Efficient dominant point detection based on discrete curve structure

Phuc Ngo
Hayat Nasser
Isabelle Debled-Rennesson

Université de Lorraine
LORIA, UMR CNRS 7503, France

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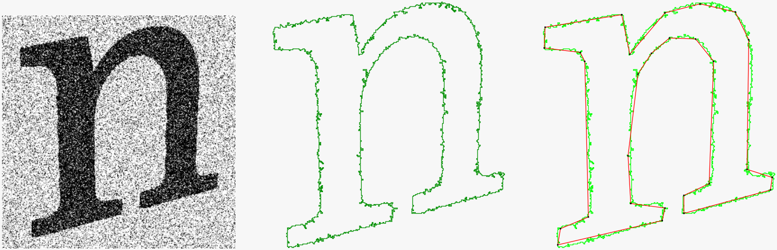
UNIVERSITÉ DE LORRAINE

Loria
Laboratoire lorrain de recherche en informatique et ses applications

Motivation

- Shape analysis
- Pattern recognition
- Polygonal approximation



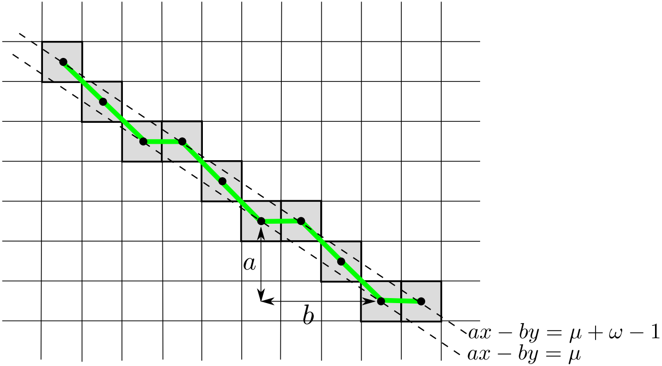
Nguyen and Debled-Rennesson, PR - 2010

Contents

Discrete line and segment

Definition

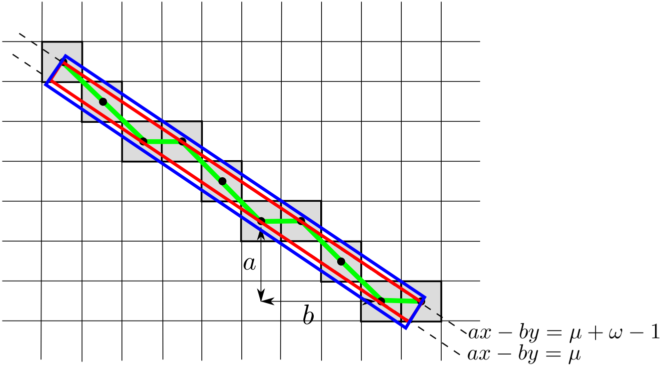
A **discrete line** $\mathcal{D}(a, b, \mu, \omega)$ is the set of integer points (x, y) verifying $\mu \leq ax - by < \mu + \omega$ where a, b, μ, $\omega \in \mathbb{Z}$ and $gcd(a, b) = 1$.



Discrete line and segment

Definition

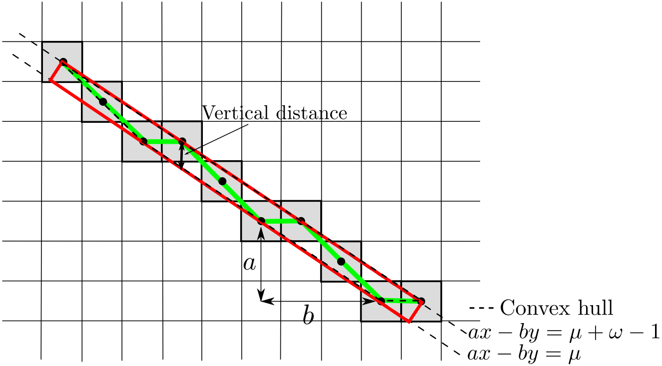
A **bounding discrete segment** of a set $\mathcal{S}_f$ of integer points is the discrete line $\mathcal{D}(a, b, \mu, \omega)$ containing all points of $\mathcal{S}_f$.



