Internship subject

Subject. Computability over partially ordered sets

Topic. Computability, partial orders, topology

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Context. Computability theory was initially developed on finite objects like the natural numbers, and was later extended to infinite objects such as real numbers, sets or functions. The two main approaches to computations with infinite objects are domain theory and representations. In the first one, the objects are endowed with a partial order, thought as an approximation relation, and an infinite object is handled by successive finite approximations. In the second one, the objects are encoded as infinite binary sequences that can be manipulated by Turing machines.

Domain theory imposes more structure than representations (namely, a partial order enjoying certain properties) and is therefore more restrictive.

Goals. The goal of this project is to bring closer together domain theory and representations, by better understanding which partially ordered sets are amenable to computations, beyond the current scope of domain theory.

There are simple examples of partially ordered sets admitting a natural representation, but that do not satisfy the usual conditions required by domain theory. We would like to characterize the partially ordered sets that

can be endowed with a representation, or at least to identify sufficient and necessary conditions on the partial order, in order to admit a representation.

The starting point will be to study concrete partial orders admitting a representation and to extract the properties that make such a representation exist.

More details. Domain theory mainly deals with partially ordered sets that are *directed complete*, i.e. in which every directed set has a supremum, and that are *ω-continuous*, i.e. which contain a countable set satisfying a certain form of density. We will still consider directed complete partial orders, but relax the ω-continuity condition.

Prerequisite. We require basic knowledge in computability (computable function, computable set, recursively enumerable set). The relevant knowledge in partially ordered sets and topology will be acquired during the internship.

Reference. The article [1] is an introduction to computability on real numbers and other infinite objects. The book [2] contains all the needed concepts about partially ordered sets.

References

[1] Vasco Brattka, Peter Hertling, and Klaus Weihrauch. A tutorial on computable analysis. In S. Barry Cooper, Benedikt Löwe, and Andrea Sorbi, editors, *New Computational Paradigms*, pages 425–491. Springer New York, 2008.

[2] Jean Goubault-Larrecq. *Non-Hausdorff Topology and Domain Theory: Selected Topics in Point-Set Topology*. New Mathematical Monographs. Cambridge University Press, 2013.