Adaptive Tangential Cover for Noisy Digital Contours

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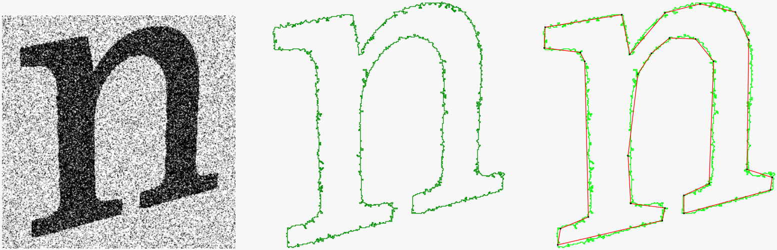
20 April 2016

Motivation

- Shape analysis
- Pattern recognition
- Polygonal approximation



Nguyen and Debled-Rennesson, PR - 2010

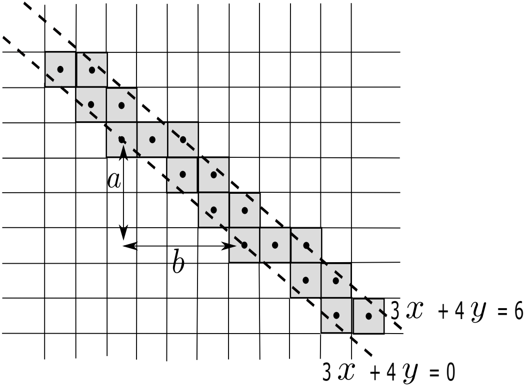
Content

Content

Discrete line and segment

Definition

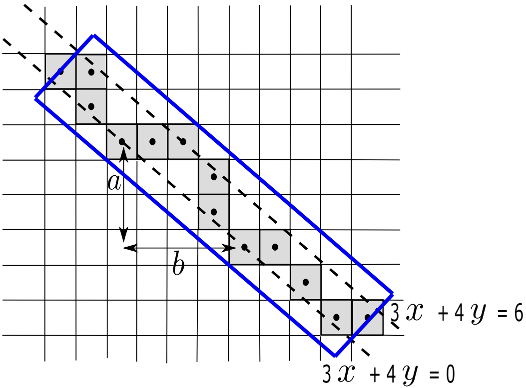
A **discrete line** $\mathcal{D}(a, b, \mu, \omega)$ is the set of integer points (x, y) verifying $\mu \leq ax - by < \mu + \omega$ where $a, b, \mu, \omega \in \mathcal{Z}$ and $gcd(a, b) = 1$.



Discrete line and segment

Definition

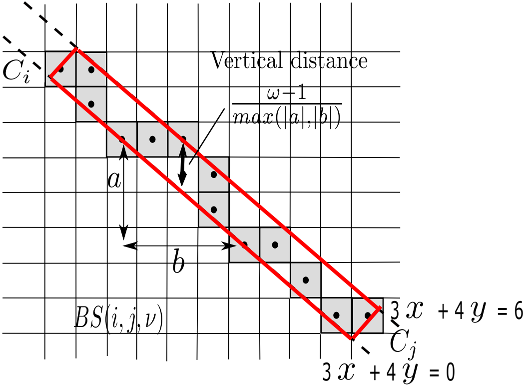
- $\mathcal{S}_f$ is a sequence of integer points.
- A discrete line $\mathcal{D}(a, b, \mu, \omega)$ is said to be **bounding** for $\mathcal{S}_f$ if all points of $\mathcal{S}_f$ belong to $\mathcal{D}$.



Discrete line and segment

Definition

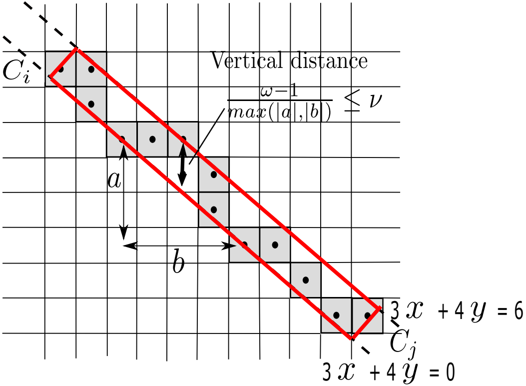
A bounding discrete segment $\mathcal{D}(a, b, \mu, \omega)$ of $\mathcal{S}_f$ is **optimal** if its vertical (or horizontal) distance $\frac{\omega-1}{max(|a|,|b|)}$ is equal to the vertical (or horizontal) thickness of the convex hull of $\mathcal{S}_f$.



Blurred segment

Definition

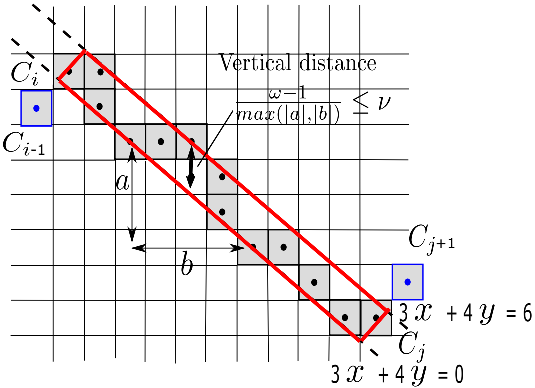
A sequence integer points $\mathcal{S}_f$ is a **blurred segment of width** ν if its optimal bounding discrete segment $\mathcal{D}(a, b, \mu, \omega)$ has the vertical or horizontal distance less than or equal to ν.



Blurred segment

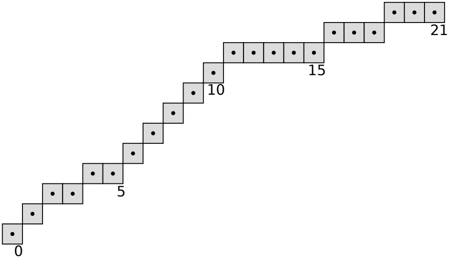
Definition

A blurred segment of width ν is **maximal**, and noted $MBS(i, j, \nu)$, iff $\neg BS(i, j+1, \nu)$ and $\neg BS(i-1, j, \nu)$.



