Journées Calculabilités 2013*
Program and abstracts
LORIA, Nancy - April 11 and 12, 2013

Thursday 11

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13:30 - 14:20	Neil Jones	**Programs = data = first-class citizens in a computational world**
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Friday 12

9:50 - 10:40	Hugo Férée	**On the query complexity of real functionals**
10:40 - 11:30	Christopher Porter	**Randomness and semi-measures**
11:30 - 13:20		*Lunch*
13:20 - 14:10	Grégory Lafitte	**The necessary use of determinacy assumptions in a computability result**
14:10 - 15:00	Ludovic Patey	**New results in the reverse mathematics analysis of Ramsey theory**
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15:30 - 16:20	Bruno Bauwens	**The Kolmogorov complexity of on-line predicting odd and even bits**

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Thursday 11

10:10 - 11:00. Laurent Bienvenu. **Higher randomness and triviality.**

Higher randomness theory investigates the notions of effective randomness one obtains when replacing "computable" by "hyperarithmetic" and "c.e." by "Π^1_1" in the usual definitions (Martin-Löf randomness, Schnorr randomness, computable randomness, etc). After recalling the basics of the theory, I will present some recent work in collaboration with Noam Greenberg and Benoit Monin. The main question we will address is the following: do the (very rich) interactions between randomness and Turing degrees have a counterpart in higher computability? We will argue that this is indeed the case, provided one correctly translates the notion of Turing reduction in the higher setting. We will thus introduce the notion of higher Turing reduction and show that a significant part of the classical theory translates accordingly. However, we will also see that the two landscapes (classical and "higher") differ dramatically on some key aspects, such as the existence of a uniform oracle tests and measures. If time permits, I will discuss the impact this has on the study of lowness and triviality, and will ask some open questions.

11:00 - 11:50 Benjamin Hellouin. **Characterizing typical asymptotic behaviours of cellular automata.**

A cellular automaton (CA) is a discrete dynamical system that can be seen as a physical or a computational model. Computer simulations of various CA show that, starting from a random configuration, they can exhibit a wide variety of typical asymptotic behaviours. In this talk, I will characterize the behaviours that can be observed in this way. This follows the trend of recent results characterizing possible parameters of discrete dynamical systems by computability conditions. More precisely, typical asymptotic behaviour is well-described by the limit probability measure(s). When the initial measure is computable, computability obstructions appear on the limit measure(s). Conversely, every measure satisfying those obstructions can be reached at the limit by a cellular automaton that we build explicitly, using auxiliary states. This result has consequences in decidability, and this method can be further extended so that the limit measures depend on the initial measure, considered as an argument or an oracle. Furthermore, additional hypotheses allow us to remove the need for auxiliary states in some of those results.

This is a joint work with Mathieu Sablik (Marseille, LATP).

11:50 - 13:30 *Lunch*

13:30 - 14:20 Neil Jones. **Programs = data = first-class citizens in a computational world.**

From a programming perspective, Alan Turing's epochal 1936 paper on computable functions introduced several new concepts, originated a great many now-common programming techniques, including the invention of what are today known as self-interpreters, using programs as data.

The 'blob' model of computation is a recent stored-program computational model, biologically motivated and without pointers or memory addresses. Novelties of the blob model: programs are truly first-class citizens, capable of being automatically executed, compiled or interpreted. The model is Turing complete in a strong sense: a universal interpretation algorithm exists, able to run any program in a natural way and without arcane data encodings. The model appears closer to being physically realisable than earlier computation models. In part this owes to strong finiteness due to early binding; and a strong adjacency property: the active instruction is always adjacent to the piece of data on which it operates.

