

Web Semantics – Exercises session

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1 First order logic

(Michael Freund) Let \mathcal{L} be the language built on $\{a, P, f\}$ where a is a constant, P is a binary predicate and f a unary predicate. Write one or several models for each of the following formulae:

1. $F = \forall x \forall y (P(x, y) \vee P(y, x))$
2. $G = \forall x \forall y (P(x, y) \rightarrow \neg P(y, x))$
3. $H = \forall x \neg P(x, a)$
4. $K = \forall x P(x, f(x))$
5. $L = \forall x \forall y (P(x, f(y)) \rightarrow P(x, y))$

2 Relations & models

(René Cori & Daniel Lascar) Let \mathcal{L} be a language composed of a unary predicate P and a binary predicate symbol R . Consider the following formulae:

1. $\exists x \forall y \exists z ((P(x) \rightarrow R(x, y)) \wedge P(y) \wedge \neg R(y, z))$
2. $\exists x \exists z ((R(z, x) \rightarrow R(x, z)) \rightarrow \forall y R(x, y))$
3. $\forall y (\exists z \forall t R(t, z) \wedge \forall x (R(x, y) \rightarrow \neg R(x, y)))$
4. $\exists x \forall y ((P(y) \rightarrow R(y, x)) \wedge (\forall u (P(u) \rightarrow R(u, y) \rightarrow R(x, y)))$
5. $\forall x \forall y ((P(x) \wedge R(x, y)) \rightarrow ((P(y) \wedge \neg R(y, x)) \rightarrow \exists z (\neg R(z, x) \wedge \neg R(y, z))))$
6. $\forall z \forall u \exists x \forall y ((R(x, y) \wedge P(u)) \rightarrow (P(y) \rightarrow R(z, x)))$

For each of these formulae, write down whether or not it is satisfied in each of the following models:

- (a) The basic set is \mathbb{N} , R is interpreted as standard \leq , P is interpreted as the set of even natural numbers.
- (b) The basic set is $\mathcal{P}(\mathbb{N})$ (the set of all subsets of \mathbb{N}), R is interpreted as the inclusion relation \subseteq , P is interpreted as the set of finite subsets of \mathbb{N} .
- (c) The basic set is \mathbb{R} , R is interpreted as the set of couples $(a, b) \in \mathbb{R}^2$ such that $b = a^2$, P is interpreted as subset of rational numbers.

3 Building ontologies

(Odile Papini) Build the ontology of accomodations.

Basic concepts: Room, Kitchen, Bathroom, Living room, Bedroom, Garden, Terrace.

Concepts to appear in the ontology: Housing, Apartment, Individual House, Small House (2 rooms at most, not counting the bathroom), Large House (4 rooms at most, not counting the bathroom), Prestigious House, Villa, Studio

4 Description logics

(Odile Papini) Give the representation in description logics of the following concepts:

1. People who have neither a dog nor a cat.
2. Vegetarian people who live in the countryside.
3. People who do not like cats.

5 Formal Concept Analysis

(Classic example) Build a table representing the formal context linking the following objects and the following attributes.

Objects fish leech, bream, frog, dog, water weeds, reed, bean, corn

Attributes needs water to live, lives in water, lives on land, needs chlorophyll, dicotyledon, monocotyledon, can move, has limbs, breast feeds