Maximal blurred segment decomposition

- Input: A discrete curve C and a width ν
- Output: The decomposition $MBS_{\nu}(C)$ of C
- Algorithm: Computation of the sequence of maximal blurred segments by incrementally adding (resp. removing) a pixel to (resp. from) the considering blurred segment



Feschet, F., Tougne, L.: Optimal time computation of the tangent of a discrete curve: Application to the curvature. In: DGCI. LNCS, vol. 1568, pp. 31-40 (1999)

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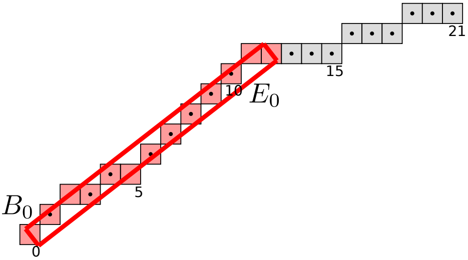

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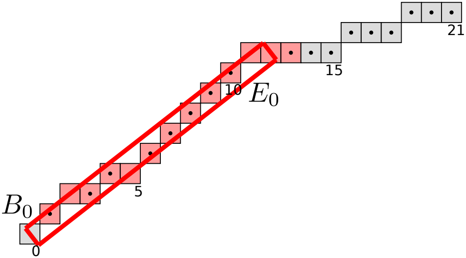

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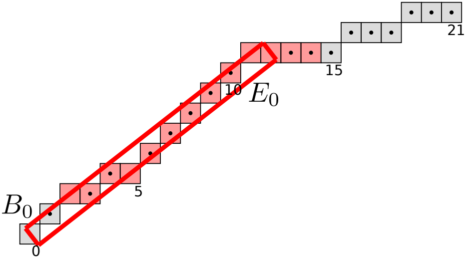

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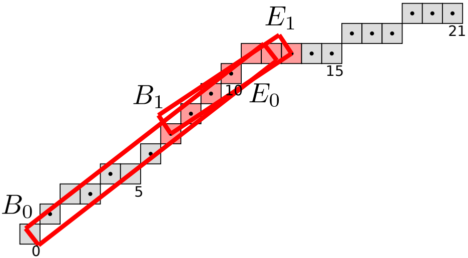

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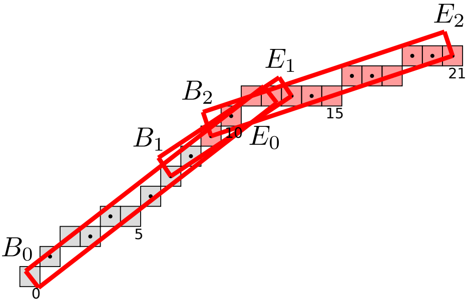

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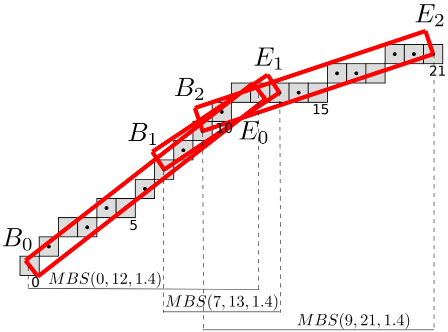
Maximal blurred segment decomposition

Property

Let $MBS_\nu(C)$ be the maximal blurred segment decomposition of witdth ν of C:
$MBS_\nu(C) = \{MBS(B_0, E_0, \nu), \ldots, MBS(B_{m-1}, E_{m-1}, \nu)\}$.
Then, $B_0 < B_1 < ... < B_{m-1}$ and $E_0 < E_1 < ... < E_{m-1}$.



Width estimator (Meaningful scale, Meaningful Thickness)

- **Meaningful scale**[1, 2] was designed to locally estimate what is the best scale to analyze a digital contour.
  - asymptotic properties of the discrete length L of maximal segments
- Meaningful scale detection has been extended to the detection of the **Meaningful Thickness (MT)**[3]

[1] Kerautret, B., Lachaud, J.O.: Meaningful Scales Detection along Digital Contours for Unsupervised Local Noise Estimation. IEEE Transactions on Pattern Analysis and Machine Intelligence 34(12), 2379-2392 (Dec 2012)
[2] Kerautret, B., Lachaud, J.O.: Meaningful Scales Detection: an Unsupervised Noise Detection Algorithm for Digital Contours. Image Processing On Line 4, 98-115 (2014)
[3] Kerautret, B., Lachaud, J.O., Said, M.: Meaningful Thickness Detection on Polygonal Curve. In Proceedings of the 1st International Conference on Pattern Recoginition Applications and Methods. pp. 372-379. SciTePress (2012)

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In our work :

- maximal blurred segment
- thickness/width parameter of the maximal blurred segment

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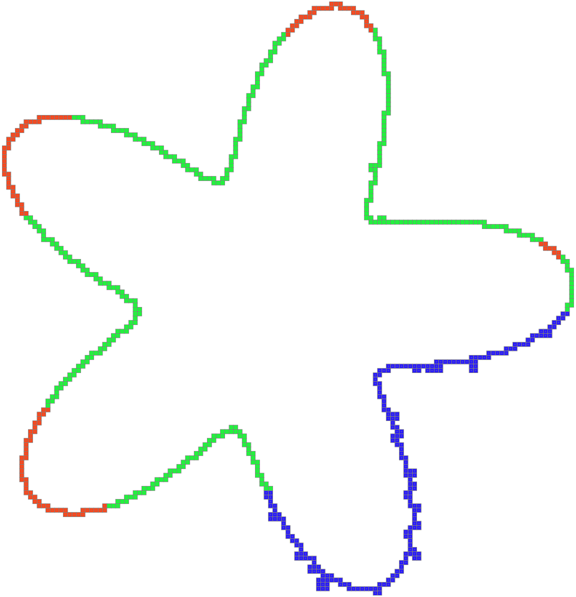
Meaningful Thickness



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Width ν tangential cover

Definition

The sequence of all maximal blurred segments with a constant width ν along a digital contour C is called the **width ν tangential cover of C**.

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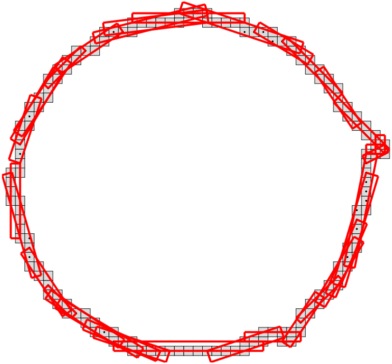

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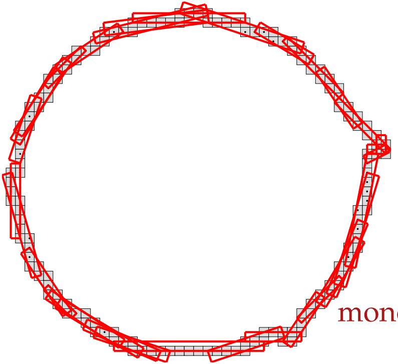

mono-width value

Issues

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- value of ν is manually adjusted
- inadequate in case of noisy curve (noise can be random along the contour)

Adaptive Tangential Cover

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- value of ν is manually adjusted
- inadequate in case of noisy curve (noise can be random along the contour)

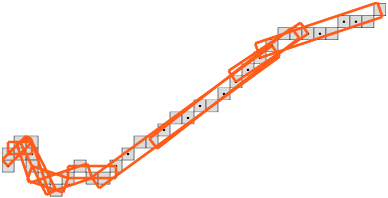
Solution

we introduce a new notion, **Adaptive Tangential Cover (ATC)**, tangential cover with different width values based on the Meaningful Thickness detection.

