

Presentations of topological spaces

Mathieu Hoyrup

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Introduction

Computable analysis:

- How to represent mathematical objects,
- Difficulty of computing them,
- Difficulty of extracting information.

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- How to represent mathematical objects,
- Difficulty of computing them,
- Difficulty of extracting information.

In this talk, the objects are **topological spaces**.

We mostly focus on **compact metrizable spaces**, represented in **three** ways.

Introduction: presentations

Let $(B_i)_{i \in \mathbb{N}}$ be the rational boxes in $[0, 1]^n$ ($n = 1, 2, \dots, \infty$).

A compact metrizable space X can be represented as a compact set $K \subseteq [0, 1]^n$ homeomorphic to X , with:

1. Positive information

An enumeration of $\{i \in \mathbb{N} : B_i \cap K \neq \emptyset\}$.

2. Negative information

An enumeration of $\{i \in \mathbb{N} : \overline{B_i} \cap K = \emptyset\}$.

3. Both

Introduction

General questions

How difficult is it to recognize the topological shape?

Can shape encode information?

Computable type

Encoding information in a space

Countably-based spaces

Computable type

Knowing the shape can be used to derive positive information from negative information.

Computable type

Let $(B_i)_{i \in \mathbb{N}}$ be the rational boxes in $[0, 1]^n$.

Definition

A compact set $K \subseteq [0, 1]^n$ is

- **Semicomputable** if $\{i \in \mathbb{N} : K \cap \overline{B_i} = \emptyset\}$ is c.e.,
- **Computable** if moreover $\{i \in \mathbb{N} : K \cap B_i \neq \emptyset\}$ is c.e.

Computable type

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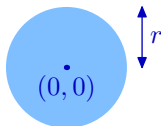
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Example

A disk $D \subseteq \mathbb{R}^2$ that is semicomputable but not computable.



$$r = 1 - \sum_{n: M_n \text{ halts}} 2^{-n-1}.$$

Computable type

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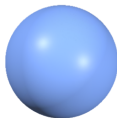
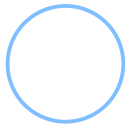
What are the possible shapes of sets that are semicomputable but not computable? What about the circle?

Computable type

Theorem (J. Miller, 2002)

If $K \subseteq \mathbb{R}^n$ is homeomorphic to a sphere $\mathbb{S}_m$, then

K is semicomputable $\iff K$ is computable.

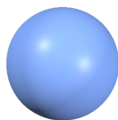
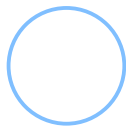


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Theorem (Iljazović, 2013)

If $K \subseteq [0, 1]^n$ is homeomorphic to a closed manifold, then

K is semicomputable $\iff K$ is computable.

Computable type

Definition (Iljazović)

A space has **computable type** if for every $K \subseteq [0, 1]^n$ that is homeomorphic to X ,

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have computable type:



Manifolds with boundary
do not have computable type:



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What about other simple spaces? (simple=triangulable)

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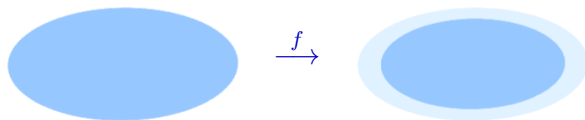
A simple space does not have computable type $\iff$ it can be “slightly squeezed”.

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Theorem (Amir, H. 2022)

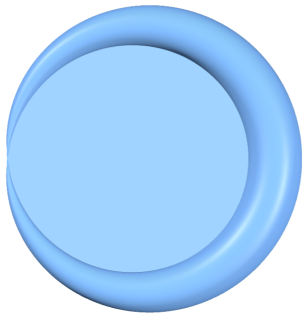
A simple space X does not have computable type $\iff \forall \epsilon > 0$, there is a non-surjective continuous function $f : X \rightarrow X$ such that $d(f(x), x) < \epsilon$ for all $x \in X$.



Computable type

Example

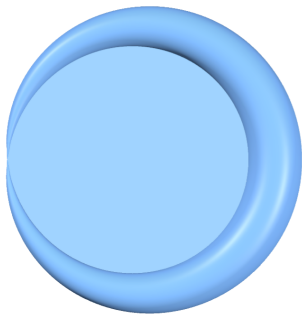
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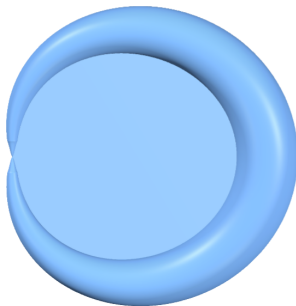
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Computable type

Example

A semicomputable, non-computable realization:



- Let $(n_0, n_1, n_2, \dots)$ be a computable enumeration of the halting set H ,
- Let the k th hole have size $\frac{1}{2^{n_k}}$,
- One has $n \in H \iff$ there is a hole of size $\frac{1}{2^n}$.

Computable type: links

Question

How to know whether a simple space can be “slightly squeezed”?

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It is determined by its **links**. Link of a point=boundary of a small neighborhood of the point.



