A polynomial algorithm for deciding the finiteness of the number of simple permutations in permutation classes *

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Our goal is to provide automatic methods to enumerate permutation classes. More precisely, in [1], the authors explain how to compute the generating function of a permutation class whenever this class contains a finite number of simple permutations. In [5, 4, 6], the authors prove that determining if a class given by its basis contains a finite number of simple permutations is decidable. The proof relies on a subclass of permutations called pin-permutations. In [3], we characterize the pin-permutations in terms of decomposition trees and give their generating function. In [2], we use this characterization to give a $\mathcal{O}(n \ln n)$ algorithm to determine if a wreath-closed class of permutations given by its basis contains a finite number of simple permutations. This result was obtained by proving that determining if a simple permutation contains a given simple pin-permutation can be reduced to a problem on words, namely determining if a word is a factor of another one.

In this article, we prove a polynomial algorithm to determine if a general permutation class contains a finite number of simple permutations. In [6], the authors reduce the problem of containing a finite number of simple permutation to a co-finiteness problem on regular languages. These languages are given by nondeterministic finite automata leading to a exponential complexity in the decision procedure. We use the same encoding of permutations by words. Our characterization of pin-permutations obtained in [3] allows us to provide an explicit description of the set of words associated to any pin-permutation. This description is essential in the efficient construction of complete deterministic automata -instead of nondeterministic ones- for which the time complexity of the co-finiteness problem is linear.

Putting all together we can prove the following theorem and also provide an effective algorithm:

Theorem 1. *Deciding if a permutation class* $\mathcal{C} = Av(\sigma^{(1)}, \sigma^{(2)}, \ldots, \sigma^{(k)})$ *contains a finite number of simple permutations can be done in time* $\mathcal{O}(n^{3k})$ *where* n *is the size of the largest permutation in the basis.*

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