Internship subject

Subject. Descriptive complexity of topological invariants

Topic. Computability, topology

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Context. A central theme in topology is to find topological invariants separating non-equivalent (non-homeomorphic) topological spaces. From a logical or computational viewpoint, most natural topological invariants are complex, in the sense that they are difficult to describe using a logical formula, or difficult to test with an algorithm (see [2] for instance). This internship is part of a research project exploring the expressiveness of topological invariants having low complexity.

Goals. We propose several directions. If one fixes two concrete spaces, what is the minimal complexity of an invariant that separates them? If one fixes a particular class of spaces, what level of complexity is required to distinguish all these spaces? If one fixes a particular space, what is the complexity of the invariant "being homeomorphic to that space"?

We recently obtained some answers to these questions, for spaces such as finite topological graphs [1]. The student will focus on spaces of dimension 2 or higher. For instance, he/she will try to determine the optimal complexity of separating the disk from the space consisting of two disks attached together (Figure 1), and the complexity of recognizing the circle.

(a) The disk (b) Two attached disks

Figure 1: What is the complexity of separating these two spaces ?

More details. A way of measuring the complexity of an invariant is to express it by a logical formula using the predicates "intersecting some open set" and "being contained in some open set", and to count the number of alternating quantifiers. The methods allowing to obtain such formulas and to show their optimality lie at the interaction between computability and topology.

Prerequisite. We require basic knowledge is computability (computable function, computable set, recursively enumerable set). Despite the topological nature of the subject, we do not require knowledge in topology.

References

[1] Djamel Eddine Amir and Mathieu Hoyrup. Descriptive complexity of topological invariants. Preprint, 2023.

[2] Gabriel Debs and Jean Saint Raymond. The descriptive complexity of connectedness in Polish spaces. *Fundamenta Mathematicae*, 249(3):261–286, 2020.