# Generalized Multi-Contact Planning

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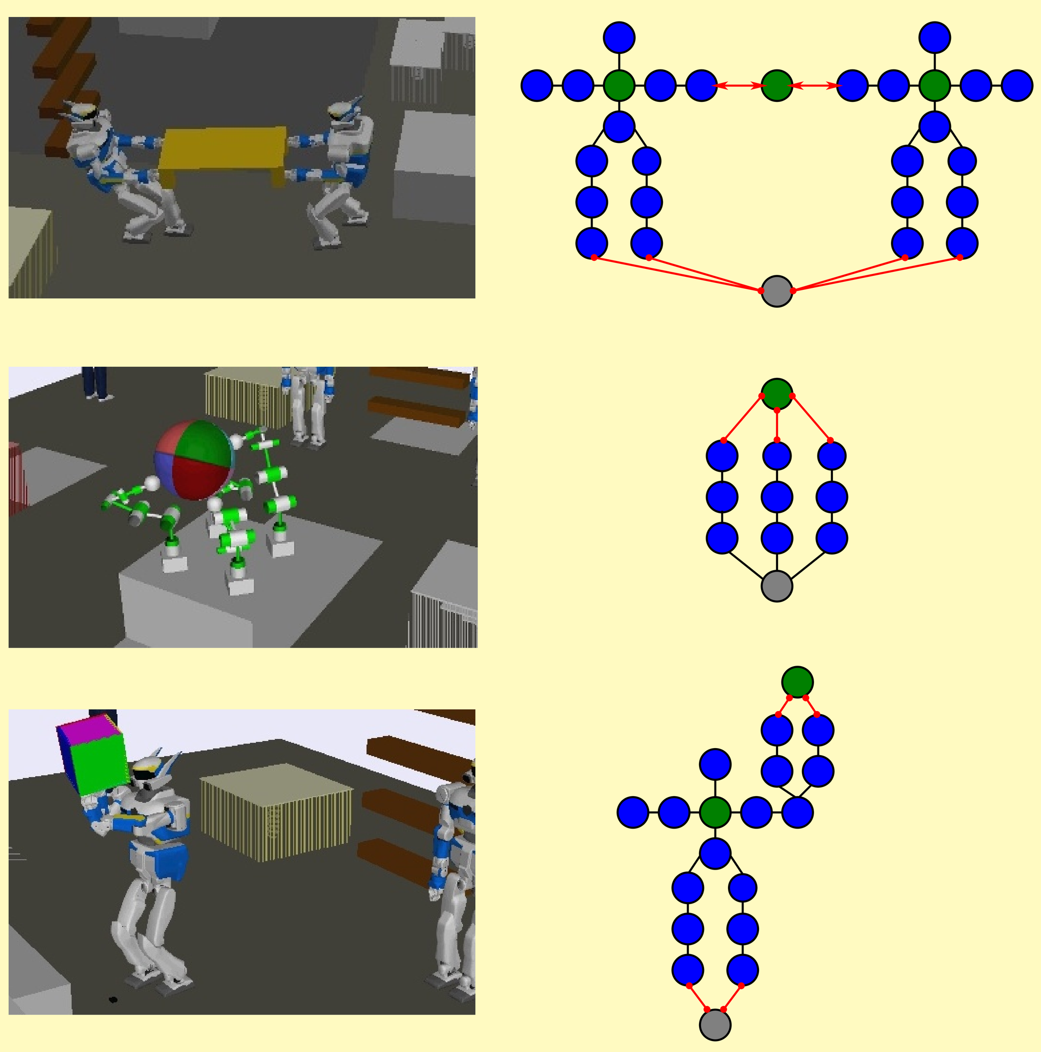
## Contribution

We propose a generalized framework that allows us to easily extend existing contact-before-motion algorithms from single-agent to mutli-agent systems. The broad range of applications of our generic implementation includes legged locomotion planning, whole-body manipulation planning, dexterous manipulation planning, as well as any multi-contact-based motion planning problem that might combine several of these sub-problems. The versatility of our planner is demonstrated through example scenarios representative of these classes of problems.

