright and legal conditions for details.

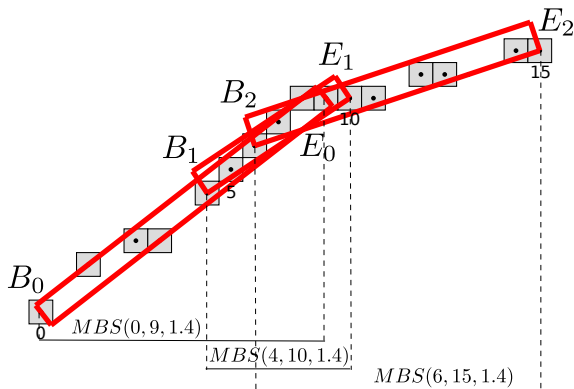
# Plan de la présentation

- 1 Notions de base
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- 3 Application : Détection de points dominants**
- 4 Application : Décomposition de courbe en arcs et segments
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# Dominant point

## Definition

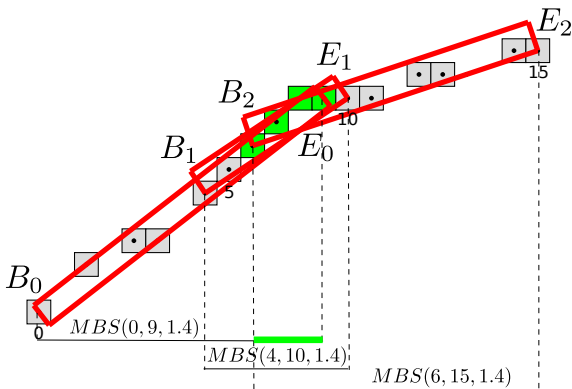
A **dominant point** (corner point) on a curve is a point of local maximum curvature.



# Dominant point detection [2]

## Proposition

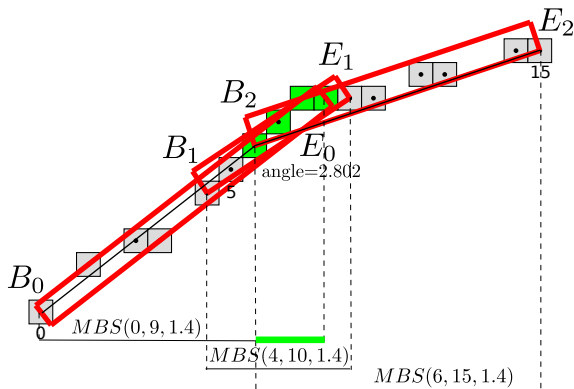
Dominant points of the curve is located in the **common zones** of successive maximal blurred segments.



# Dominant point detection [2]

## Strategy

Dominant point is detected as the point with **minimum angle measure** estimated with extremities of the MBS composing the common zone.

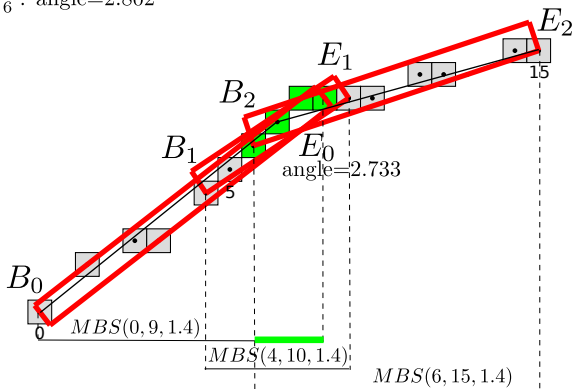


# Dominant point detection [2]

## Strategy

Dominant point is detected as the point with **minimum angle measure** estimated with extremities of the MBS composing the common zone.