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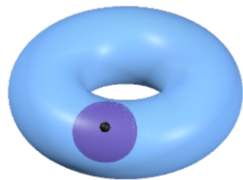


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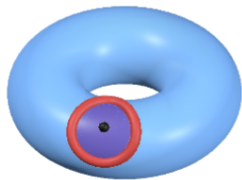


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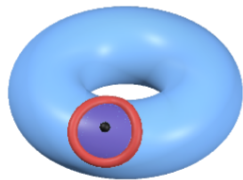


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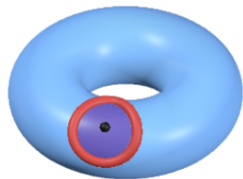


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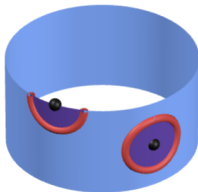
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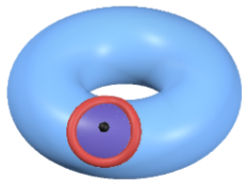


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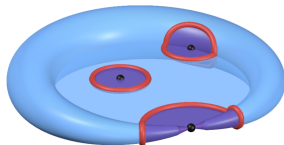
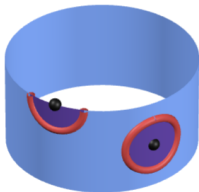
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How to know whether a simple space can be “slightly squeezed”?

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Links:



Computable type: links

In a two-dimensional simple space, the links are finite graphs.

Theorem (Amir, H. 2022)

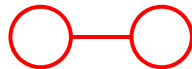
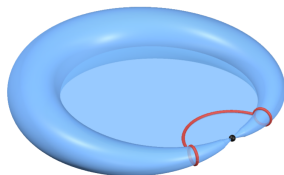
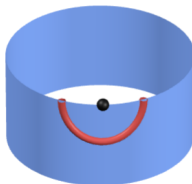
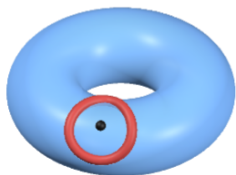
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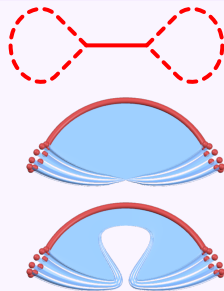
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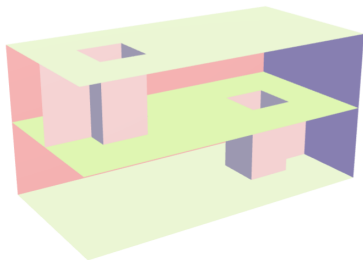
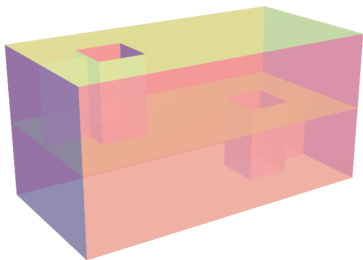
Proof idea of $\implies$.

- If a link is not a union of cycles:
- Then locally, the space looks like:
- So it can be distorted this way:
- We can encode the halting set.



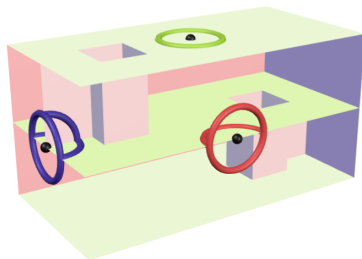
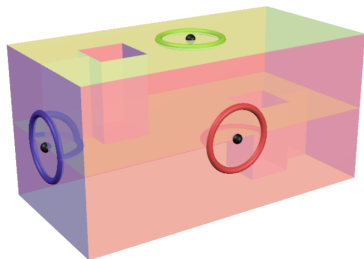
Computable type: links

Example: Bing's house



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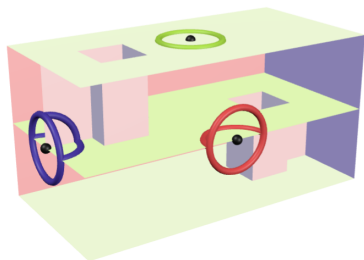
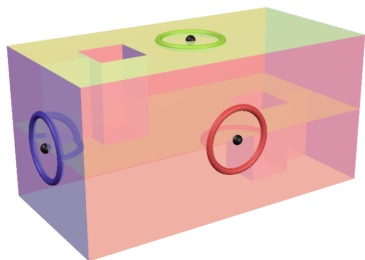


Bing's house has computable type, because its links are unions of cycles:



Computable type: links

Example: Bing's house



Bing's house has computable type, because its links are unions of cycles:



Bing's house is contractible, but cannot be “slightly squeezed”.

Computable type: links

Higher dimensions

In a space of dimension n , the links have dimension $n - 1$.

Theorem (Amir, H. 2024)

A purely n -dimensional simple space has computable type $\iff$ its links are unions of cycles¹.

Examples of links of dimension 2:



¹homology cycles

Computable type

Encoding information in a space

Countably-based spaces

Encoding information in a space

Can shape encode information?

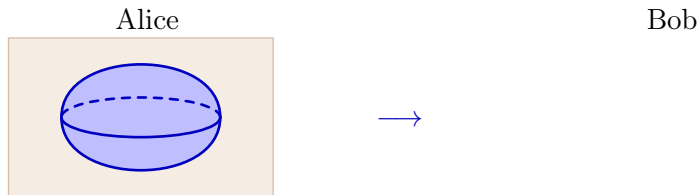
Encoding information in a space

- Alice wants to send a message $M = 0100101110\dots$ to Bob,



Encoding information in a space

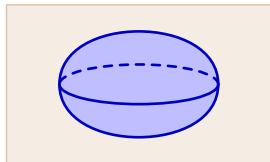
- Alice wants to send a message $M = 0100101110\dots$ to Bob,
- Alice encodes M in a topological space X_M and sends X_M ,



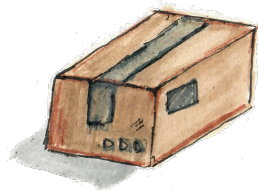
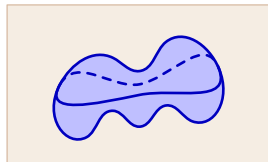
Encoding information in a space

- Alice wants to send a message $M = 0100101110\dots$ to Bob,
- Alice encodes M in a topological space X_M and sends X_M ,
- Bob receives a continuous deformation of X_M .