Discrete line and segment

Definition

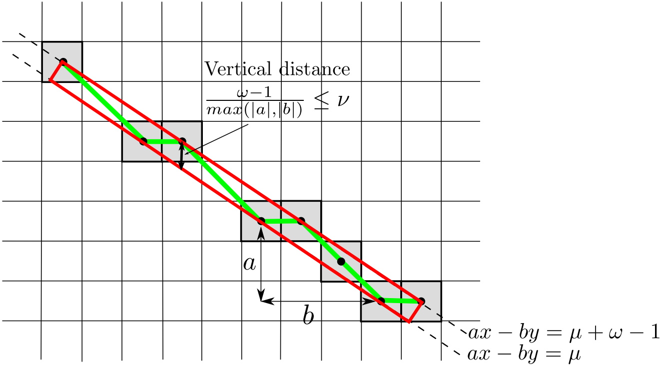
A bounding discrete segment $\mathcal{D}(a, b, \mu, \omega)$ of $\mathcal{S}_f$ is **optimal** if its vertical (or horizontal) distance $\frac{\omega-1}{max(|a|,|b|)}$ is equal to the vertical (or horizontal) thickness of the convex hull of $\mathcal{S}_f$.



Blurred segment

Definition

A sequence integer points $\mathcal{S}_f$ is a **blurred segment of width** ν if its optimal bounding discrete segment $\mathcal{D}(a, b, \mu, \omega)$ has the vertical or horizontal distance less than or equal to ν.



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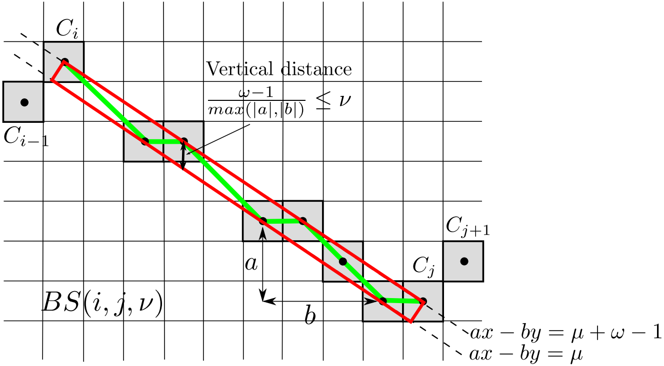
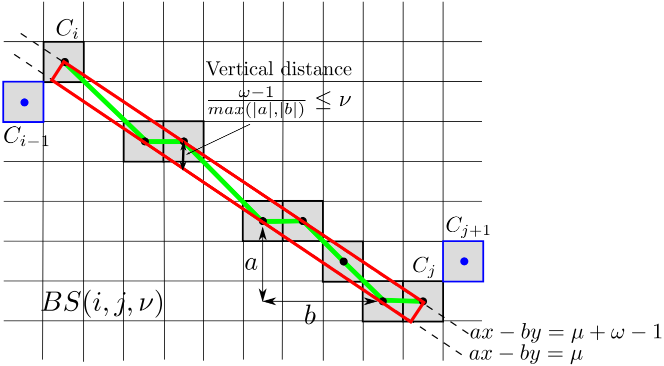


Blurred segment

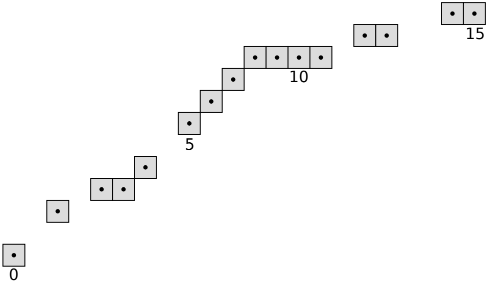
Definition

A blurred segment ν $BS(i,j,\nu)$ is **maximal**, and noted $MBS(i,j,\nu)$, iff $\neg BS(i,j+1,\nu)$ and $\neg BS(i-1,j,\nu)$.