$P_6$  : angle=2.802



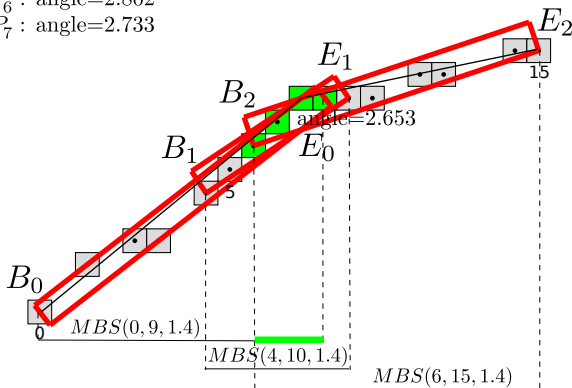
# Dominant point detection [2]

## Strategy

Dominant point is detected as the point with **minimum angle measure** estimated with extremities of the MBS composing the common zone.

$$P_6 : \text{angle} = 2.802$$

$$P_7 : \text{angle} = 2.733$$





# Dominant point detection [2]

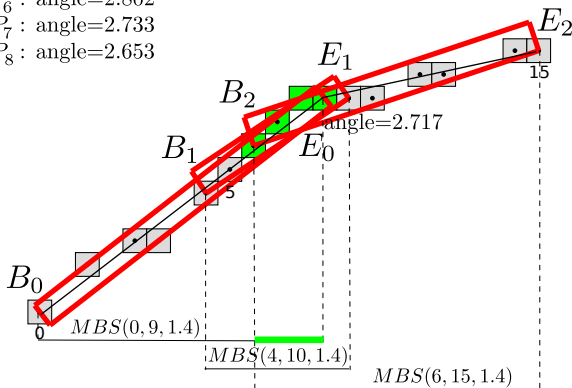
## Strategy

Dominant point is detected as the point with **minimum angle measure** estimated with extremities of the MBS composing the common zone.

$$P_6 : \text{angle}=2.802$$

$$P_7 : \text{angle}=2.733$$

$$P_8 : \text{angle}=2.653$$



# Dominant point detection [2]

## Strategy

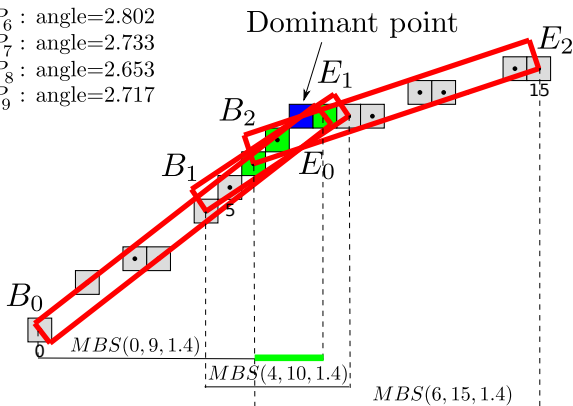
Dominant point is detected as the point with **minimum angle measure** estimated with extremities of the MBS composing the common zone.

$$P_6 : \text{angle}=2.802$$

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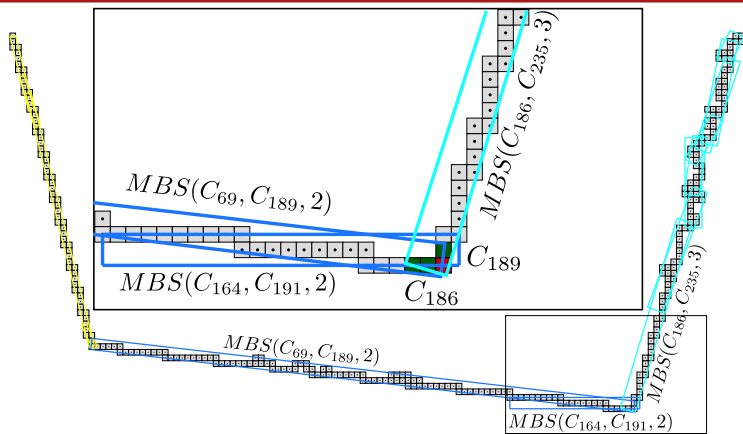
$$P_9 : \text{angle}=2.717$$



# Dominant point detection [2]

## Strategy

Dominant point is detected as the point with **minimum angle measure** estimated with extremities of the MBS composing the common zone.