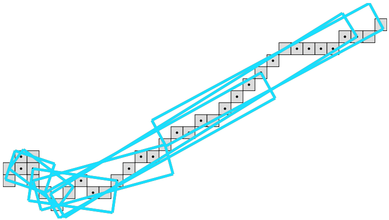
- parameter free
- adequate in case of noisy curves

Definition

- MBS_j is said to be **included** in MBS_i if $B_i \leq B_j$ and $E_i \geq E_j$, and noted by $MBS_j \subseteq MBS_i$
- a maximal blurred segments is said **largest** if for all $MBS_j \in MBS(C)$ with $i \neq j$, $MBS_j \nsubseteq MBS_i$



ν=1



ν=2

Definition

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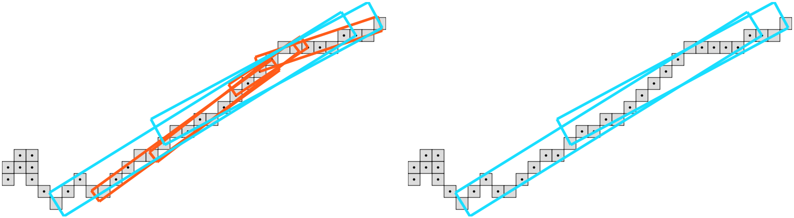
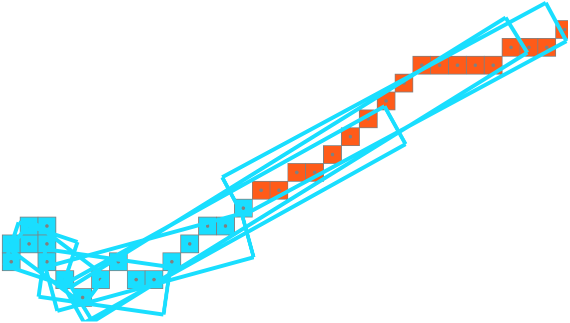


Definition

- $\eta = (\eta_i)_{0 \leq i \leq n-1}$ vector of Meaningful Thickness associated to each C_i of C
- An **Adaptive Tangential Cover associated to Meaningful Thickness (**ATC_{MT}**)** of C is defined as the set of the largest MBS of $\{MBS_j = MBS(B_j, E_j, \nu_k) \in MBS(C) \mid \nu_k = \max\{\eta_t \mid t \in [\![B_j, E_j]\!]\}\}$

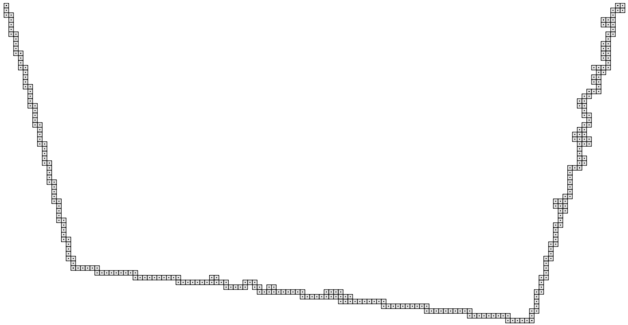


Adaptive Tangential Cover

Strategy

Input:

- $C = (C_i)_{0 \leq i \leq n-1}$ discrete curve of n points



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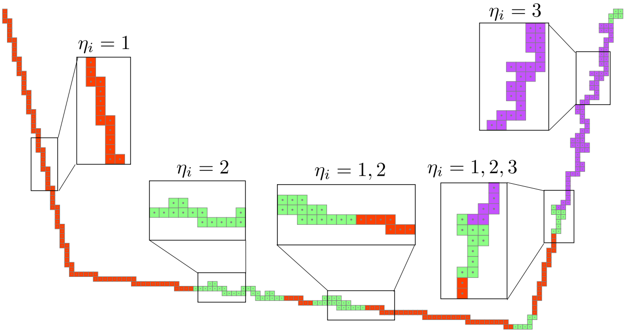

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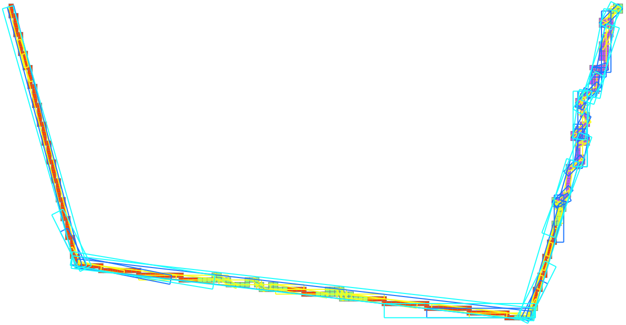
- $C = (C_i)_{0 \leq i \leq n-1}$ discrete curve of n points
- $\eta = (\eta_i)_{0 \leq i \leq n-1}$ vector of Meaningful Thickness associated to each point C_i of C
- $MBS(C) = \{MBS_{\nu_k}(C)\}_{k=0}^{m-1}$ sets of maximal blurred segments of C for each width value $\nu_k \in \nu$ and $\nu = \{\nu_k \mid \nu_k \in \eta\}$



Adaptive Tangential Cover

Strategy

The method for computing ATC_{MT} is divided into two steps :

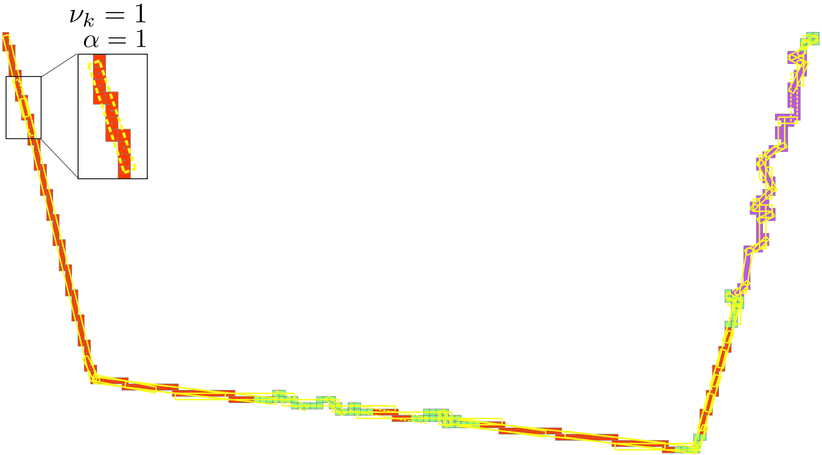
- label the point from the Meaningful Thickness values
- build the ATC_{MT} of the curve from the labels previously obtained

Adaptive Tangential Cover

Step1:

Level 1: $\nu_k = 1$

$\alpha = \max\{\eta_i \mid i \in [\![B_i, E_i]\!]\}$



Adaptive Tangential Cover

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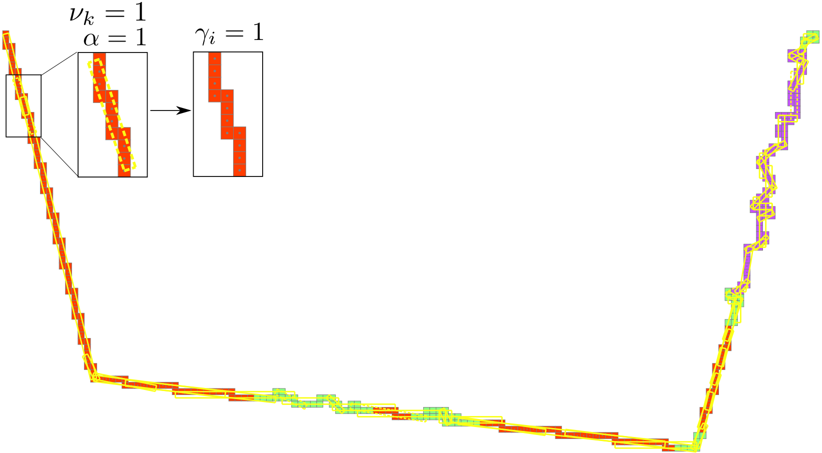
$\gamma = (\gamma)_{0 \leq i \leq n-1}$ vector of labels



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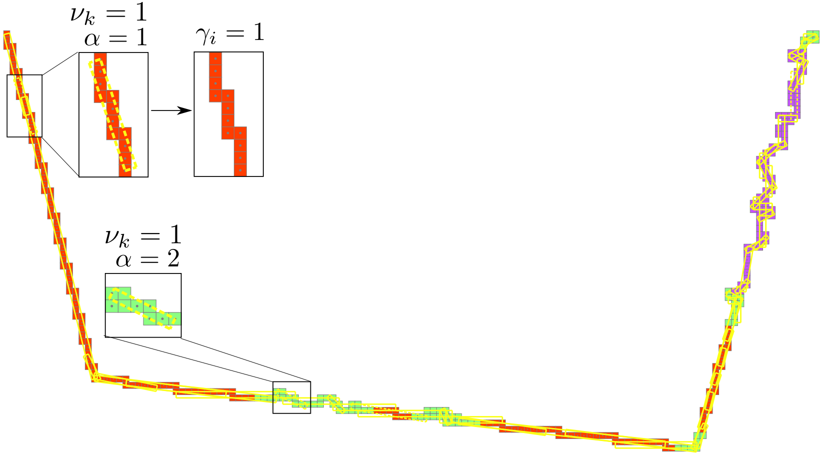

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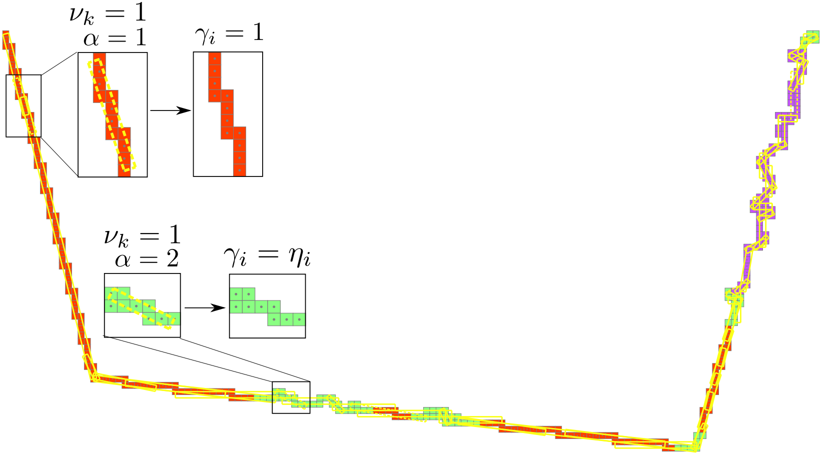

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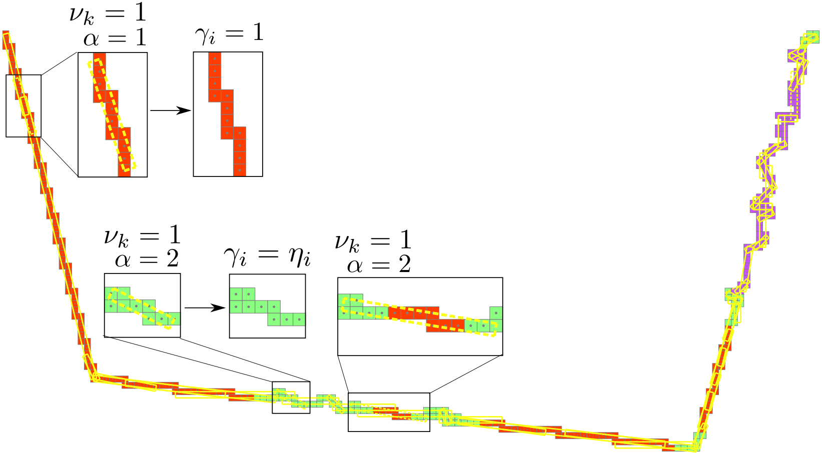

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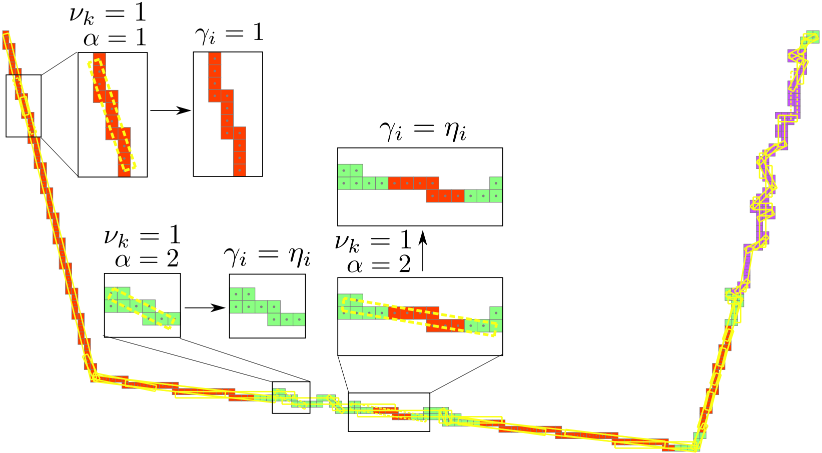

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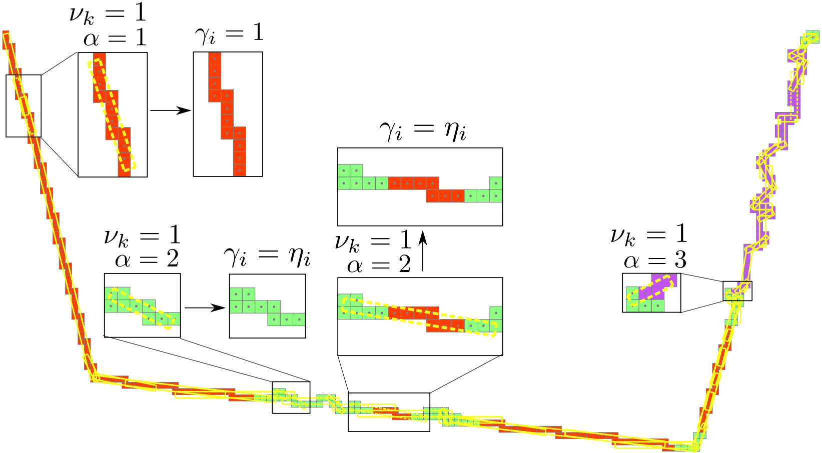