## Very general systems

We take a centralized multi-agent planning approach. A system can be made of an arbitrary number of interacting entities. A humanoid robot, a dexterous hand, a fixed-base manipulator, a manipulated object, and the environment itself, are examples of such entities. Each entity $r \in \{1, \ldots, N\}$ yields a configuration space $\mathscr{C}_r$. The configuration space of the system is the Cartesian product of all respective configuration spaces of the entities that constitute the system

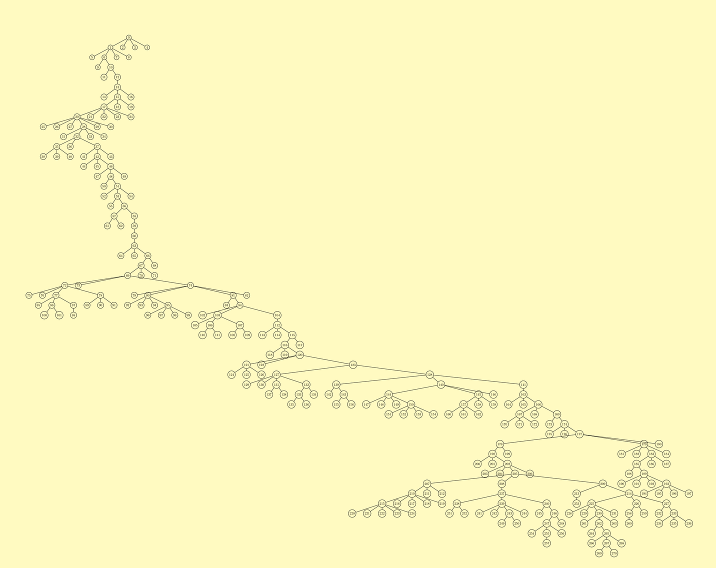
$$\mathscr{C} = \prod_{r=1}^{N} \mathscr{C}_r\,.$$



- Articulated link (arms, legs, fingers, etc. )
- Free-flying link (manipulated objects, root of humanoids, etc.)
- Fixed link (environment, palm of dexterous hands, etc.)
- Articulated joint (revolute, prismatic, spherical, etc.)
- Unilateral contact (Coulomb friction cone)
- Bilateral contact (for humanoid grasp with gripper)

## Search tree

Example for locomotion using hands and feet



## Definitions and notations

Contacts are possible between any two plane surfaces from two different nodes of the generalized kinematic tree representing the whole system. A contact is a 7-tuple

$$c = (r_1, s_1, \quad r_2, s_2, \quad x, y, \theta) \in \mathbb{N}^4 \times SE(2)$$

| | |
|---|---|
| $r_1$ | first entity (eg. humanoid robot) |
| $s_1$ | surface on $r_1$ (eg. right foot sole) |
| $r_2$ | second entity (eg. environment) |
| $s_2$ | surface on $r_2$ (eg. floor) |
| $(x, y, \theta)$ | relative position of the surfaces |

We define the following objects

| | |
|---|---|
| set of contacts | $E_{\text{ct}} \subset \mathbb{N}^4 \times SE(2)$ |
| stance | $\sigma = \{c_1, \ldots, c_m\} \subset E_{\text{ct}}$ |
| stances space | $\Sigma \subset 2^{E_{\text{ct}}}$ |
| forward kinematics | $p_\Sigma : \mathscr{C} \to \Sigma$ |
| inverse kinematics | $\mathscr{Q}_\sigma = p_\Sigma^{-1}(\{\sigma\})$ |
| feasible spaces | $\mathscr{F}_\sigma \subset \mathscr{Q}_\sigma$ |

We define binary relations on $\Sigma$

$$\sigma' \in \text{Adj}^-(\sigma) \quad \Leftrightarrow \quad \sigma' \subset \sigma \text{ and } |\sigma'| = |\sigma| - 1$$
$$\sigma' \in \text{Adj}^+(\sigma) \quad \Leftrightarrow \quad \sigma' \supset \sigma \text{ and } |\sigma'| = |\sigma| + 1$$
$$\sigma' \in \text{Adj}(\sigma) \quad \Leftrightarrow \quad \sigma' \in \text{Adj}^-(\sigma) \cup \text{Adj}^+(\sigma)$$