# Algorithm of dominant point detection

**Input:**  $C$  discrete curve of  $n$  points

**Output:**  $D$  set of dominant points

**Begin**

Build  $ATC = \{MBS(B_i, E_i, \cdot)\}_{i=0}^{m-1}$

$n = |C|; m = |ATC|$

$q = 0; p = 1; D = \emptyset$

**while**  $p < m$  **do**

**while**  $E_q > B_p$  **do**

$p++$

**end while**

$D = D \cup \min\{Angle(C_{B_q}, C_i, C_{E_{p-1}}) \mid i \in \llbracket B_{p-1}, E_q \rrbracket\}$

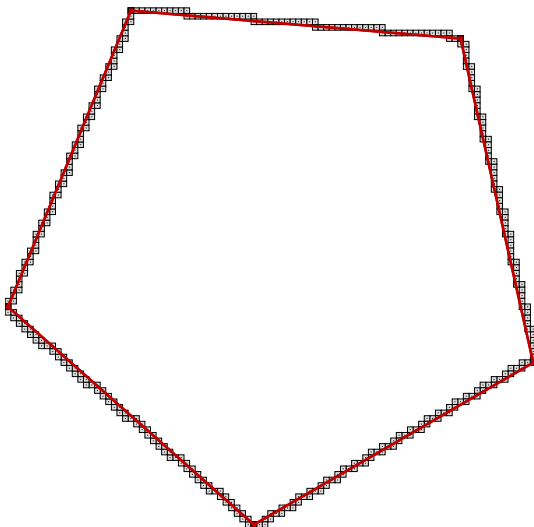
$q = p - 1$

**end while**

**End**

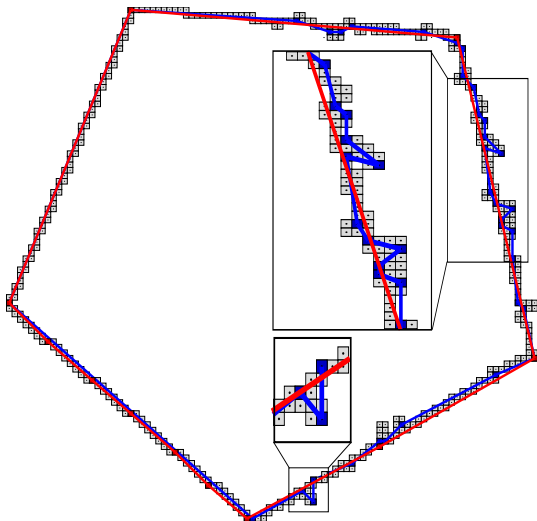
# Experimental results

- Mean tangential cover [6]
- Adaptive tangential cover



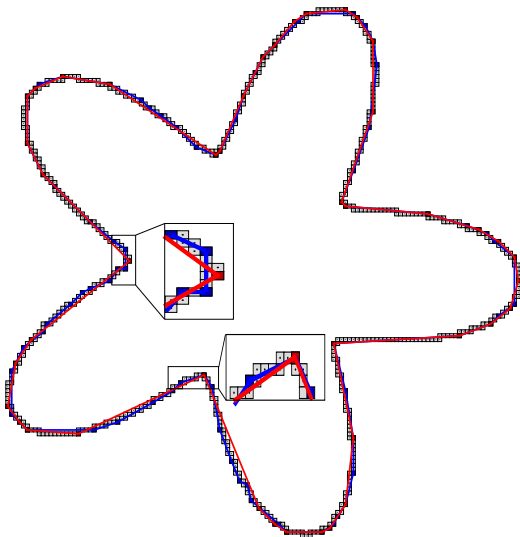
# Experimental results

- Mean tangential cover [6]
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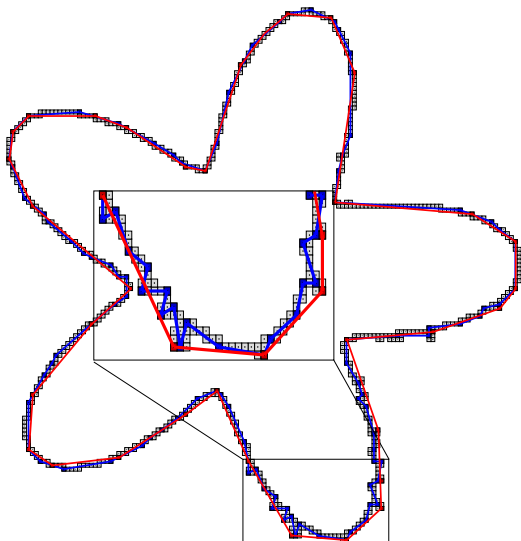
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- Mean tangential cover [6]
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# Experimental results

- Mean tangential cover [6]
- Adaptive tangential cover





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# Arcs and segments decomposition