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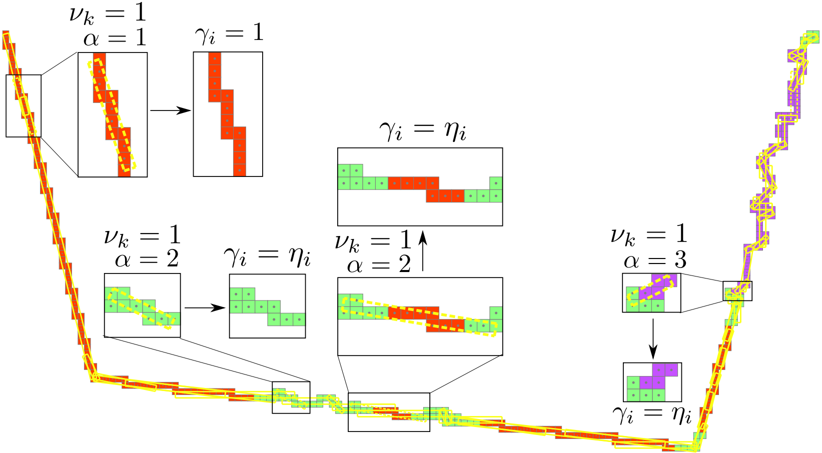

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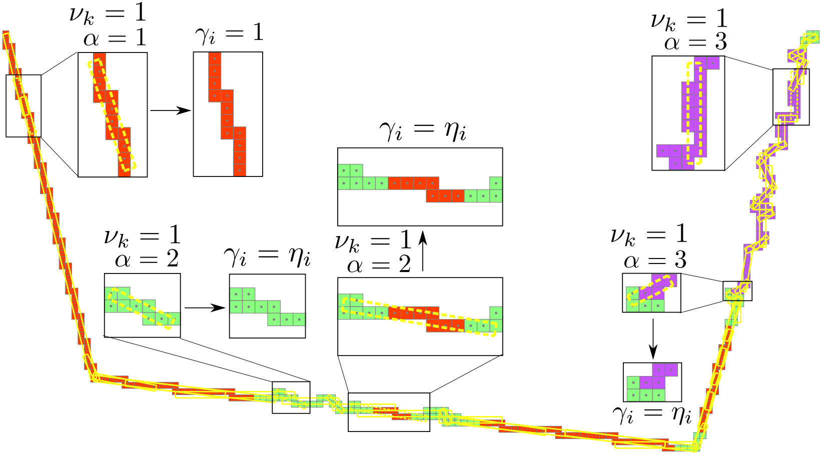

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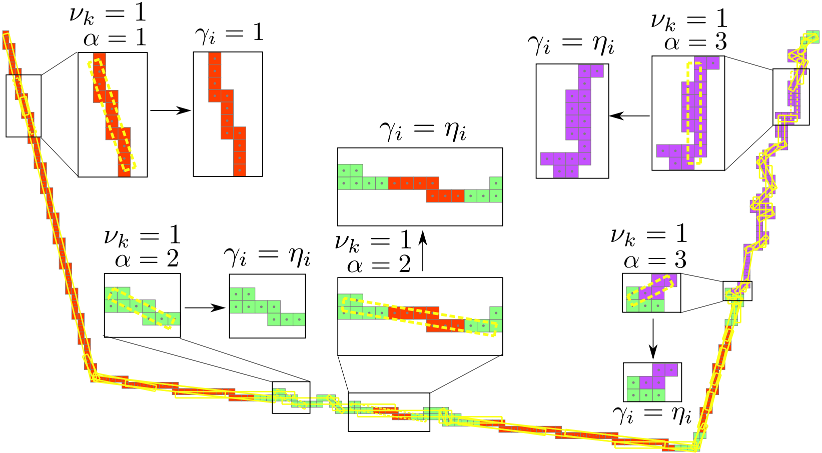

Adaptive Tangential Cover

Step1:

Level 2: $\nu_k = 2$

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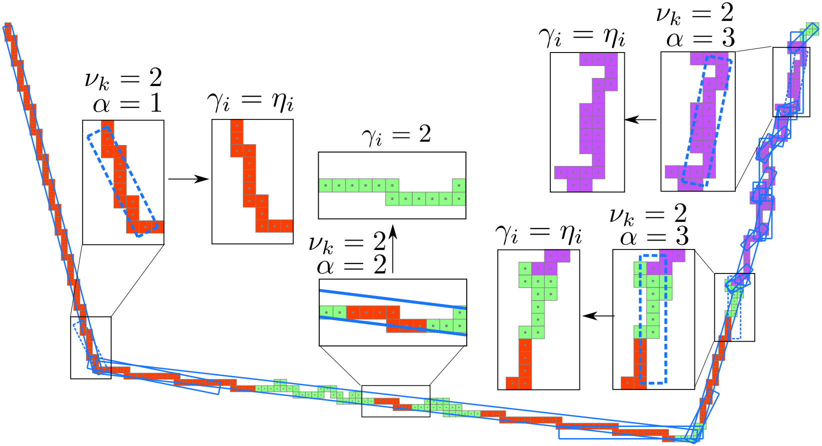

Adaptive Tangential Cover

Step1:

Level 3: $\nu_k = 3$

$\alpha = \max\{\eta_i \mid i \in [\![B_i, E_i]\!]\}$

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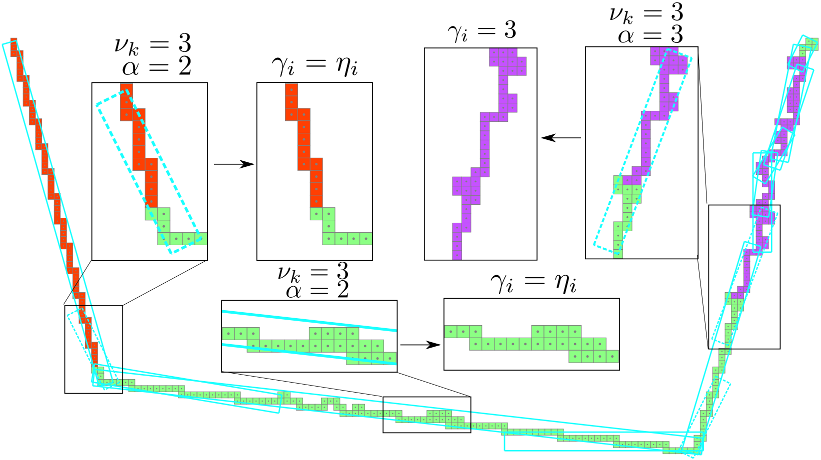