Alice



Bob



Presentations

We only consider compact metrizable spaces.

Definition

A **presentation** of a space X is:

- A compact set $K \subseteq [0, 1]^n$ homeomorphic to X ,
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Alice sends a presentation of X_M , Bob receives another presentation of X_M .

Sending messages: naive attempt

Alice sends this message:



0|00|011|0...

Sending messages: naive attempt

Alice sends this message:

0|0 0|0||0 ...

Bob may receive this message:

0|0|0|0|0| ...

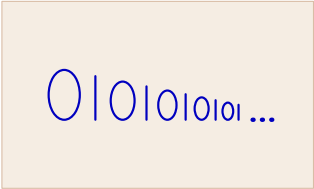
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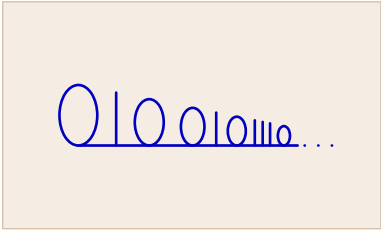
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Problem

All messages with infinitely many 0's and 1's are homeomorphic.

Sending messages

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0 1 0 0 1 0 1 1 1 0 ...

Sending messages

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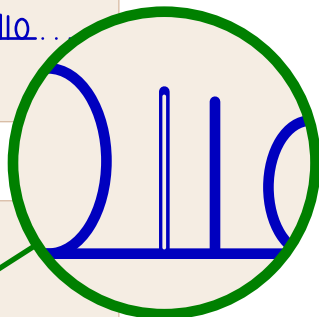


010010110...

Bob may receive this message:



0101101001...



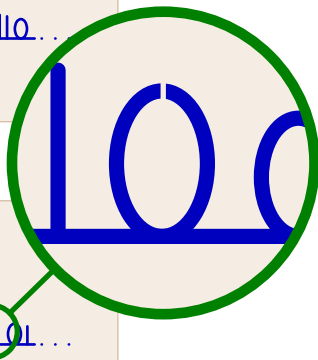
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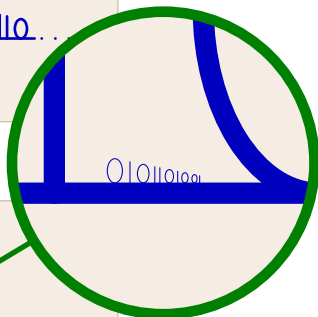
Sending messages

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0 1 0 0 1 0 1 1 0 ...

Bob may receive this message:

0 1 0 1 1 0 1 0 0 1 ...



Binary sequences vs sets

Now the message is a set $A \subseteq \mathbb{N}$.

Computing a binary sequence $x_0x_1x_2 \dots$

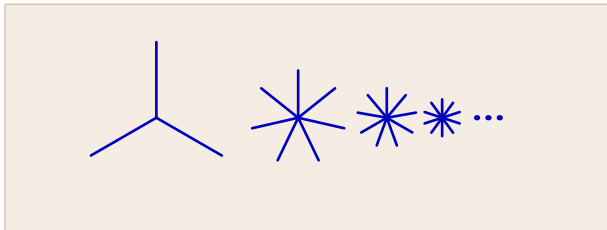


Enumerating the set $\{2n : x_n = 0\} \cup \{2n + 1 : x_n = 1\}$.

Stars

Message: $\{3, 7, 9, 10, \dots\}$.