Maximal blurred segment decomposition

- Input: A discrete curve C and a width ν
- Output: The decomposition $MBS_\nu(C)$ of C
- Algorithm: Computation the sequence of maximal blurred segments by incrementally adding (resp. removing) a pixel to (resp. from) the considering blurred segment



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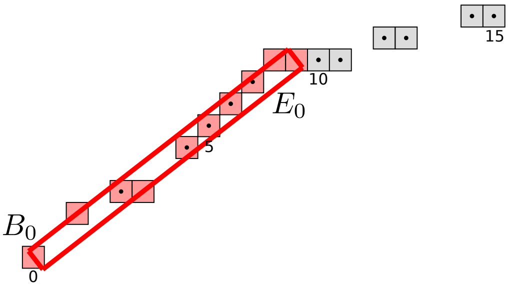

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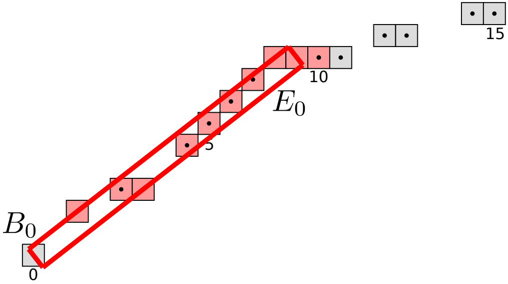

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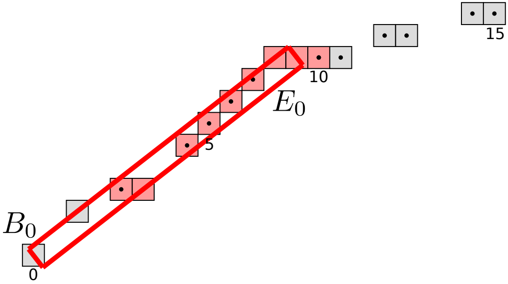

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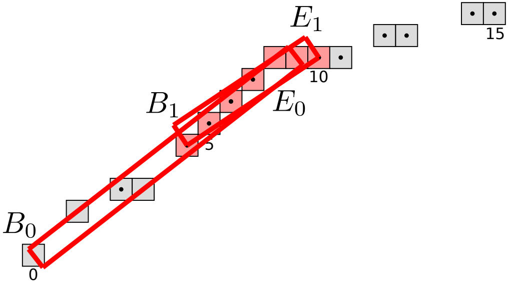

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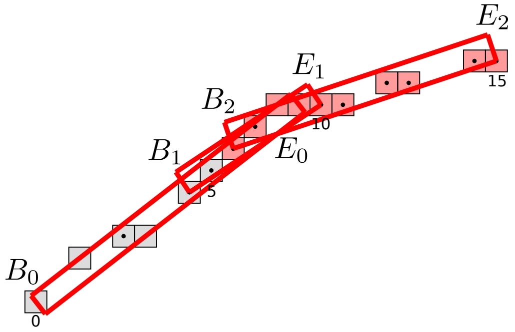

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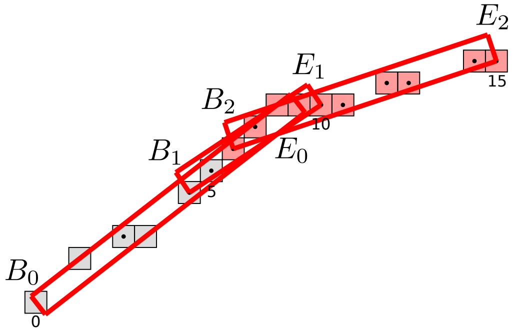
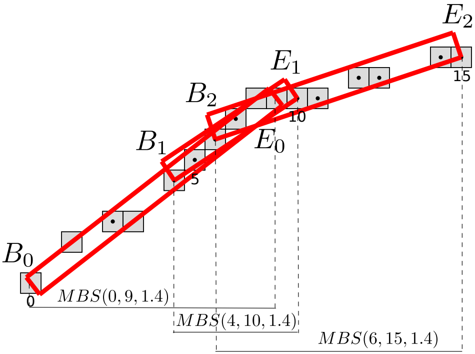


$\Longrightarrow$ Algorithm of decomposition is in quasi-linear time [1]