We define an additional structure on $\text{Adj}^+(\sigma)$

| | |
|---|---|
| projection map | $p_{\mathbb{N}^4} : \mathbb{N}^4 \times SE(2) \to \mathbb{N}^4$ |
| equivalence rel. | $\sim_\sigma$ on $\text{Adj}^+(\sigma)$ |
| $\sigma_1 \sim_\sigma \sigma_2 \quad \Leftrightarrow$ | $p_{\mathbb{N}^4}(\sigma_1 \setminus \sigma) = p_{\mathbb{N}^4}(\sigma_2 \setminus \sigma)$ |
| equivalence class | $[\sigma']_{\sim_\sigma}$ (short. $[\sigma']$) |
| quotient space | $\text{Adj}^+(\sigma)_{/\sim_\sigma}$ |

## Problem statement

Given $q_{\text{init}} \in \mathscr{F}_{\sigma_{\text{init}}}$ and $q_{\text{goal}} \in \mathscr{F}_{\sigma_{\text{goal}}}$, find a sequence of stances $(\sigma_1, \ldots, \sigma_n)$ such that

$$\begin{cases} \sigma_1 = \sigma_{\text{init}}, \sigma_n = \sigma_{\text{goal}} & \\ \forall\, i \in \{1, \ldots, n-1\} & \sigma_{i+1} \in \text{Adj}(\sigma_i) \\ \forall\, i \in \{1, \ldots, n-1\} & \mathscr{F}_{\sigma_i} \cap \mathscr{F}_{\sigma_{i+1}} \neq \varnothing \end{cases}$$

## Best-First Algorithm

initialize priority queue $Q \leftarrow \{\sigma_{\text{init}}\}$
**repeat**
  pop best stance $\sigma$ from $Q$;
  **for** $[\sigma']_{\sim_\sigma} \in \text{Adj}^+(\sigma)_{/\sim_\sigma}$ **do**
    call the iss to find $q$ in $\mathscr{F}_\sigma \cap \mathscr{F}_{[\sigma']}$;
    **if** *found* $q$ **then**
      push $\sigma' = p_\Sigma(q)$ into $Q$;
  **for** $\sigma' \in \text{Adj}^-(\sigma)$ **do**
    call the iss to find $q$ in $\mathscr{F}_\sigma \cap \mathscr{F}_{\sigma'}$;
    **if** *found* $q$ **then**
      push $\sigma'$ into $Q$;
**until** $\sigma$ *is close enough to* $\sigma_{\text{goal}}$;

## Inverse stance solver

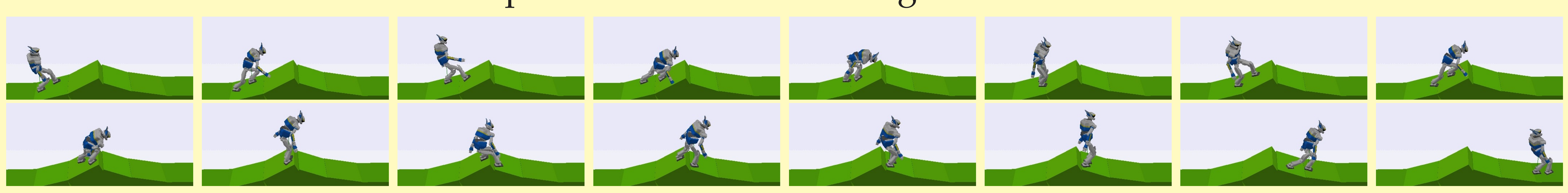
$$\min_{\lambda, q \in \mathscr{F}_\sigma} \text{obj}(q, \lambda)$$

$$\min_{(x,y,\theta), \lambda, q \in \mathscr{F}_{[\sigma]}} \text{obj}(q, \lambda, x, y, \theta)$$