14:20 - 15:10 Paul Shafer. **A low DNR(k) function that computes no DNR(2) function.**

A classic result of Jockusch states that, for every $k \geq 2$, every DNR(k) function computes a DNR(2) function. Jockusch's proof can be formalized in RCA_0 + $\mathrm{I}\Sigma_2$, thus the corresponding reverse mathematics result is that the principles "exists k DNR(k)" (i.e., there is a k such that for every X there is a function that is DNR(k) relative to X) and WKL are equivalent over RCA_0 + $\mathrm{I}\Sigma_2$. The question then, perhaps first articulated by Simpson, is whether or not $\mathrm{I}\Sigma_2$ is necessary. In a step toward separating the principles "exists k DNR(k)" and WKL over RCA_0, we show that if $\mathrm{B}\Sigma_2$ holds and $\mathrm{I}\Sigma_2$ fails, then there is a DNR(k) function (for some necessarily non-standard k) that computes no DNR(2) function.

This is joint work with Francois Dorais (Dartmouth College) and Jeffry Hirst (Appalachian State University).

15:10 - 15:40 *Coffee break*

15:40 - 16:30 Amaury Pouly. **The General Purpose Analog Computer (GPAC) and computable analysis.**

On the one hand, Computable Analysis is a framework based on classical computability to study real functions. On the other hand, the General Purpose Analog Computer is an idealisation of the Differential Analyser, an analog computer much used in the 1960's. Despite the initial appearance, a few years ago, Bournez, Graça, Campagnolo & Hainry showed that these models are equivalent from a computability point of view. This very interesting result shows that some realistic digital and analog computer have the same computational power. However, this result does not shed any light on what happens at the complexity level: analog computers cannot solve the halting problem but maybe they can compute faster than digital computers. Our goal is to understand this equivalence at the complexity level. We introduce a notion of complexity for the GPAC and show some results toward a similar equivalence. In this talk I will mostly focus on the simulation of Computable Analysis by the GPAC and mention some results for the converse direction.

16:30 - 17:20 Antoine Taveneaux. **Randomized algorithms to compute completions of Peano arithmetic.**

The famous Gödel's theorem asserts that there is no algorithmic way to produce a completion of Peano arithmetic (PA). This does not answer the question: is there a probabilistic algorithm which produces such a completion with positive probability? Soare and Jockusch answered this question negatively. As suggested by Levin, one can refine Soare and Jockusch's analysis by looking at the probability of generating a completion of PA for formulas with a "size" up to a constant n (for a notion of size we will introduce). We show that the best algorithm computing a consistent extension of PA for one quantifier formulas with a size up to a constant n achieves this with probability 2^{-n}. If we look at the case of 2 or more quantifiers, we show that the situation is different if we want to produce some completion of PA or the standard completion of PA. If time permits, we will discuss the algorithmic difficulty of generating other classes of objects, such as shift-complex sequences, instead of completions of PA.

17:20 - 19:30 *Free time*

19:30 - 02:00 Dinner at the restaurant *La Table de Bacchus*, 15 rue des Maréchaux, Nancy, which is close to place Stanislas. The prices range from 19 to 32 euros.

Friday 12

9:50 - 10:40 Hugo Férée. **On the query complexity of real functionals.**

Recently Kawamura and Cook developed a framework to define the computational complexity of operators arising in analysis. Our goal is to understand the effects of complexity restrictions on the analytical properties of the operator. We focus on the case of norms over $\mathscr{C}[0,1]$ and introduce the notion of dependence of a norm on a point and relate it to the query complexity of the norm. We show that the dependence of almost every point is of the order of the query complexity of the norm. A norm with small complexity depends on a few points but, as compensation, highly depends on them. We characterize the functionals that are computable using one oracle call only and discuss the uniformity of that characterization.

This is joint work with Walid Gomaa and Mathieu Hoyrup.