Maximal blurred segment decomposition

Property

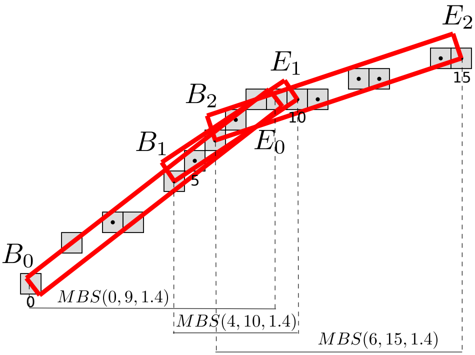
Let $MBS_\nu(C)$ be the maximal blurred segment decomposition of witdth ν of C:
$MBS_\nu(C) = \{MBS(B_0, E_0, \nu), \ldots, MBS(B_{m-1}, E_{m-1}, \nu)\}$.
Then, $B_0 < B_1 < \ldots < B_{m-1}$ and $E_0 < E_1 < \ldots < E_{m-1}$.



Dominant point

Definition

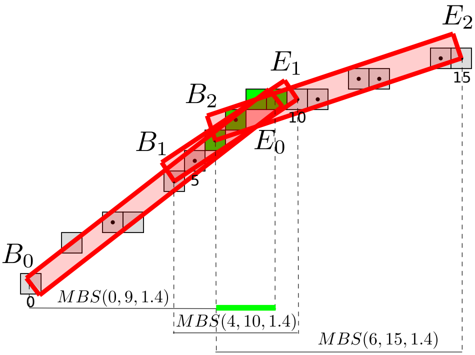
A **dominant point** (corner point) on a curve is a point of local maximum curvature.



Dominant point detection [2]

Proposition

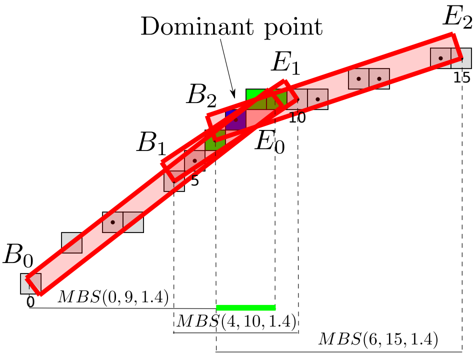
Dominant points of the curve is located in the **common zones** of successive maximal blurred segments.



Dominant point detection [2]

Heuristic strategy

Dominant point is detected as the **middle point** of each common zone of successive maximal blurred segments.



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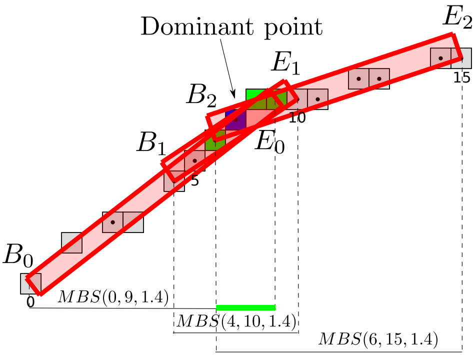
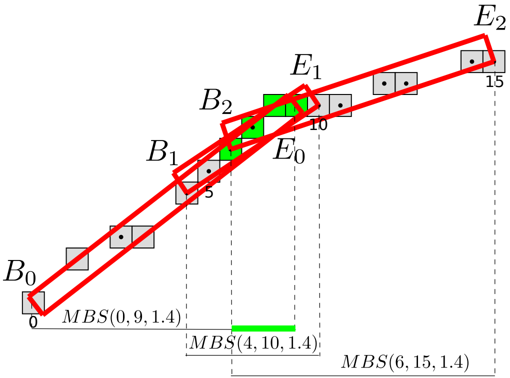


⟹ This heuristic is effective, but not always optimal!

Modified algorithm of dominant point detection

Pseudo curvature

A **measure of angle** is estimated for points in the common zone. Such angle is defined by the considered point and the two left and right extremities of the left and right maximal blurred segments composing the common zone.