Adaptive Tangential Cover

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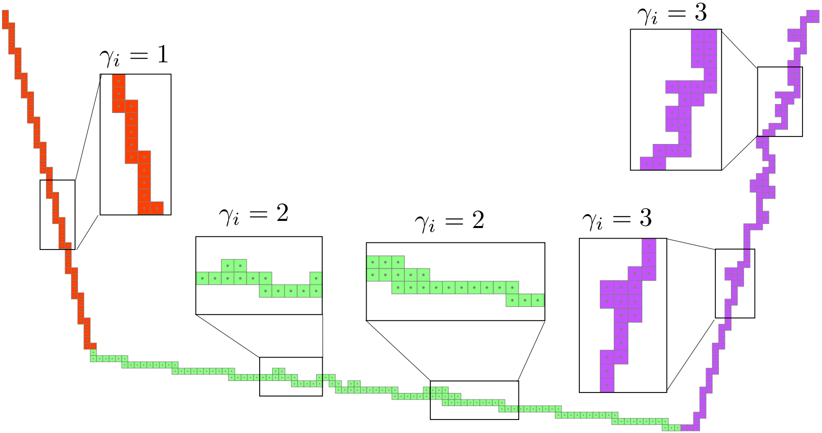
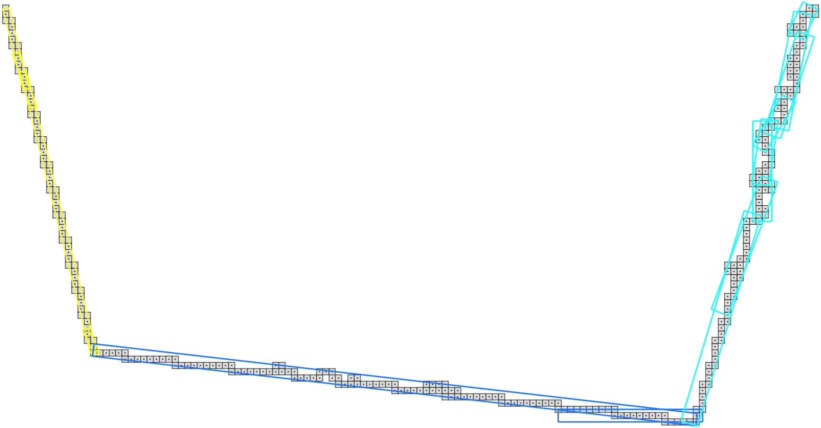


Adaptive Tangential Cover

Step2:

ATC_{MT} is composed of the MBS with widths being the label associated to points constituting the MBS



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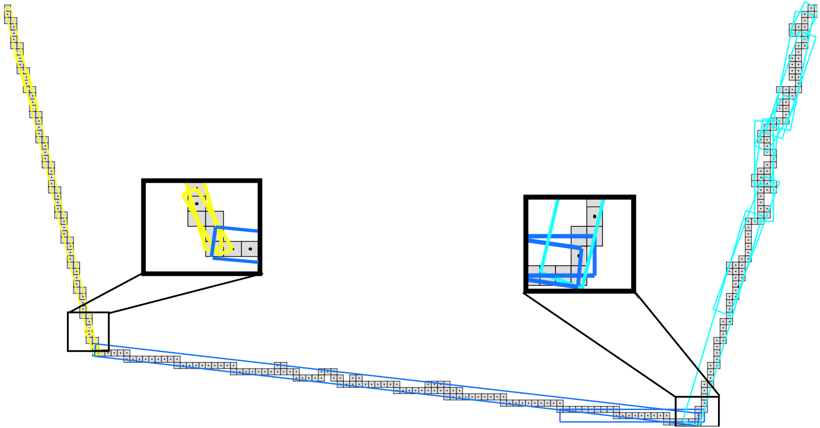
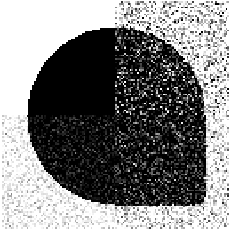
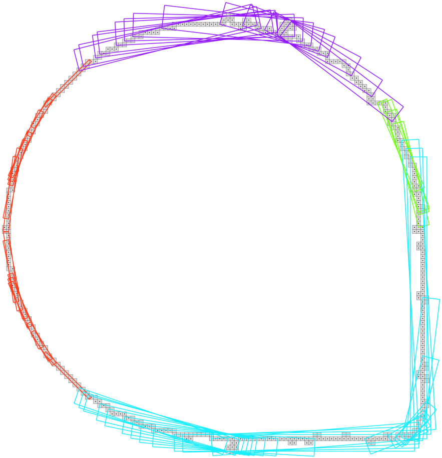


Examples of Adaptive Tangential Cover





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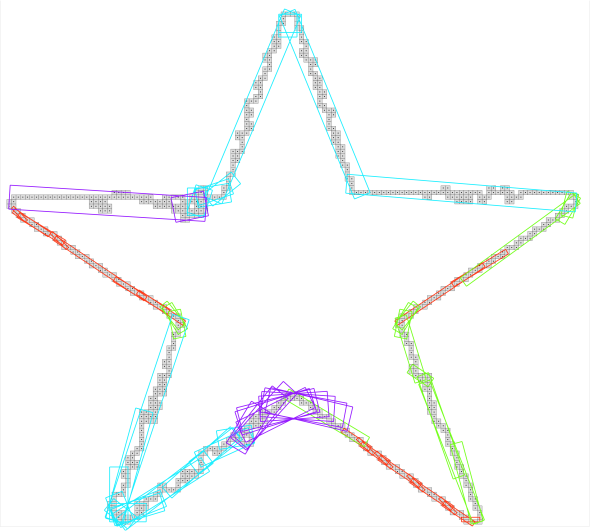
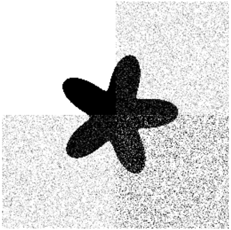
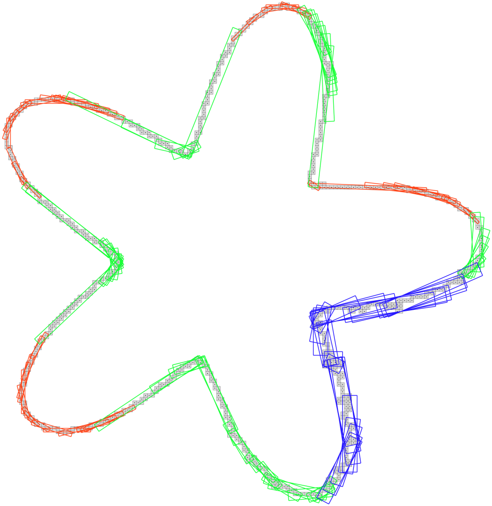


Examples of Adaptive Tangential Cover



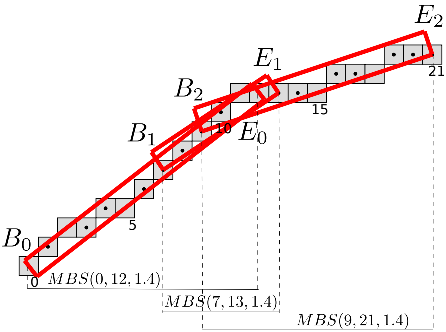


Content

Application to dominant point detection

Definition

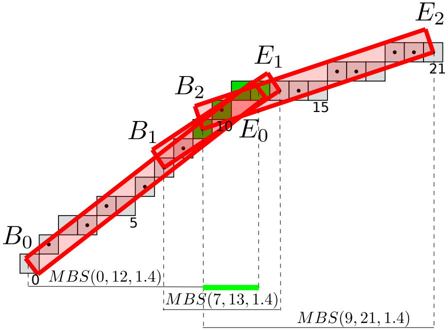
A **dominant point** (corner point) on a curve is a point of local maximum curvature.