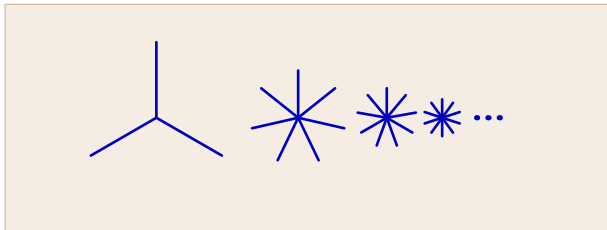
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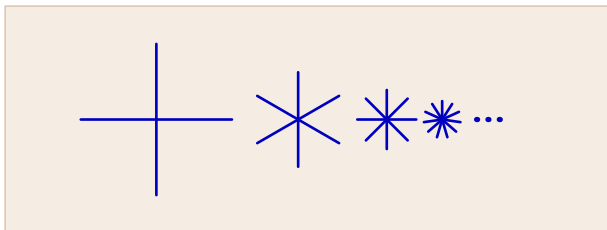
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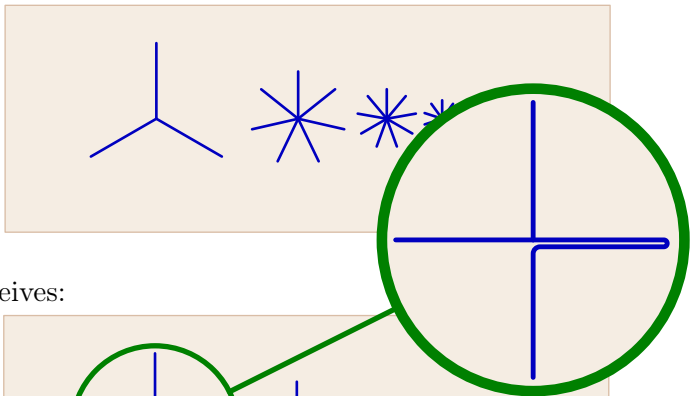
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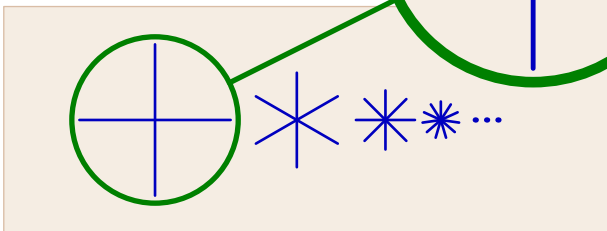
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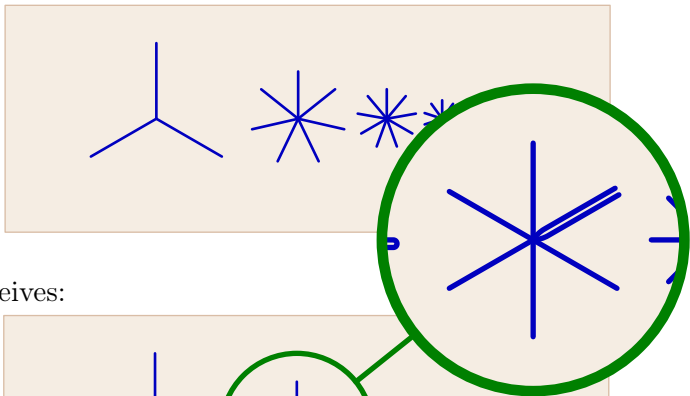
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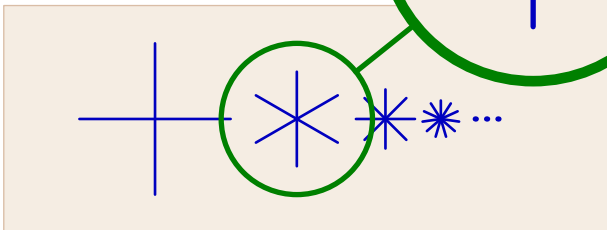
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Alice sends:



Bob receives:



Stars

Decyphering

Bob can only know A in a Σ_3^0 fashion: from the received message he can compute a set $B \subseteq \mathbb{N}^4$ such that

$$n \in A \iff \exists i, \forall j, \exists k, (n, i, j, k) \in B.$$

This is optimal.

Stars

Precise result

Let X_A be the disjoint union of:

- One star S_{2k} for each $k \in A$,
- Infinitely many stars S_{2k+1} for each $k \in \mathbb{N}$.

Theorem (Melnikov, Harrison-Trainor, Ng, 2020)

An oracle Z computes a presentation of $X_A \iff A$ is Σ_3^0 relative to Z .

Stars

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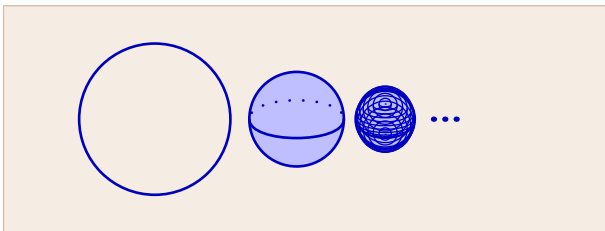
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Question

Can we encode A into a space Y_A in a Σ_2^0 way? in a Σ_1^0 way?

Spheres

Let $Y_A = \bigsqcup_{k \in A} \mathbb{S}_{2k} \sqcup \bigsqcup_{k \notin A} \mathbb{S}_{2k+1}$.

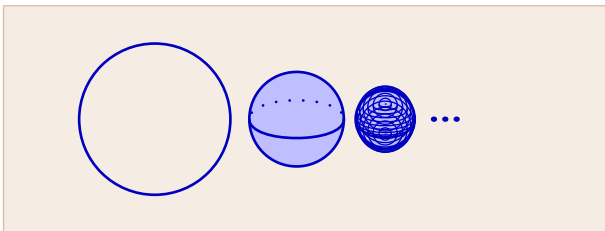


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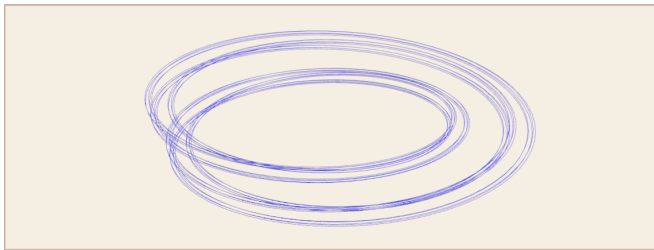
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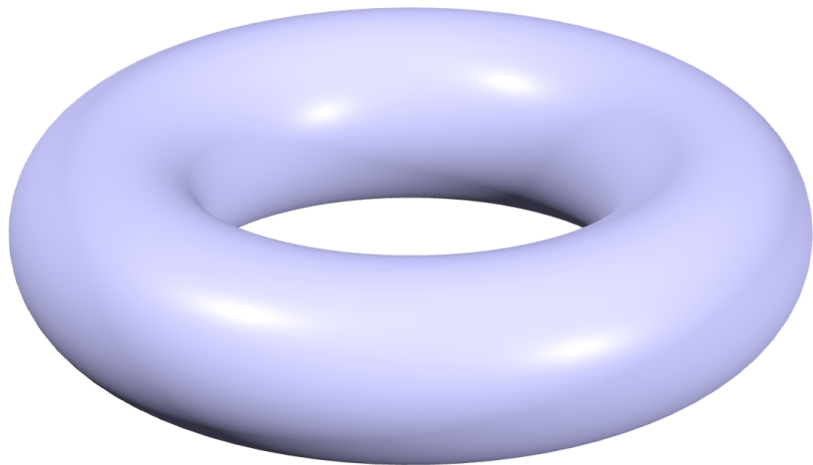
Question

What about Σ_1^0 ?

