1 First order logic

(Michael Freund) Let \( L \) be the language built on \( \{a, P, f\} \) where \( a \) is a constant, \( P \) is a binary predicate and \( f \) a unary predicate. Write one or several models for each of the following formulae:

1. \( F = \forall x \forall y \ (P(x, y) \lor P(y, x)) \)
2. \( G = \forall x \forall y \ (P(x, y) \rightarrow \neg P(y, x)) \)
3. \( H = \forall x \neg P(x, a) \)
4. \( K = \forall x \ P(x, f(x)) \)
5. \( L = \forall x \forall y \ (P(x, f(y)) \rightarrow P(x, y)) \)

2 Relations & models

(René Cori & Daniel Lascar) Let \( L \) be a language composed of a unary predicate \( P \) and a binary predicate symbol \( R \). Consider the following formulae:

1. \( \exists x \forall y \exists z \ ((P(x) \rightarrow R(x, y)) \land P(y) \land \neg R(y, z)) \)
2. \( \exists x \exists z \ ((R(z, x) \rightarrow R(x, z)) \rightarrow \forall y \ R(x, y)) \)
3. \( \forall y \ (\exists z \forall t \ R(t, z) \land \forall x \ (R(x, y) \rightarrow \neg R(x, y))) \)
4. \( \exists x \forall y \ ((P(y) \rightarrow R(y, x)) \land (\forall u \ (P(u) \rightarrow R(u, y) \rightarrow R(x, y))) \)
5. \( \forall x \forall y \ ((P(x) \land R(x, y)) \rightarrow ((P(y) \land \neg R(y, x)) \rightarrow \exists z \ (\neg R(z, x) \land \neg R(y, z)))) \)
6. \( \forall z \forall u \exists x \forall y \ ((R(x, y) \land P(u)) \rightarrow (P(y) \rightarrow R(z, x))) \)

For each of these formulae, write down whether or not it is satisfied in each of the following models:
(a) The basic set is $\mathbb{N}$, $R$ is interpreted as standard $\leq$, $P$ is interpreted as the set of even natural numbers.

(b) The basic set is $\mathcal{P}(\mathbb{N})$ (the set of all subsets of $\mathbb{N}$), $R$ is interpreted as the inclusion relation $\subseteq$, $P$ is interpreted as the set of finite subsets of $\mathbb{N}$.

(c) The basic set is $\mathbb{R}$, $R$ is interpreted as the set of couples $(a, b) \in \mathbb{R}^2$ such that $b = a^2$, $P$ is interpreted as subset of rational numbers.

### 3 Building ontologies

(Odile Papini) Build the ontology of accommodations.

**Basic concepts:** Room, Kitchen, Bathroom, Living room, Bedroom, Garden, Terrace.

**Concepts to appear in the ontology:** Housing, Apartment, Individual House, Small House (2 rooms at most, not counting the bathroom), Large House (4 rooms at most, not counting the bathroom), Prestigious House, Villa, Studio

### 4 Description logics

(Odile Papini) Give the representation in description logics of the following concepts:

1. People who have neither a dog nor a cat.
2. Vegetarian people who live in the countryside.
3. People who do not like cats.

### 5 Formal Concept Analysis

(Classic example) Build a table representing the formal context linking the following objects and the following attributes.

**Objects** fish, leech, bream, frog, dog, water weeds, reed, bean, corn

**Attributes** needs water to live, lives in water, lives on land, needs chlorophyll, dicotyledon, monocotyledon, can move, has limbs, breast feeds