Application to dominant point detection

Proposition

Dominant points of the curve are located in the **common zones** of successive maximal blurred segments[4].

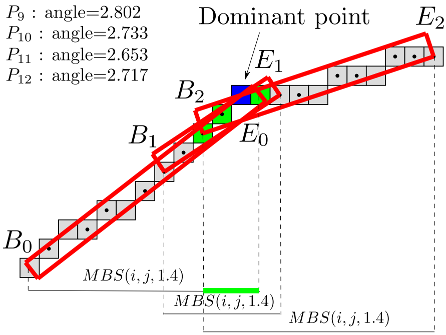


[4] Nguyen, T.P., Debled-Rennesson, I.: A discrete geometry approach for dominant point detection. Pattern Recognition 44(1), 32-44(2011)

Application to dominant point detection

Strategy

Dominant point is detected as the point in the common zone with **minimum angle measure**[5].



[5] Ngo, P., Nasser, H., Debled-Rennesson, I.: Efficient dominant point detection based on discrete curve structure. In: International Workshop on Combinatorial Image Analysis (IWCIA), Kolkata, India, November. LNCS, vol. 9448 (2015)

- Dominant point detection algorithm was based on the mono value tangential cover
- Now, we use it with the Adaptive Tangential Cover

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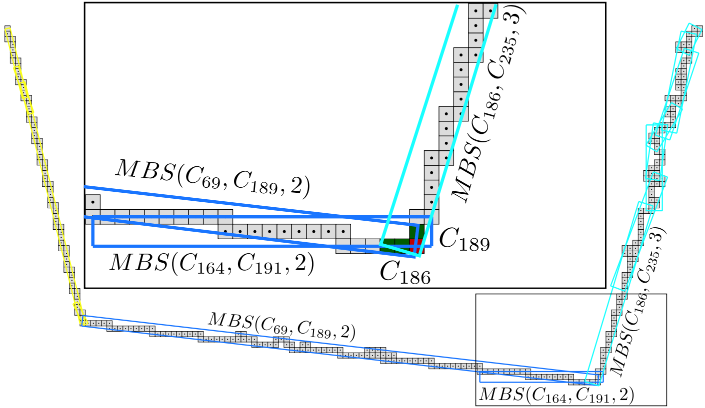
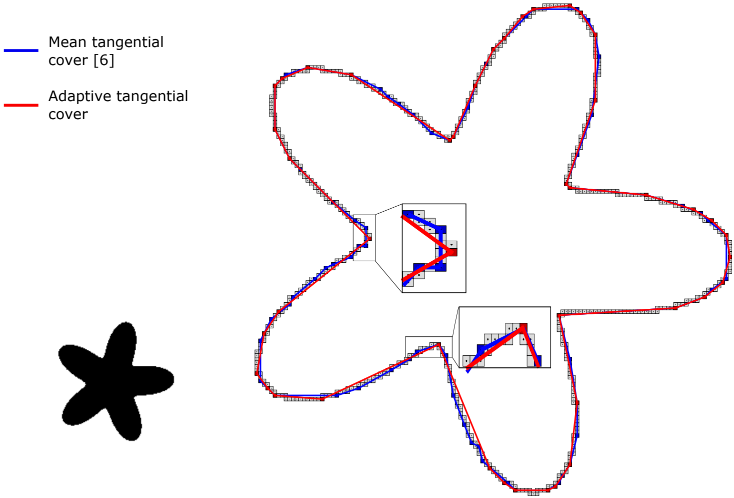


Comparison

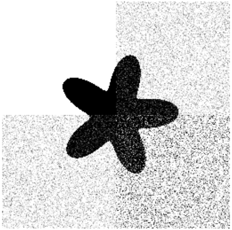


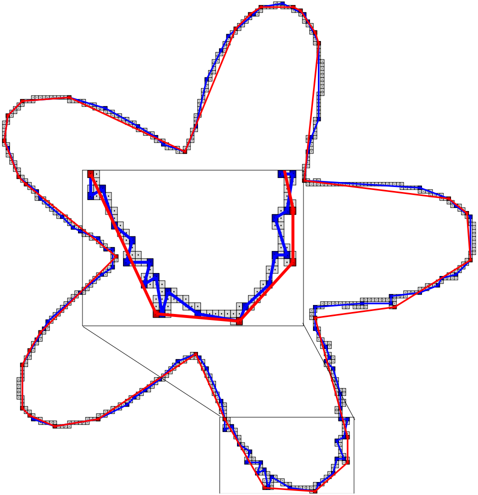
[6] Nguyen, T.P., Kerautret, B., Debled-Rennesson, I., Lachaud, J.: Unsupervised, fast and precise recognition of digital arcs in noisy images. In: Computer Vision and Graphics - International Conference, ICCVG 2010, Warsaw, Poland, September. LNCS, vol. 6374 (2010)

Comparison

Mean tangential cover [6]

Adaptive tangential cover





[6] Nguyen, T.P., Kerautret, B., Debled-Rennesson, I., Lachaud, J.: Unsupervised, fast and precise recognition of digital arcs in noisy images. In: Computer Vision and Graphics - International Conference, ICCVG 2010, Warsaw, Poland, September. LNCS, vol. 6374 (2010)

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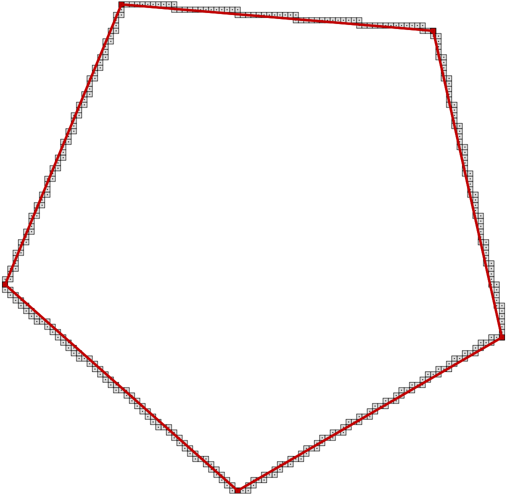

[6] Nguyen, T.P., Kerautret, B., Debled-Rennesson, I., Lachaud, J.: Unsupervised, fast and precise recognition of digital arcs in noisy images. In: Computer Vision and Graphics - International Conference, ICCVG 2010, Warsaw, Poland, September. LNCS, vol. 6374 (2010)