Solenoid

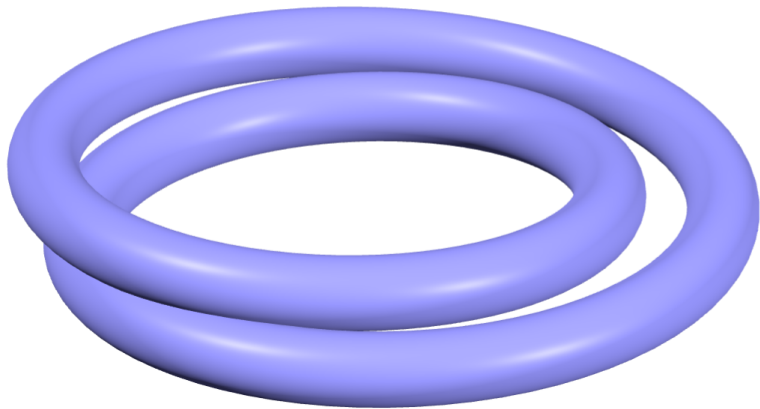


Solenoid



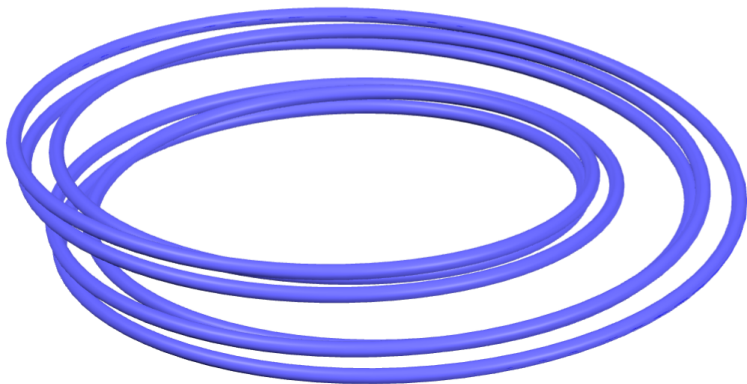
$$A = \{2, 3, 5, \dots\}$$

Solenoid



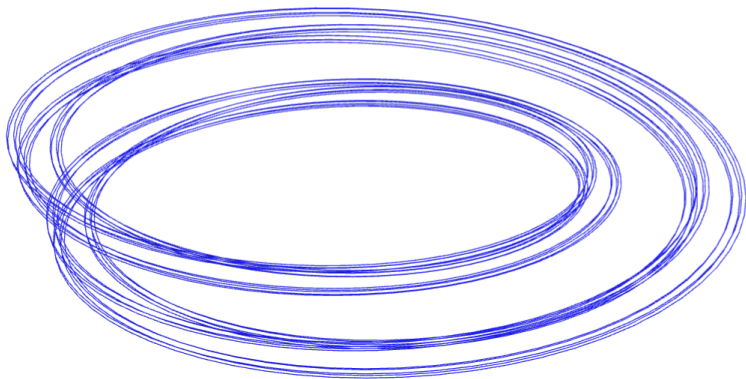
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$$A = \{2, \mathbf{3}, 5, \dots\}$$

Solenoid

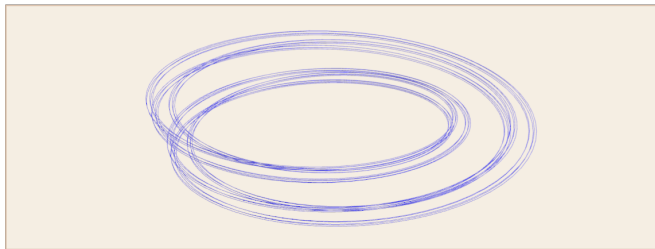


$$A = \{2, 3, 5, \dots\}$$

Solenoid

$$A = \{2, 3, 5, \dots\}$$

Alice sends $\text{Solenoid}(2, 3, 5, \dots)$:



Bob can enumerate A from any presentation of that space.

Solenoid

Let $p_0, p_1, \dots$ be the prime numbers. For $A = \{a_0, a_1, \dots\} \subseteq \mathbb{N}$, let

$$X_A = \text{Solenoid}(p_{a_0}, p_{a_1}, \dots).$$

Theorem (Lupini, Melnikov, Nies 2023)

An oracle Z computes a presentation of $X_A \iff A$ is Σ_1^0 relative to Z .

Proof.

Use cohomology...

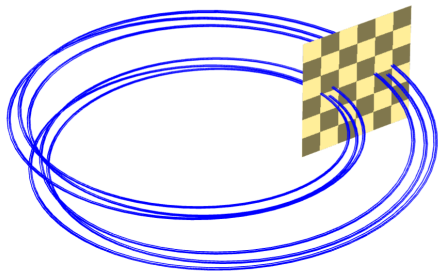


Solenoid: cohomology

To each space X is associated a group $\check{H}^1(X)$, the **Čech cohomology** group.

Elements of $\check{H}^1(X)$:

Ways of counting turns. Operators assigning a number to each path.