## Motivation

Arcs and segments are the most appearing primitives in images

- ▶ Detection of shapes
  - ▶ medical imaging, technical images, manual drawings
- ▶ Automatic character recognition
  - ▶ sketch, scanned documents



# Arcs and segments decomposition

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Arcs and segments are the most appearing primitives in images

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## Tools

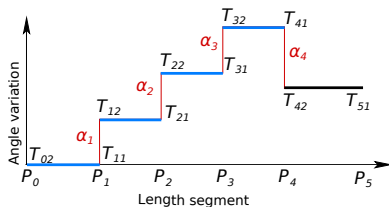
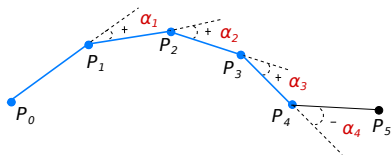
- ▶ Adaptive tangent cover
- ▶ Dominant point detection
- ▶ Tangent space representation

# Tangent space representation

## Definition

Let  $P = \{P_i\}_{i=0}^m$  be a polygon,  $l_i = |\overrightarrow{P_i P_{i+1}}|$  and  $\alpha_i = \angle(\overrightarrow{P_{i-1} P_i}, \overrightarrow{P_i P_{i+1}})$  s.t.  $\alpha_i > 0$  if  $P_{i+1}$  is on the right side of  $\overrightarrow{P_{i-1} P_i}$  and  $\alpha_i < 0$  otherwise. A tangent space representation  $T(P)$  of  $P$  is a step function which is constituted of segments  $T_{i2} T_{(i+1)1}$  and  $T_{(i+1)1} T_{(i+1)2}$  for  $0 \leq i < m$  with

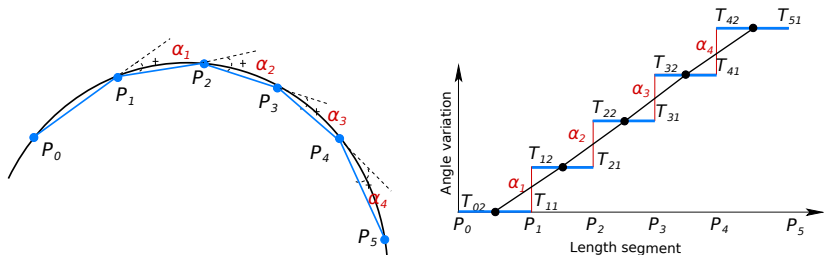
- ▶  $T_{02} = (0, 0)$ ,
- ▶  $T_{i1} = (T_{(i-1)2}.x + l_{i-1}, T_{(i-1)2}.y)$  for  $1 \leq i \leq m$ ,
- ▶  $T_{i2} = (T_{i1}.x, T_{i1}.y + \alpha_i)$ ,  $1 \leq i \leq (m - 1)$ .



# Tangent space representation

## Proposition

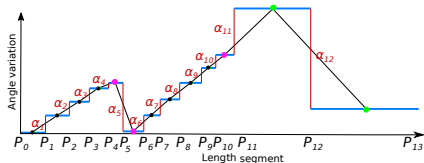
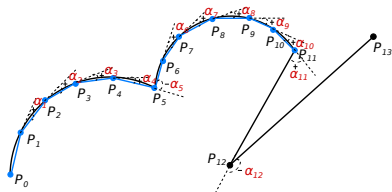
Let  $P = \{P_i\}_{i=0}^m$  be a polygon,  $l_i = |\overrightarrow{P_i P_{i+1}}|$ ,  $\alpha_i = \angle(\overrightarrow{P_{i-1} P_i}, \overrightarrow{P_i P_{i+1}})$  s.t.  $\alpha_i \leq \alpha \leq \frac{\pi}{4}$  for  $0 \leq i < n$ ,  $T(P)$  the tangent space representation of  $P$  and  $T(P)$  constitutes of segments  $T_{i2} T_{(i+1)1}$ ,  $T_{(i+1)1} T_{(i+1)2}$  for  $0 \leq i < m$ ,  $M = \{M_i\}_{i=0}^{m-1}$  the midpoint set of  $\{T_{i2} T_{(i+1)1}\}_{i=0}^{m-1}$ .  $P$  is a polygon whose vertices are on a real arc only if  $M$  is a set of quasi collinear points.