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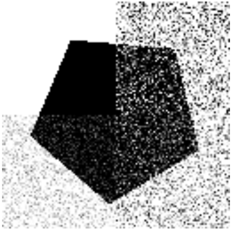
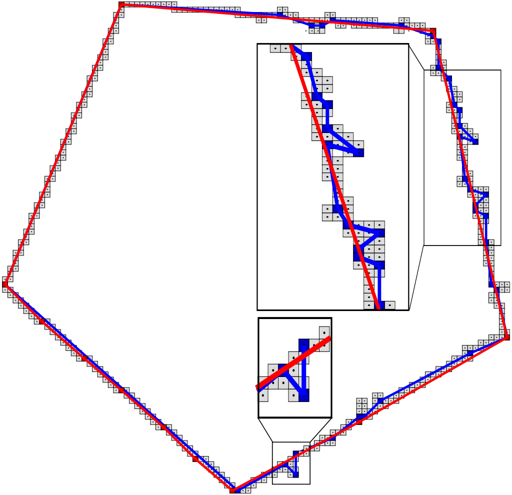
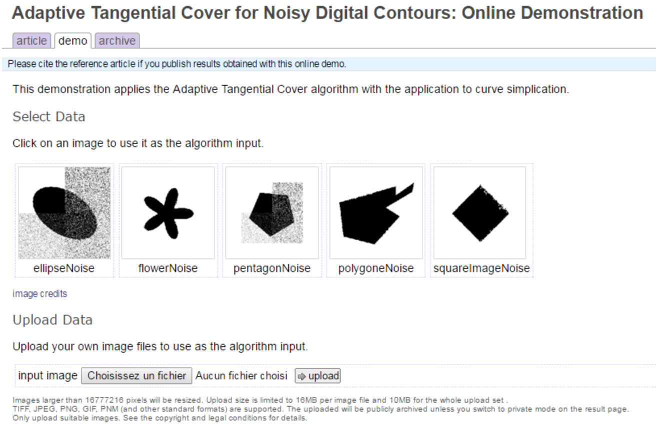




[6] Nguyen, T.P., Kerautret, B., Debled-Rennesson, I., Lachaud, J.: Unsupervised, fast and precise recognition of digital arcs in noisy images. In: Computer Vision and Graphics - International Conference, ICCVG 2010, Warsaw, Poland, September. LNCS, vol. 6374 (2010)

Online Demonstration

An online demonstration based on the DGtal and ImaGene library, is available at the following website: `http://ipol-geometry.loria.fr/~kerautre/ipol_demo/ATC_IPOLDemo/`



Content

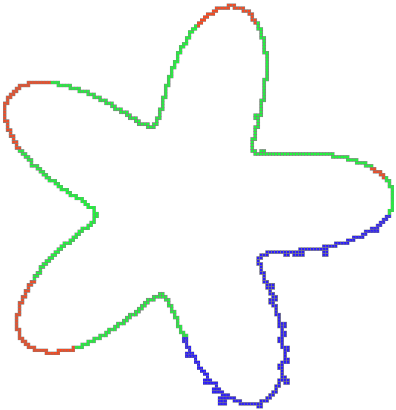
Contributions

- Adaptive Tangential Cover deduced from the Meaningful Thickness
- Method to compute the ATC is parameter free
- ATC is used in a dominant point detection algorithm
  - very good results on the polygonal shapes
  - the algorithm simplifies the shapes in a polygonal way for the shapes with convex and/or concave parts

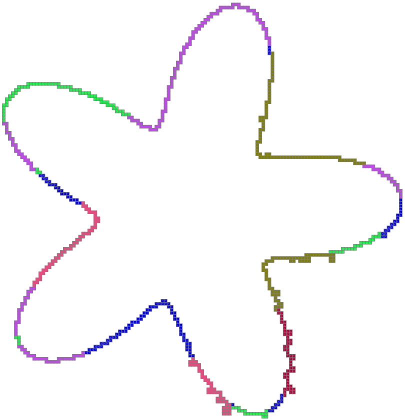
Perspectives

- Generalization of ATC using other width estimations
- Use of ATC in geometric estimators:
  - curvature
  - decomposition of a curve into arcs and segments
- Modify the method to compute the Meaningful Thicknesses (Input)
  - take a step less than 1 (for example 0.2)
  - obtain best results for polygonization

Perspectives



Input value of Meaningful Thicknesses with step 1



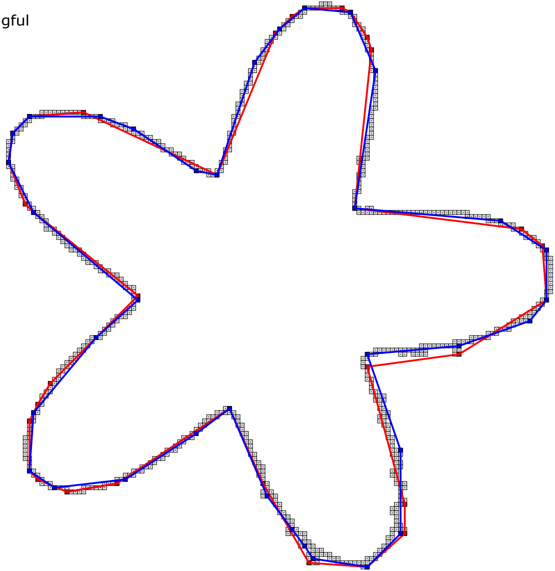
Input value of Meaningful Thicknesses with step 0.2

Perspectives

to compute the Meaningful thicknesses (Input)

Step 0.2

Step 1

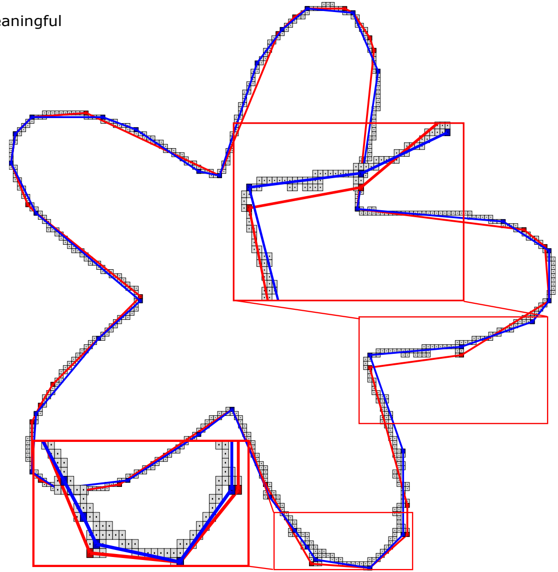


Perspectives

to compute the Meaningful thicknesses (Input)

Step 0.2

Step 1



Thank you for your attention!

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- B. Kerautret, J.-O. Lachaud, and M. Said, "Meaningful Thickness Detection on Polygonal Curve," in *Proceedings of the 1st International Conference on Pattern Recognition Applications and Methods*, pp. 372–379, SciTePress, 2012.
- T. P. Nguyen and I. Debled-Rennesson, "A discrete geometry approach for dominant point detection," *Pattern Recognition*, vol. 44, no. 1, pp. 32–44, 2011.