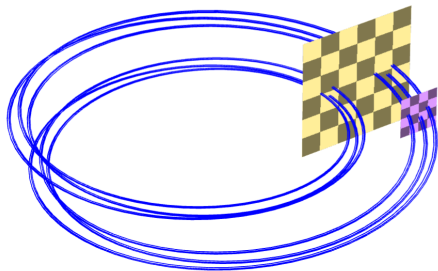


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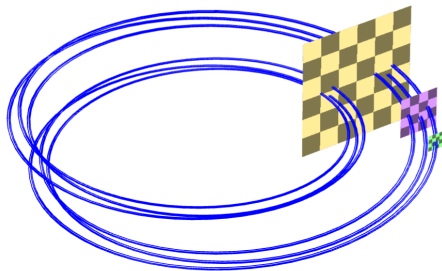


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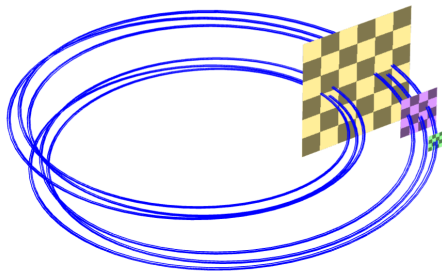


Solenoid: cohomology

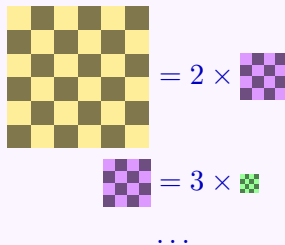
To each space X is associated a group $\check{H}^1(X)$, the **Čech cohomology** group.

Elements of $\check{H}^1(X)$:

Ways of counting turns. Operators assigning a number to each path.



Group relations:



Solenoid: cohomology

$$n \in A \iff \begin{array}{|c|c|c|c|c|} \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \end{array} \text{ is divisible by } p_n$$

$$\iff \exists x \in \check{H}^1(X_A) \text{ such that } \begin{array}{|c|c|c|c|c|} \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\ \hline \end{array} = p_n \times x.$$

Solenoid: cohomology

$$n \in A \iff \begin{array}{|c|c|c|c|c|} \hline \text{grid} \\ \hline \end{array} \text{ is divisible by } p_n$$

$$\iff \exists x \in \check{H}^1(X_A) \text{ such that } \begin{array}{|c|c|c|c|c|} \hline \text{grid} \\ \hline \end{array} = p_n \times x.$$

Theorem (Lupini, Melnikov, Nies 2023)

A presentation of $\check{H}^1(X)$ can be computed given X .

Corollary

A can be enumerated given X_A .

Alice goes pointillist

Summary

Information can be encoded in spaces presented using positive + negative information.

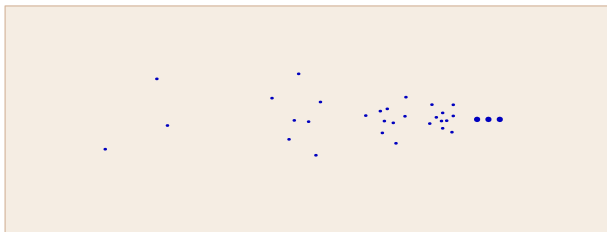
Alice goes pointillist

Summary

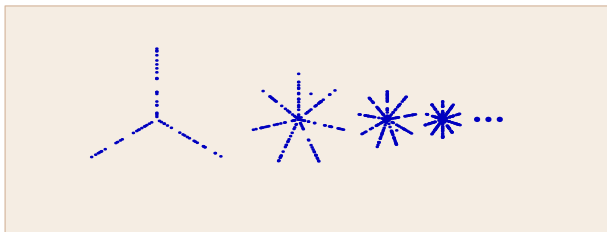
Information can be encoded in spaces presented using positive + negative information.

What about using positive information only?

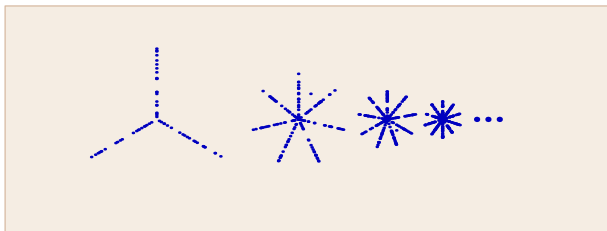
Alice goes pointillist



Alice goes pointillist



Alice goes pointillist



Polish presentation of X :

- A compact set $K \subseteq [0, 1]^n$ homeomorphic to X ,
- Enumeration of $\{i \in \mathbb{N} : B_i \cap K \neq \emptyset\}$ ($\simeq$ a dense sequence in K).

Alice goes pointillist

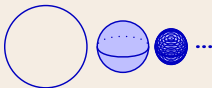
Σ_3^0



Σ_4^0

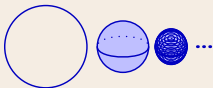
Alice goes pointillist

 Σ_3^0

 Σ_4^0
 Σ_2^0

 Σ_3^0

Alice goes pointillist

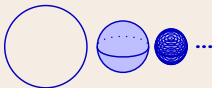
 Σ_3^0

 Σ_4^0
 Σ_2^0

 Σ_3^0
 Σ_1^0

 Σ_2^0

Alice goes pointillist

 Σ_3^0

 Σ_4^0
 Σ_2^0

 Σ_3^0
 Σ_1^0

 Σ_2^0

???

 $\Sigma_1^{0?}$

Alice goes pointillist

Polish presentations cannot encode information in a Σ_1^0 way.

Theorem (H., Kihara, Selivanov, 2020)

Let $A \subseteq \mathbb{N}$ not Σ_1^0 . There is no space X such that A is Σ_1^0 relative to any Polish presentation of X .