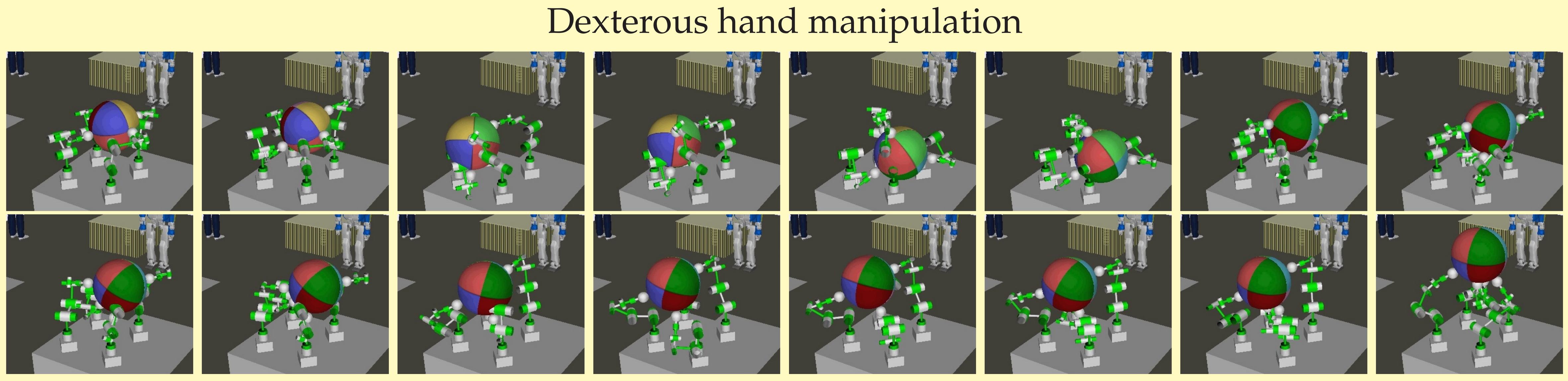
An inverse kinematics solver under static equilibrium constraints. Solves an optimization problem for the configuration variables $q$ and the contact forces variables $\lambda$ simultaneously. Also decides the best added contact location.

## Results

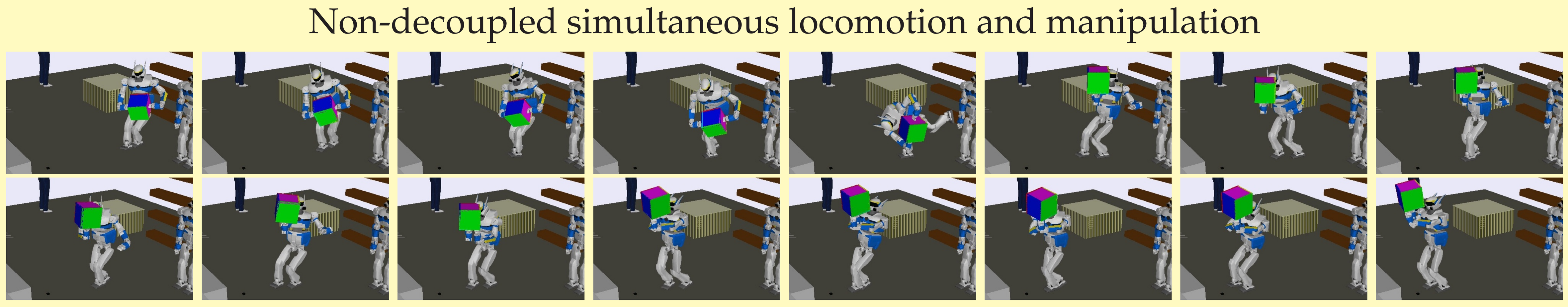
Biped locomotion over irregular terrain



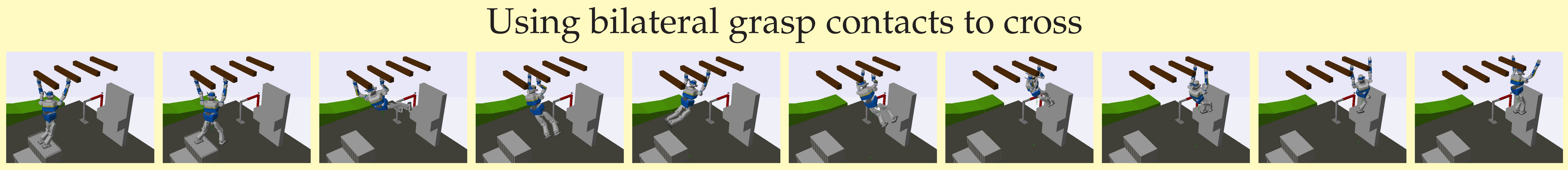
Dexterous hand manipulation



Non-decoupled simultaneous locomotion and manipulation



Using bilateral grasp contacts to cross



Collaborative task