Analysis, Randomness and Applications (ARA) 2013
Program and abstracts

LORIA, Nancy - June 27 and 28, 2013

Thursday 27

9:30		*Coffee*
10:00	Johanna Franklin	**Uniform-distribution randomness and genericity**
	Laurent Bienvenu	**Recent progress on effective Brownian motion**
	Kenshi Miyabe	L^1**-computability and the computability of conditional probability**
		Lunch
14:00	Jason Rute	**Transformations which preserve computable randomness**
	Rupert Hölzl	**Algorithmic randomness and semimeasures**

Friday 28

9:30		*Coffee*
10:00	Noam Greenberg, Benoit Monin	**Higher randomness**
	Joseph S. Miller	**Highness properties close to being of PA degree**
		Lunch
14:00	Chris Porter	**Demuth and randomness**
	Dan Turetsky	**Weak 2-Randomness and Higher Dimensional Differentiability**
	Andre Nies	**Density-one points, martingale convergence, and differentiability**
		Open problems

Thursday 27

9:30 *Coffee*

10:00 Johanna Franklin (Univ. of Connecticut)

Uniform-distribution randomness and genericity.

Jeremy Avigad developed the concept of UD-randomness based on a theorem of Weyl on uniform distribution. Every Schnorr random real is UD-random, but not every UD-random is weakly 1-random (and vice versa), so this is a very weak randomness notion. I will discuss the relationship between the Turing degrees of UD-random reals and generic reals.

- Laurent Bienvenu (Université Paris 7)

Recent progress on effective Brownian motion.

This work is a contribution to the study of Martin-Löf randomness for Brownian motion, a line of study initiated by Asarin and Prokovsky in the 1980's, and continued by Kjos-Hanssen, Fouché and others in recent years. We study the zero sets of one-dimensional Martin-Löf random Brownian paths. It is well-known classically that with probability 1, the zero set of a Brownian path has dimension 1/2. It is not true however that the zero set of a Martin-Löf random Brownian path has constructive dimension 1/2 (indeed, it always has constructive dimension 1). We take a different approach and ask the question: for which reals t does there exists some Martin-Löf random path having t as a zero? We show that:

  - There is no such real of constructive dimension $< 1/2$.
  - All reals of constructive dimension $> 1/2$ have that property.
  - Among reals of constructive dimension exactly $1/2$, some have that property, some do not.

We also study the general structure of the zero set of a Martin-Löf random path and show in particular that the zero set of a random path B is a recursive closed set of reals relative to the oracle B. This has an elegant and surprising application to computable analysis, namely the following theorem: A computable instance of an n-dimensional Dirichlet problem always admits a computable solution.

This is joint work with Kelty Allen and Ted Slaman.

- Kenshi Miyabe (Tokyo University)

L^1-computability and the computability of conditional probability.

I will present a hierarchy of variants of L^1-computability that corresponds to the hierarchy of the notions of randomness. This hierarchy also has a connection with differentiability and the convergence of martingales. As one application, I will give a sufficient condition for van Lambalgen's theorem to hold for ML-randomness for non-uniform measures.

- *Lunch*

14:00 Jason Rute (Carnegie-Mellon)

Transformations which preserve computable randomness.

Preservation of randomness says that if x is random, and T is a computable map, then $T(x)$ is random on the measure P_T (the distribution of T). Unlike Schnorr and Martin-Löf randomness, preservation of randomness does not hold for computable randomness (Bienvenu, Porter; Rute). However, one can get interesting partial results. My main result is that preservation of computable randomness holds for computable maps T such that the conditional expectation $P[|T]$ is computable. My main question is, are these the only computable maps which preserve computable randomness?

The proof of the main result uses computable analysis in interesting ways. I will give some review of conditional probability and of effectively measurable functions, as well as other applications of computable conditional probability to randomness.

- Rupert Hölzl (Universitaet der Bundeswehr, Munich)

  Algorithmic randomness and semimeasures.

  The topic of the field of algorithmic randomness is to study what it means for an object to be random. This is well-studied in the context of randomness relative to various kinds of probability measures, both computable and non-computable. The general consensus in the field is that Martin-Löf randomness is the most natural randomness notion to consider here. Much less well understood is randomness relative to semimeasures. Semimeasures can be seen as defective measures, as they are not required to be additive: When looking at a set more and more fine-grainedly measure can be "lost" – that is, the sum of measures of a partition of a set can be much smaller than the measure of that set as a whole. While this may at first seem like an artificial notion, it is in fact a very natural object to study in algorithmic randomness, as lower semicomputable semimeasures are exactly the measures induced by Turing functionals. In this talk, we will discuss some ongoing work on the problem of providing a natural and useful definition of (Martin-Löf) randomness with respect to lower semicomputable semimeasures. We first agree on some desirable properties we expect from such a definition. Then by varying in several ways the old definitions for randomness, we will generate a number of candidates for the new definition. Possible variations include trying to transplant directly the classical definition of Martin-Löf randomness to semimeasures; blind or non-blind randomness; trying to "cut back" semimeasures until they become measures and to then apply a randomness definition for those; trying to "inflate" semimeasures with the same intention.

  Joint with L. Bienvenu, C. Porter and P. Shafer.

Friday 28

9:30 *Coffee*

10:00 Noam Greenberg and Benoit Monin.

Higher Randomness.

- Joseph S. Miller (Univ. Wisconsin)

  Highness properties close to being of PA degree

- *Lunch*

14:00 Chris Porter (Univ. Paris 7)

Demuth and randomness

- Dan Turetsky (Kurt Gödel Center Vienna)

  Weak 2-Randomness and Higher Dimensional Differentiability.

  Brattka, Miller and Nies investigated the relationship between algorithmic randomness and differentiability of effectively given functions. I will review some of their results and prove a higher dimensional analog of one of them.

- Andre Nies (Univ. Auckland)

  Density-one points, martingale convergence, and differentiability