Computable type

Encoding information in a space

Countably-based spaces

Countably-based spaces

- Compact metrizable spaces embed in $[0, 1]^{\mathbb{N}}$,
- Countably-based T_0 -spaces embed in $\{0, 1\}^{\mathbb{N}} = P(\mathbb{N})$ endowed with the **Scott topology**, whose basis is:

$$B_{\langle i_0, \dots, i_n \rangle} = \{A \subseteq \mathbb{N} : \{i_0, \dots, i_n\} \subseteq A\}.$$

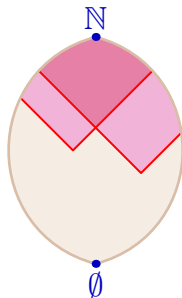


Figure: The Scott domain

Countably-based spaces

Let X be a countably-based T_0 -space.

Definition

A **presentation** of X is given by:

- A set $Y \subseteq \{0, 1\}^{\mathbb{N}}$ homeomorphic to X ,
- Positive + negative information: characteristic function of $\{i \in \mathbb{N} : B_i \cap Y \neq \emptyset\}$.

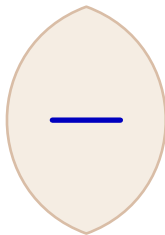
Countably-based spaces

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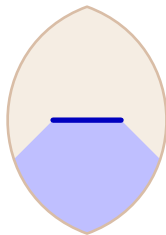
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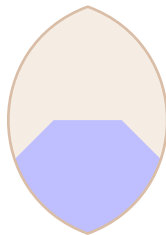
- A set $Y \subseteq \{0, 1\}^{\mathbb{N}}$ homeomorphic to X ,
- Positive + negative information: characteristic function of $\{i \in \mathbb{N} : B_i \cap Y \neq \emptyset\}$.



(a) Y



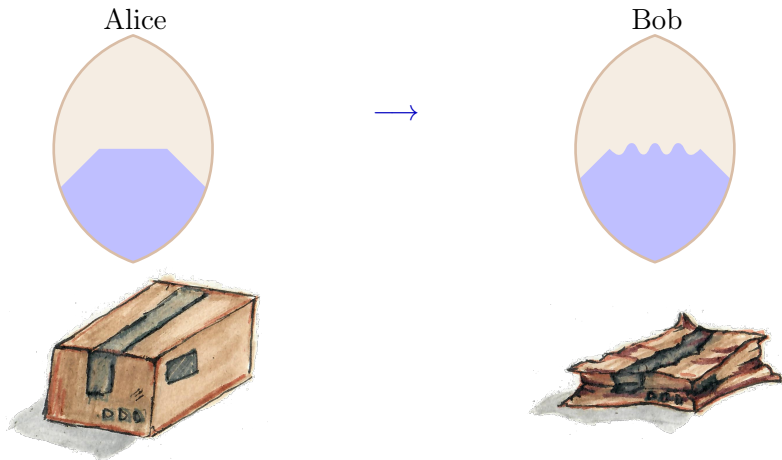
(b) Y and its closure



(c) Presentation

Countably-based space

Can we encode information in a countably-based space presented this way?



Countable-based spaces

Theorem (H., Melnikov, Ng 2025)

Every countably-based space has a computable presentation.

Countable-based spaces

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Theorem (H., Melnikov, Ng 2025)

All the perfect compact metrizable spaces share a computable presentation.

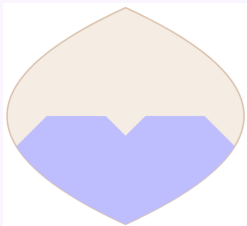
Proof.

They all “look like” the Cantor space.



Countably-based spaces

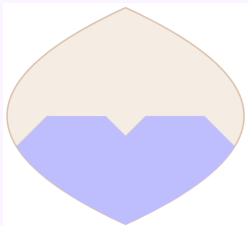
Presentation



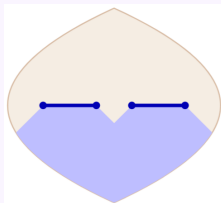
Actual space?

Countably-based spaces

Presentation

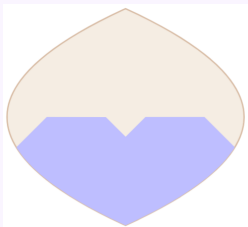


Actual space? two segments?

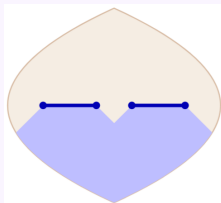


Countably-based spaces

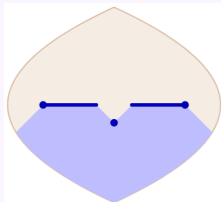
Presentation



Actual space? two segments?

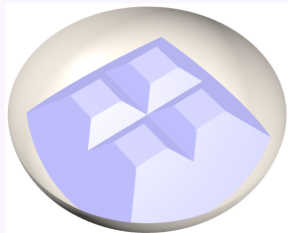


or one segment?



Countably-based spaces

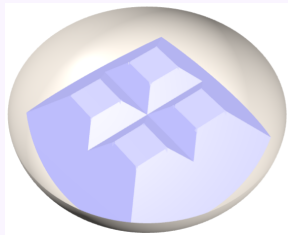
Presentation



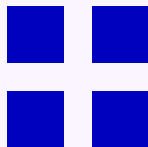
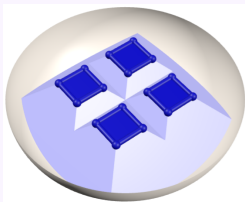
Actual space?

