Computational Geometry Algorithms Library

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## Course Outline

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Overview

Andreas Fabri
GeometryFactory
Mission Statement

“Make the large body of geometric algorithms developed in the field of computational geometry available for industrial applications”

CGAL Project Proposal, 1996
Algorithms and Datastructures

Bounding Volumes
Polyhedral Surface
Boolean Operations
Triangulations
Voronoi Diagrams
Mesh Generation
Subdivision
Simplification
Parametrisation
Streamlines
Ridge Detection
Neighbor Search
Kinetic Datastructures
Lower Envelope
Arrangement
Intersection Detection
Minkowski Sum
PCA
Polytope distance
QP Solver
CGAL in Numbers

500,000 lines of C++ code
10,000 downloads/year (+ Linux distributions)
3,500 manual pages
3,000 subscribers to cgal-announce
1,000 subscribers to cgal-discuss
120 packages
  60 commercial users
  20 active developers
12 months release cycle
  2 licenses: Open Source and commercial
Some Commercial Users

cadence™
pulsic
VDRC
TOSHIBA
Orbotech
INDUSTRIAL RESEARCH LIMITED
Agilent Technologies
BAE SYSTEMS
QinetiQ
Leica Geosystems
SAFE SOFTWARE
rm DATA
inpho
The MathWorks
TGS
SAFE SOFTWARE
Data
NN weathernews
Midland Valley
TruePosition
France Telecom
BP
BSAP

Image Processing
VLSI
CAD/CAM
Digital maps
GIS
Geophysics (Oil&Gas)
Telecom
Scientific visualization
Medical
Why They Use CGAL

“ I recommended to the senior management that we start a policy of buying-in as much functionality as possible to reduce the quantity of code that our development team would have to maintain.

This means that we can concentrate on the application layer and concentrate on our own problem domain.”

Senior Development Engineer
& Structural Geologist

Midland Valley Exploration
Why They Use CGAL

“ My research group JYAMITI at the Ohio State University uses CGAL because it provides an efficient and robust code for Delaunay triangulations and other primitive geometric predicates. Delaunay triangulation is the building block for many of the shape related computations that we do. [...] 

Without the robust and efficient codes of CGAL, these codes could not have been developed. ”

Tamal Dey
Professor, Ohio State University
CGAL Open Source Project
Project = « Planned Undertaking »

- Institutional members make a long term commitment: Inria, MPI, Tel-Aviv U, Utrecht U, Groningen U, ETHZ, GeometryFactory, FU Berlin, Forth, U Athens
- Editorial Board
  - Steers and animates the project
  - Reviews submissions
- Development Infrastructure
  - Gforge: svn, tracker, nightly test suite, ...
  - 120p developer manual and mailing list
  - Two 1-week developer meetings per year
Contributions

- Submission of specifications of new contributions
- Review and decision by the Editorial Board

- Value for contributor
  - Integration in the CGAL community
  - Gain visibility in a mature project
  - Publication value for accepted contributions
Exact Geometric Computing
Predicates and Constructions

Predicates

- orientation
- in_circle

Constructions

- intersection
- circumcenter
Robustness Issues

- Naive use of floating-point arithmetic causes geometric algorithms to:
  - Produce [slightly] wrong output
  - Crash after invariant violation
  - Infinite loop

- There is a gap between
  - Geometry in theory
  - Geometry with floating-point arithmetic
Geometry in Theory

\[
ccw(s,q,r) \land ccw(p,s,r) \land ccw(p,q,s) \Rightarrow ccw(p,q,r)
\]

Correctness proofs of algorithms rely on such theorems
The Trouble with Double

orientation(p,q,r) = sign((p_x - r_x)(q_y - r_y) - (p_y - r_y)(q_x - r_x))
Exact Geometric Computing  [Yap]

Make sure that the control flow in the implementation corresponds to the control flow with exact real arithmetic
Filtered Predicates

- Generic functor adaptor \texttt{Filtered\_predicate<>}
  - Try the predicate instantiated with intervals
  - In case of uncertainty, evaluate the predicate with multiple precision arithmetic

- Refinements:
  - Static error analysis
  - Progressively increase precision
Filtered Constructions

Lazy number = interval and arithmetic expression tree

\[(3.2 + 1.5) \times 13\]

Test that may trigger an exact re-evaluation:

if \((n' < m')\)

Lazy object = approximated object and geometric operation tree

if \((\text{collinear}(a', m', b'))\)
The User Perspective

- **Convenience Kernels**
  - `Exact_predicates_inexact_constructions_kernel`
  - `Exact_predicates_exact_constructions_kernel`
  - `Exact_predicates_exact_constructions_kernel_with_sqrt`

- **Number Types**
  - `double, float`
  - `CGAL::Gmpq (rational), Core (algebraic)`
  - `CGAL::Lazy_exact_nt<ExactNT>`

- **Kernels**
  - `CGAL::Cartesian<NT>`
  - `CGAL::Filtered_kernel<Kernel>`
  - `CGAL::Lazy_kernel<NT>`
Merits and Limitations

- Ultimate robustness inside the black box
- The time penalty is reasonable, e.g. 10% for 3D Delauny triangulation of 1M random points
- Limitations of Exact Geometric Computing
  - Topology preserving rounding is non-trivial
  - Construction depth must be reasonable
  - Cannot handle trigonometric functions
Generic Programming
STL Genericity

template <class Key, class Less>
class set {
    Less less;

    insert(Key k) {
        if (less(k, treenode.key))
            insertLeft(k);
        else
            insertRight(k);
    }
};
CGAL Genericity

template < class Geometry >
class Delaunay_triangulation_2 {
    Geometry::Orientation orientation;
    Geometry::In_circle in_circle;

    void insert(Geometry::Point t) {
        ...
        if(in_circle(p,q,r,t)) {...}
        ...
        if(orientation(p,q,r){...}
    }
};
CGAL Genericity

template < class Geometry, class TDS >
class Delaunay_triangulation_2 {

};

template < class Vertex, class Face >
class Triangulation_data_structure_2 {

};
Iterators

template <class Geometry>
class Delaunay_triangulation_2 {

    typedef .. Vertex_iterator;
    typedef .. Face_iterator;

    Vertex_iterator vertices_begin();
    Vertex_iterator vertices_end();

    template <class OutputIterator>
    incident_faces(Vertex_handle v, OutputIterator it);
};

std::list<Face_handle> faces;
dt.incident_faces(v, std::back_inserter(faces));
Iterators

template <class Geometry>
class Delaunay_triangulation_2 {

    template <class T>
    void insert(T begin, T end); // typeof(*begin)==Point

};

list<Kernel::Point_2> points;

Delaunay_triangulation<Kernel> dt;

dt.insert(points.begin(), points.end());