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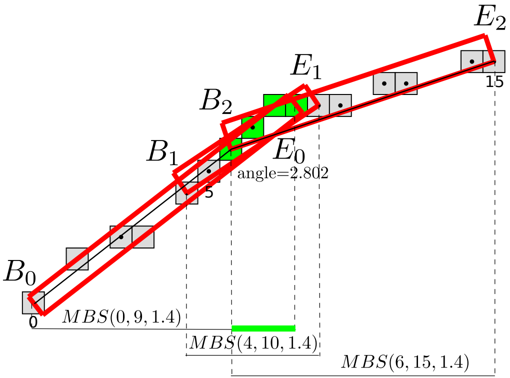

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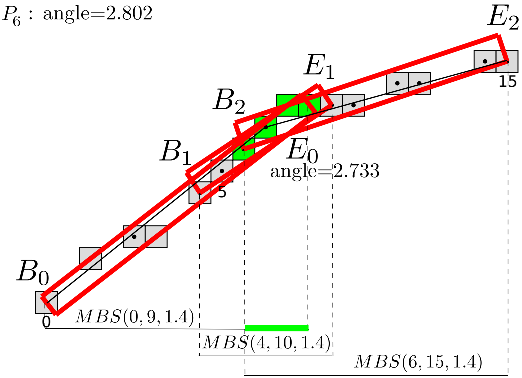

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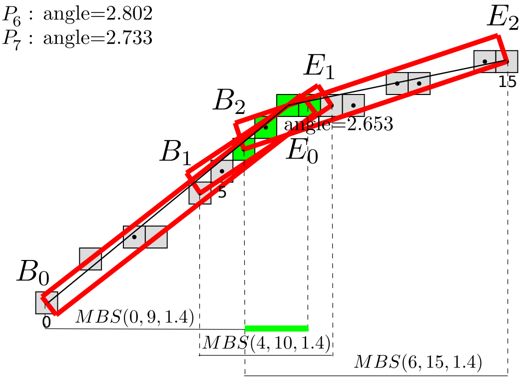

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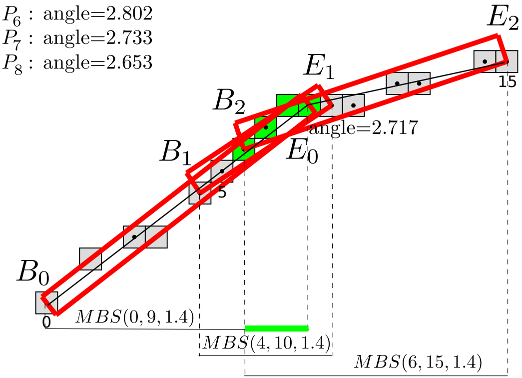
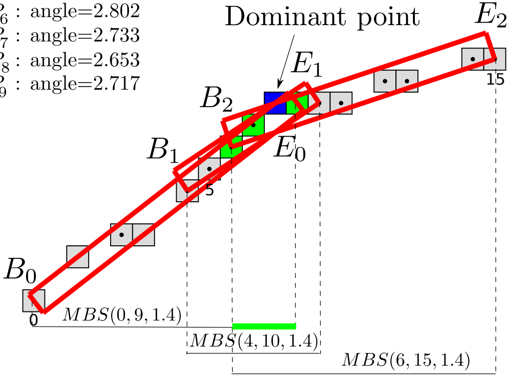


Modified algorithm of dominant point detection

New strategy

Dominant point is detected as the point in the common zone with **minimum angle measure**.

P_6 : angle=2.802
P_7 : angle=2.733
P_8 : angle=2.653
P_9 : angle=2.717



Modified algorithm of dominant point detection

Input: C discrete curve of n points and a width ν
Output: D set of dominant points

Begin
Build $MBS_\nu = \{MBS(B_i, E_i, \nu)\}_{i=0}^{m-1}$
$n = |C|; m = |MBS_\nu|$
$q = 0; p = 1; D = \emptyset$
while $p < m$ **do**
  while $E_q > B_p$ **do**
    $p++$
  end while
  $D = D \cup \min\{Angle(C_{B_q}, C_i, C_{E_{p-1}}) \mid i \in [\![B_{p-1}, E_q]\!]\}$
  $q = p - 1$
end while
End