Countably-based spaces

Presentation

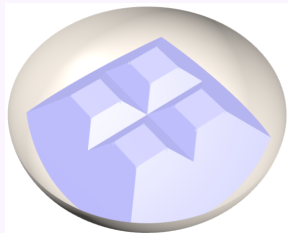


Actual space? four squares?

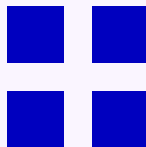
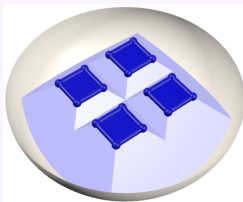


Countably-based spaces

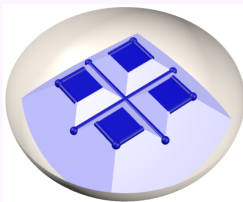
Presentation



Actual space? four squares?



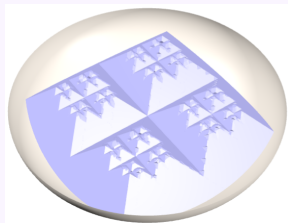
or just one?



Countably-based spaces

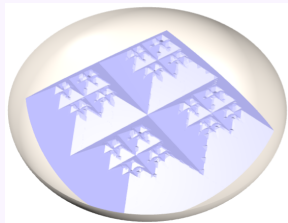
Actual space?

Presentation

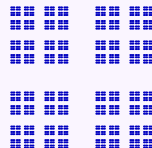
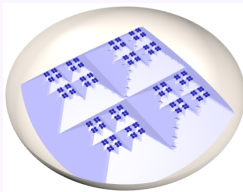


Countably-based spaces

Presentation

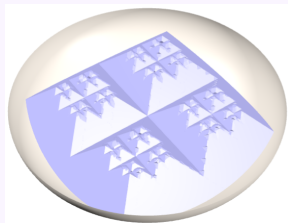


Actual space? Cantor space?

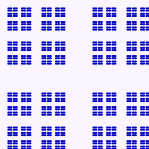
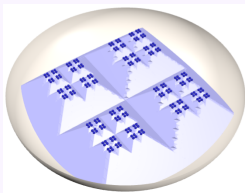


Countably-based spaces

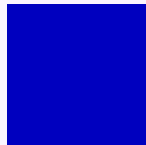
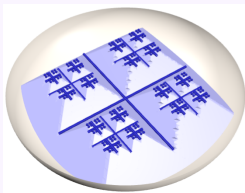
Presentation



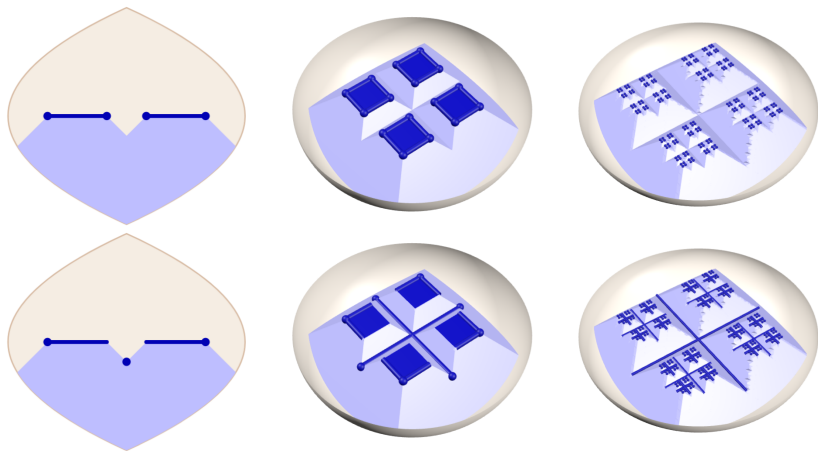
Actual space? Cantor space?



or the square?



Countably-based spaces



The lower spaces are obtained from the upper spaces by **identifying a few points**.

Countably-based spaces

Every compact metrizable space X without isolated points can be obtained from the Cantor space by identifying a few points.

Countably-based spaces

Every compact metrizable space X without isolated points can be obtained from the Cantor space by identifying a few points.

- If $f : 2^{\mathbb{N}} \rightarrow X$ is surjective continuous then X is the quotient of $2^{\mathbb{N}}$ by this equivalence relation:

$$p \sim q \stackrel{\Delta}{\iff} f(p) = f(q).$$

Countably-based spaces

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- If $f : 2^{\mathbb{N}} \rightarrow X$ is surjective continuous then X is the quotient of $2^{\mathbb{N}}$ by this equivalence relation:

$$p \sim q \stackrel{\Delta}{\iff} f(p) = f(q).$$

- There exists f such that for p in a dense set,

$$[p]_{\sim} = \{p\}.$$

Countably-based spaces

Theorem (H., Melnikov, Ng 2025)

All the perfect compact metrizable spaces share a computable presentation.

Proof idea.

- Let X be compact metrizable space without isolated points,
- X can be obtained from the Cantor space by identifying points,
- The canonical presentation of the Cantor space is also a presentation of X .



Countably-based spaces

- Too many countably-based spaces $\rightarrow$ no good notion of presentation,

Countably-based spaces

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- The quasi-Polish spaces [de Brecht, 2013] are the “complete” ones. They have many equivalent representations, studied by:
 - de Brecht (2013),
 - de Brecht (2020),
 - de Brecht, Pauly, Schröder (2020),
 - H., Rojas, Selivanov, Stull (2020),
 - de Brecht, Kihara, Selivanov (2024).

Question

What information can be encoded in such spaces?

Countably-based spaces

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Thank you!