Graphical method for matrix multiplication

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1 Graphical method for matrix multiplication

We will present here two versions of the graphical method for matrix multiplication: a complete, detailed and easy to use one, and a faster simplified one.

The full version is presented here mainly for educational purposes, and is rarely ever used in practice.

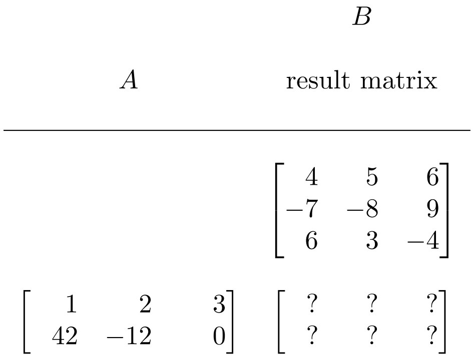
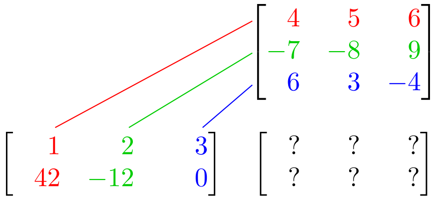
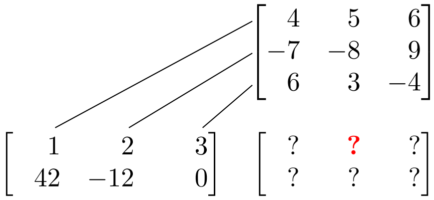
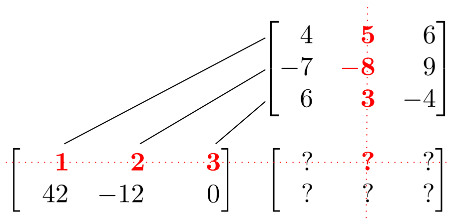
The matrices used in the following examples are:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 42 & -12 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 5 & 6 \\ -7 & -8 & 9 \\ 6 & 3 & -4 \end{bmatrix}$$

And we will try to compute the product AB.

Note: there are some dotted lines in the examples; those lines should not be drawn, contrary to the full ones.

2 Full version

Step	Explanation	Example
1	The first thing to do is to write both A and B as in the example. *A trick to remember which matrix goes where is to read the product AB: A is on the left and B on the right, so A goes to the left, and B to the right (so the top).*	
2	The second step is to check if the dimensions of the two matrices allow multiplication. To do that, you can use the empty top left corner, by drawing lines between the top of the columns of A and the left side of the lines of B. If there is a line or a column that is left alone – you can see that easily – it means that it is impossible to compute the product AB.	
3	The third step is to chose an entry from the result matrix. In the example, the entry ab_{12} (entry at row 1, column 2, of the result matrix) was chosen. *Note that which entry is chosen first does not really matter, as all the entries will be filled in the end.*	
4	Once you have chosen an entry, you must locate the corresponding row in A and the corresponding column in B. To do so, you can draw (imaginary) lines crossing at the position of the chosen entry. *If you do not keep those lines imaginary, your matrices will quickly become messy.*	

5 Now, you can “extract” the row of A and the column of B, like in the example.

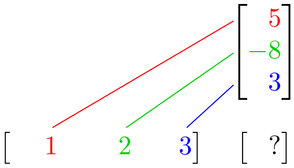
Writing those out of the matrix avoids mixing up the numbers.

This will allow you to easily compute the value of the chosen an entry.

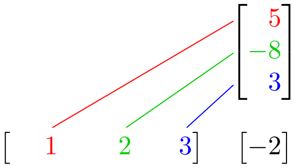
$$\begin{bmatrix}5\\-8\\3\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix} \quad \begin{bmatrix}?\end{bmatrix}$$

6 The next step is to match the corresponding entries by drawing lines in the empty corner (like in step 3).

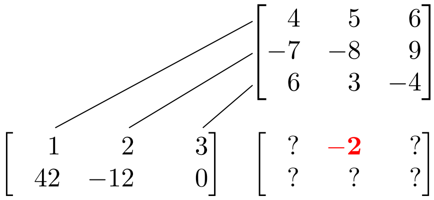


7 You can now compute the value of the result entry by following a simple rule: multiply entries linked by a line, then add-up the results of these products.

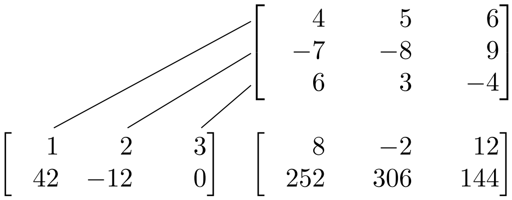


1 * 5 + 2 * (-8) + 3 * 3 = -2

8 Now, you can insert the computed entry in the result matrix.



9 Repeat from step 3 until the result matrix is full.



10 You are done!

$$AB = \begin{bmatrix}8 & -2 & 12\\252 & 306 & 144\end{bmatrix}$$

2.1 Notes

When choosing the entries, the order does not matter much. While some may prefer computing row by row or column by column, I usually do the easiest looking computations first. One of the advantages of the graphical method is that no matter what order is chosen it is easy to find the computation for each entry.