Evaluation criteria of experimental results

1. Number of dominant points (nDP)
2. Compression ratio (CR)

$$CR = \frac{n}{nDP}$$

3. Integral sum of square errors (ISSE)

$$ISSE = \sum_{i=1}^{n} d_i^2$$

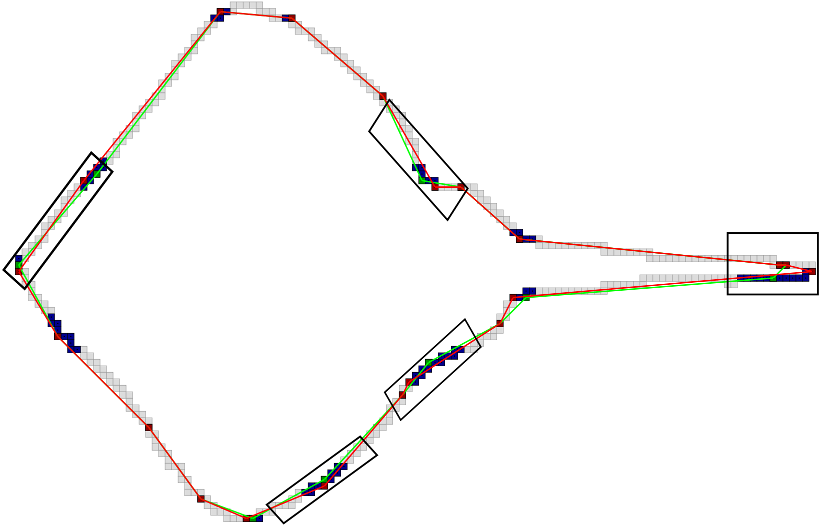
4. Maximum error (L_∞)

$$L_\infty = max\{d_i\}_{i=1}^{n}$$

5. Figure of merit (FOM)

$$FOM = \frac{CR}{ISSE}$$

Comparing results to Nguyen's method [2]



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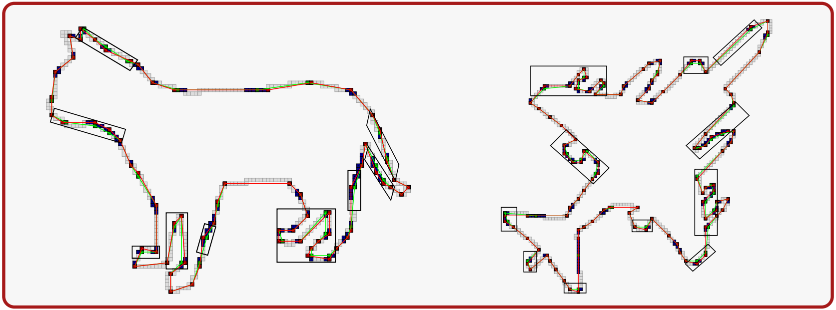

Curve	Method	nDP	CR	ISSE	L_∞	FOM
(Left) $n = 404$	Nguyen	20	20.2	236.806	3.536	0.0853
	Ours	20	20.2	**150.314**	**1.539**	**0.1344**
(Right) $n = 252$	Nguyen	43	5.86	68.896	1	0.0851
	Ours	43	5.86	**57.582**	1	**0.1018**

Polygonal simplification

Issue

Overmuch of dominant points detected due to the nature of the maximal blurred segment sequence defined on a discrete curve.

Solution

Elimination of dominant points which are less **important** with respect to the approximating polygon of the discrete curve using the ratio of the two folowing criteria :

- ISSE
- angle criterion

Polygonal simplification

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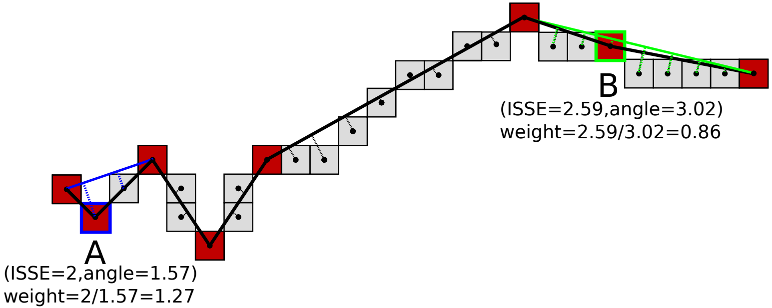
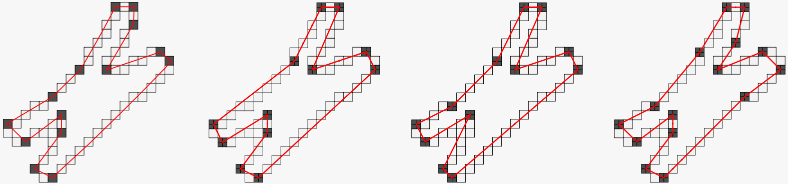