# Tangent space representation

In the tangent space representation, the midpoints can be classified as

- ▶ **isolated point** if either  $(|M_i.y - M_{i-1}.y| > \alpha)$  or  $(|M_i.y - M_{i+1}.y| > \alpha) \Rightarrow$  a junction between two primitives
- ▶ **fully isolated point** if  $(|M_i.y - M_{i-1}.y| > \alpha)$  and  $(|M_i.y - M_{i+1}.y| > \alpha) \Rightarrow$  a segment
- ▶ **arc point** otherwise  $\Rightarrow$  an arc chord



# Algorithm of arcs and segments decomposition

**Input:**  $C = (C_i)_{0 \leq i \leq n-1}$  discrete curve of  $n$  points  
 $\nu, \alpha$  test of collinear and admissible angle in tangent space  
**Output:** *ARCs* and *SEGs* sets of arcs and segments of  $C$

**Begin**

$ARCs \leftarrow \emptyset, SEGs \leftarrow \emptyset$

Detect the dominant point  $D$  of  $C$

Transform  $D$  into the tangent space  $T(D)$

Construct the midpoint curve  $\{M_i\}_{i=0}^{m-1}$  of  $T(D)$

**for**  $i \leftarrow 1$  to  $m - 2$  **do**

$C_{b_i} C_{e_i}$  the part of  $C$  corresponds to  $M_i$

**if**  $(|M_i.y - M_{i-1}.y| > \alpha) \& (|M_i.y - M_{i+1}.y| > \alpha)$  **then**

$SEGs \leftarrow SEGs \cup \{\text{find a segment from } C_{b_i} C_{e_i}\}$

$MBS_\nu \leftarrow \emptyset$

**else**

...

**end if**

**end for**

# Algorithm of arcs and segments decomposition

**Input:**  $C = (C_i)_{0 \leq i \leq n-1}$  discrete curve of  $n$  points  
 $\nu, \alpha$  test of collinear and admissible angle in tangent space

**Output:** ARCs and SEGs sets of arcs and segments of  $C$

**Begin**

...

**for**  $i \leftarrow 1$  to  $m - 2$  **do**

$C_{b_i} C_{e_i}$  the part of  $C$  corresponds to  $M_i$

...

**if**  $MBS_\nu \leftarrow MBS \cup \{M_i\}$  is a MBS of width  $\nu$  **then**

$MBS_\nu \leftarrow MBS_\nu \cup \{M_i\}$

$pARC \leftarrow pARC \cup \{C_{b_i} C_{e_i}\}$

**else**

$ARCs \leftarrow ARCs \cup \{\text{find an arc from } pARC\}$

$pARC \leftarrow \emptyset$

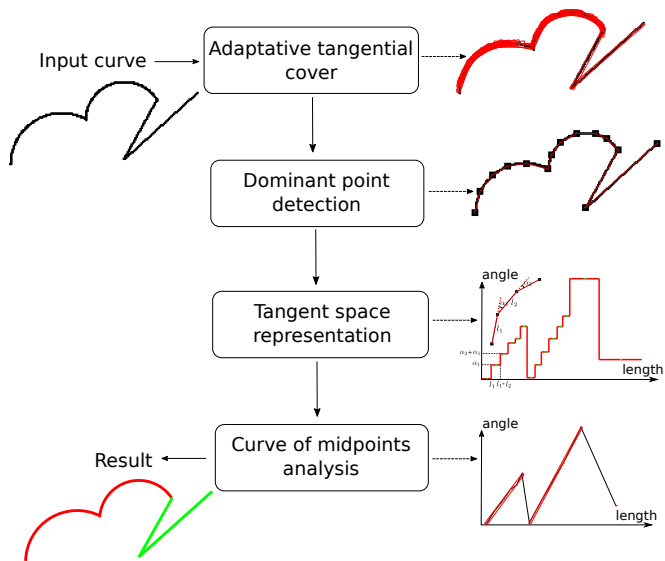
**end if**

**end for**

**End**



# Algorithm of arcs and segments decomposition



# Experimental results

...

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# Conclusion

## Contributions






- ▶ Algorithm based on discrete structure of the curve
- ▶ Algorithm without heuristic but a simple measure of angle

## Perspectives

- ▶ Parameter free method
- ▶ Adaptive-thickness

Thank you for your attention!

# References

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