This method is designed to help during computations. Thus, many of the previous steps may become unnecessary once you get used to using the method. You can omit or merge most of them – by doing them mentally for example. The simplified algorithm presented in Section 3 presents an example of such a simplified version of the full algorithm.

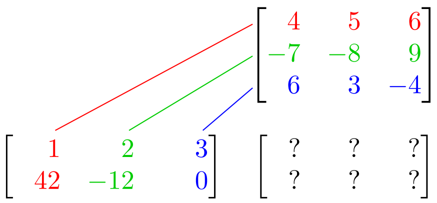
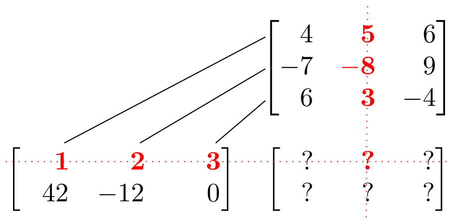
2.2 Summarized algorithm

1. Write the two matrices on their respective positions
2. Check if the dimensions of the two matrices allow the multiplication
3. Choose any entry of the result matrix, then:
    (a) Locate in the two multiplied matrices the row and column corresponding to the chosen entry
    (b) Extract the row and column and write them separately
    (c) Find the corresponding entries in the line and the column by drawing diagonal lines
    (d) Multiply the corresponding entries and sum the results
    (e) Insert the result in the result matrix
4. Repeat 3 for the remaining entries until the result matrix is full

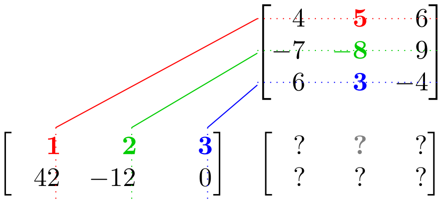
3 Simplified version

The simplified version of the graphical method for matrix multiplication is very similar to its full version. The major changes are:

- the removal of the extraction of the line and the column;
- the use of a single set of lines both to associate the numbers and check if the multiplication is possible.

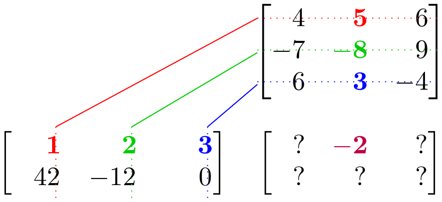
Step	Explanation	Example
1	As in the full version, the first thing to do is to write both A and B as in the example. *Still the same trick to remember which matrix goes where, by reading the product AB: A is on the left and B on the right, so A goes on the left, and B on the right (so the top).*	$\begin{array}{cc} & B \\ A & \text{result matrix} \\ \hline & \begin{bmatrix}4&5&6\\-7&-8&9\\6&3&-4\end{bmatrix} \\ \begin{bmatrix}1&2&3\\42&-12&0\end{bmatrix} & \begin{bmatrix}?&?&?\\?&?&?\end{bmatrix} \end{array}$
2	The second step is, using the empty top left corner, to draw lines between the top of the columns of A and the left side of the lines of B. This will be used for two operations: to check if the matrices are multipliable, and later-on to match the number to compute together. If there is a line or a column that is alone (you can see this easily), it means the computation AB is impossible.	
3	The third step is to chose an entry from the result matrix, and locate the corresponding row in A and the corresponding column in B. In the example, the entry ab_{12} (entry at row 1, column 2, of the result matrix) was chosen. *Note that which entry is chosen does not really matter, as all the entries will be computed in the end.*	

4 The next step is to match the entries in the line and the column by using the lines in the upper left corner.



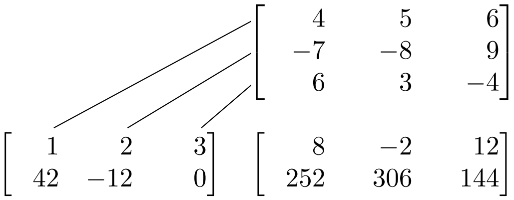
5 You can now compute the value of the result entry by following a simple rule: multiply entries linked by a line, then add-up the results of these products.

Either do the math in your head, or write it down if there are too many values to compute.



1 * 5 + 2 * (-8) + 3 * 3 = -2

6 Repeat from step 3 until the result matrix is full.



7 You are done!

$$AB = \begin{bmatrix} 8 & -2 & 12 \\ 252 & 306 & 144 \end{bmatrix}$$

3.1 Summarized algorithm

1. Write the two matrices to their respective positions
2. Match the lines and columns by drawing diagonal lines and check if the dimensions of the two matrices allow the multiplication
3. Choose any entry of the result matrix, then:
   (a) Locate in the two multiplied matrices the row and column corresponding to the chosen entry
   (b) Compute the value of the entry by multiplying the corresponding entries and summing the results
4. Repeat 3 for the remaining entries until the result matrix is full