Data Structures TD4: Graphs

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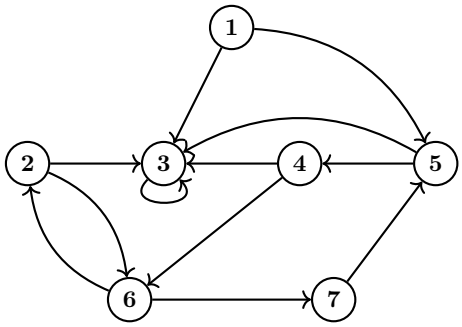
1 Introduction

1. (a) Describe how to find all the nodes accessible from a given node X in a graph?

   (b) Apply your idea (while drawing with colours, or explaining (= sentences) each step for example) on the following graph with node 2.



2. In the above graph:

   (a) Is the node 7 accessible? and 1? (**Definition**: We say that a node A is *accessible* if there exists at least one node X such that A is accessible from X.)

   (b) What are the nodes directly accessible from 3? from 2?

   (c) Is there a path (and which path is it) from 2 to 5? from 5 to 2?

   (d) Is there a path (and which path is it) from 1 to 6? from 6 to 1?

   (e) What is the shortest path from 7 to 3?

   (f) What is the shortest path from 5 to 5? from 3 to 3?

   (g) What is the resulting graph when removing the edge $4 \to 6$?

   (h) What is the resulting graph when removing the edges $4 \to 3$, $5 \to 3$, $1 \to 3$, $3 \to 3$ and $2 \to 3$?

   (i) What is the resulting graph when removing the node 4?

   (j) Draw the resulting graph when adding the node 9 to the above graph.

   (k) Draw the resulting graph when adding the edge $3 \to 8$ to the above graph.

   (l) Draw the directed graph made of a node of label 4, a node of label 7, a node of label 8, a node of label 1, an edge from the node of label 8 to the node of label 7, an edge from the node of label 1 to the node of label 8, an edge from the node of label 1 to the node of label 4, an edge from the node of label 4 to the node of label 4, a node of label 2, an edge from the node of label 2 to the node of label 8, a node of label 6, a node of label 5, an edge from the node of label 5 to the node of label 5, a node of label 3, an edge from the node of label 2 to the node of label 3, an edge from the node of label 3 to the node of label 2, an edge from the node of label 7 to the node of label 8, an edge from the node of label 1 to the node of label 2, an edge from the node of label 4 to the node of label 8, an edge from the node of label 7 to the node of label 4, an edge from the node of label 3 to the node of label 8, an an edge from the node of label 5 to the node of label 7.

2 Creating an algorithm

1. How do you delete all the nodes having an odd label in a given graph? Apply your idea on a graph to illustrate it.

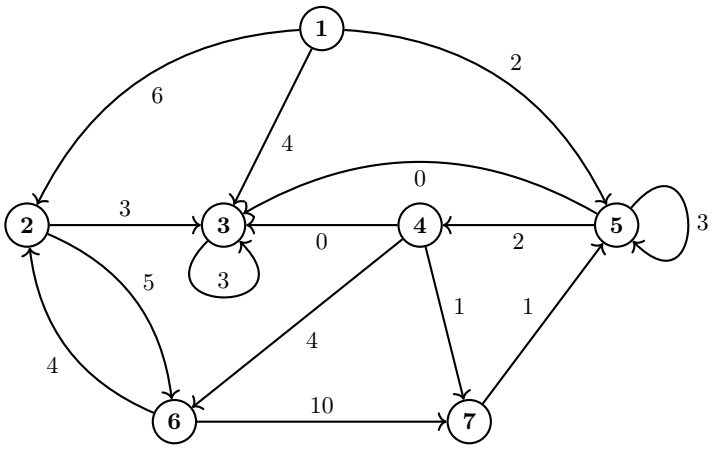
2. How do you make a given node X of a directed graph inaccessible? And in an un-directed graph? Draw an example graph and the resulting graph after applying these ideas for both the directed version and the un-directed version.

3. How do you delete all the nodes from which a given node X is directly accessible in a directed graph? Does your idea change (and how) for an un-directed graph? Draw an example graph and the resulting graph after applying these ideas for both the directed version and the un-directed version.

4. How do you check if there exists a path from a given node X to a given node Y in a directed graph?

5. A cycle is a path that starts with a node X and ends with the same node X. How do you find the cycles in a directed graph?

6. A weighted graph is a graph where each edge has a (numerical) value, called a weight (see the following weighted directed graph).



   (a) How do you find the path with the smallest weight from **1** to **7** in the above weighted graph?

   (b) In general, how do you find if there exists a path of a weight smaller than a given weight W between a node X and a node Y?

3 Idea of a formalism - for the volunteers - don't panic

We can represent a graph with a list. The nodes of a graph will each have an integer label (starting with 0), and each index of the list corresponds to the node of the same value. The value at index X of the list will be the list of all the nodes directly accessible from the node of label X. For instance, in [[0, 3, 2], [0, 2], [0, 3, 4], [1], [1, 2]], the nodes 0, 3 and 4 are directly accessible from node 2.

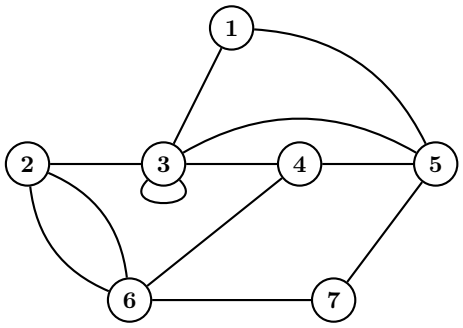
1. Draw the following graphs:
   (a) graph A: [[0, 3, 2], [0, 2], [0, 3, 4], [1], [1, 2]]
   (b) graph B: [[0, 5, 2], [], [2, 6], [5,0], [], [1], [2, 6]]

2. In the graph [[0, 5, 2, 4], [3], [2, 6], [5,0], [0, 1, 2, 3], [1, 4], [2, 6]]:
   (a) Is there a path from node 1 to node 3?
   (b) Is there a path from node 0 to node 3?

3. We consider the graph [[0, 3, 2, 4], [3], [2, 6], [5, 0], [0, 1, 2, 5], [1, 4], [2, 6]]. How would you modify this graph (*i.e.*, write the resulting list) to:
   (a) add the edge $5 \rightarrow 3$?
   (b) remove the edge $2 \rightarrow 6$?
   (c) add the node 7?
   (d) remove the node 2?

4 Optional Exercise - Due December 22th

1. How do you find all the nodes accessible from a given node X in an un-directed graph with a path of length 3 or more?
2. Apply your idea (draw and explain) to the following graph for the node **4**:



3. Does your idea change (and how) for a directed graph?