Algorithm of polygonal simplification

Input: D set of dominant points and n number of points on the simplifying polygon
Output: New set of n dominant points

Begin
for $i = 1 \to |D| - 1$ **do**
  $w(p_i) = ISSE(p_{i-1}, p_{i+1})/Angle(p_{i-1}, p_i, p_{i+1})$
end for
while $|D| > n$ **do**
  $p_j = \min\{w(p_i) \mid p_i \in D\}$
  $w(p_{j-1}) = ISSE(p_{j-2}, p_{j+1})/Angle(p_{j-2}, p_{j-1}, p_{j+1})$
  $w(p_{j+1}) = ISSE(p_{j-1}, p_{j+2}/Angle(p_{j-1}, p_{j+1}, p_{j+2})$
  $D = D \setminus \{p_j\}$
end while
End

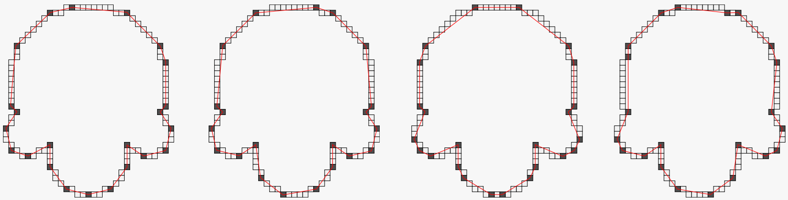
Comparing results to other methods



Ours, ν=0.7, simplification 14/19 (16%) | Masood [3] | Marji [4] | Teh [5]

Curve	Method	nDP	CR	ISSE	L_∞	FOM
Chro-mosome n=60	Ours	14	4.286	**5.116**	0.8	**0.838**
	Masood	12	5	7.76	0.88	0.65
	Marji	12	5	8.03	0.895	0.623
	Teh	15	4	7.2	**0.74**	0.556

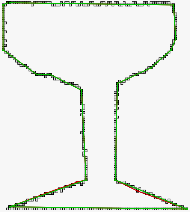
Comparing results to other methods



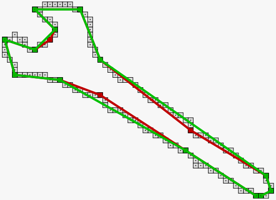
Ours, ν=0.7, simplification 23/26 (11%) | Masood [3] | Marji [4] | Teh [5]

Curve	Method	nDP	CR	ISSE	L_∞	FOM
Semicircle n=102	Ours	23	4.435	**7.639**	0.724	**0.581**
	Masood	22	4.64	8.61	**0.72**	0.54
	Marji	26	3.92	9.01	0.74	0.435
	Teh	22	4.64	20.61	1	0.225

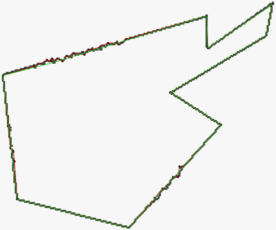
Experimental results



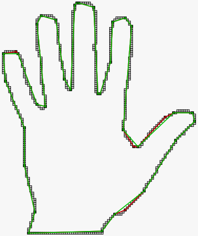
ν=1.5, nDP=16 (15%)



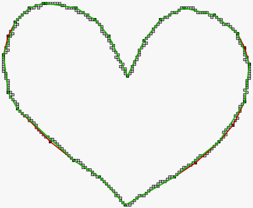
ν=1.5, nDP=13 (15%)



ν=1.5, nDP=12 (70%)



ν=1.5, nDP=33 (10%)



ν=1.5, nDP=23 (15%)

ν=1.5, nDP=27 (25%)

Conclusion

Contributions

- Algorithm based on discrete structure of the curve
- Algorithm without heuristic but a simple measure of angle

Perspectives

- Parameter free method
- Adaptive-thickness

Thank your for your attention!

References

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