10:40 - 11:30 Christopher Porter. **Randomness and semi-measures.**

Although Martin-Löf randomness for various probability measures is well-understood (for both computable and non-computable measures), there is no canonical definition of Martin-Löf randomness with respect to a semi-measure, where a semi-measure can be seen as a defective probability measure (as it need not be additive). In this talk, I will discuss some ongoing work on the problem of providing a natural and useful definition of Martin-Löf randomness with respect to a lower semicomputable semi-measure (or equivalently, a semi-measure that is induced by a Turing functional). I will introduce several candidates for such a definition, considering how they relate to another, as well as the relative strengths and weaknesses of each candidate definition.

This is joint work with Laurent Bienvenu, Rupert Hölzl, and Paul Shafer.

11:30 - 13:20 *Lunch*

13:20 - 14:10 Grégory Lafitte. **The necessary use of determinacy assumptions in a computability result.**

In 1966, Gerald Sacks asked the question of the existence of a degree-invariant relativized solution to Emil Post's problem: Does there exist e such that for all sets A, W_e^A, the e^{th} set recursively enumerable in A, is of higher Turing degree than A and of lower Turing degree than A', and such that if A and B have the same Turing degree, then W_e^A and W_e^B have the same Turing degree? Alistair Lachlan proved in 1975 that there is no uniform such degree-invariant relativized solution. Sacks' question, however, remains open. Rod Downey and Richard Shore found in 1997 how to use John Steel's 1982 work to show that any such degree-invariant solution must be low_2 or high_2 for all sufficiently large Turing degrees. Moreover, they show that there is no degree-invariant half jump operator. The part of Steel's work used is proved under the assumption of the axiom of determinacy. We show that Downey and Shore's result for operators of a certain level of definability implies Turing determinacy for similar definability levels. This leads us to the study of the consistency strength of (certain levels of) Turing determinacy (through the work of Harvey Friedman and Donald Martin), which in turn lets us show that the underlying use of determinacy is actually necessary when considering Borel operators.

This is joint work with Laurent Bienvenu (University of Paris VII) and Ted Slaman (University of California, Berkeley).

14:10 - 15:00 Ludovic Patey. **New results in the reverse mathematics analysis of Ramsey theory.**

A coloring function is n-bounded if every color is used at most n times. Rainbow Ramsey theorem for pairs (RRT^2_2) states that every 2-bounded coloring function over pairs has an infinite injective subset. We will relate this principle to Stable Ramsey theorem for pairs (SRT^2_2), Erdös-Moser Principle (EM) and Thin Sets for pairs ($\mathsf{TS}(2)$).

To this purpose, we will use the characterization proven by J. Miller of RRT^2_2 as the existence of a diagonally non-computable function for Σ^0_2 functions, the Ramsey-Type Königs lemmas principles introduced by S. Flood and the recent result about the non implication of RT^2_2 by SRT^2_2 for non-standard models by C. T. Chong & al.

This work has been done in collaboration with Laurent Bienvenu and Paul Shafer.

15:00 - 15:30 *Coffee break*

15:30 - 16:20 Bruno Bauwens. **The Kolmogorov complexity of on-line predicting odd and even bits.**

Symmetry of information states that $C(x) + C(y|x) = C(x,y) + O(\log C(x))$. We show that a similar relation for on-line complexity Kolmogorov complexity does not hold. Let the even on-line Kolmogorov complexity of a $2n$-bitstring $x_1 y_1 \ldots x_n y_n$ be the length of a shortest program that computes y_1 on input x_1, computes y_2 on input $x_1 y_1 x_2$, etc; and similar for odd complexity. We show that the sum of odd and even complexity may exceed the full Kolmogorov complexity by a linear term. We also show that there exist strings of arbitrarily large complexity such that both odd and even on-line complexity are almost as large as the complexity of the full string. Moreover, after flipping odd and even bits the sum of odd and even on-line complexity $y_1 x_1 \ldots y_n x_n$ decreases by a linear term. Our result has an interpretation in terms of causality: there exist n-bitstrings x, y such that the order of generation $y_1 \to x_1 \to \ldots \to y_n \to x_n$ is much more likely than $x_1 \to y_1 \to \ldots \to x_n \to y_n$.