Root solver using subdivision and Fast Fourier Transform

General information

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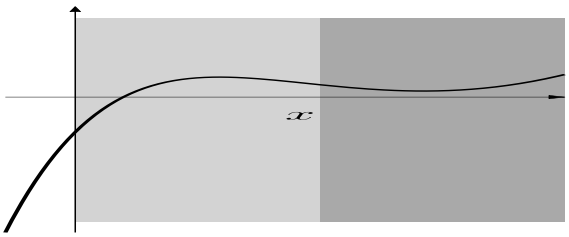
Motivations

In scientific computation, finding the roots of polynomial equations is a fundamental problem arising in robotics or visualization for example. Bisection is a standard numerical method to solve this problem [1,§2.1]. On the other hand, it requires to evaluate the input polynomials at each bisection step, and for a high degree polynomial p, evaluating p can be costly. Approaches based on Fast Fourier Transform can be used to evaluate a polynomial on several point for the cost of one evaluation [2,§8.2, Cor.10.8]. These approaches have never been used in numerical root-finding methods. The goal of this internship is to use multipoint evaluation approaches to improve significantly root-finding methods for univariate and bivariate polynomial equations.

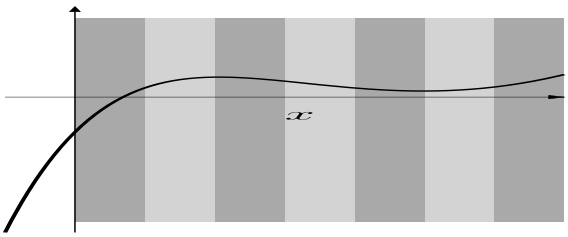
Subject

Let $p(x)$ be a polynomial of degree d with regular roots. For a interval I small enough, it is possible to decide if p has a root or not in I using evaluations. The bisection method consists in splitting the initial domain in two, recursively, until the intervals are small enough to decide if they contain a root of p or not.

Instead of splitting the initial domain in two parts (left figure), we would like to split it in d parts (right figure). Indeed, using Fast Fourier Transform, evaluating p in one point costs almost the same as evaluating p in d points [2, Cor. 10.8].



Bisecting the x-domain in two parts



Splitting the x-domain in d parts

During the internship, the candidate will first need to become familiar with interval arithmetic, subdivision and Fast Fourier Transform methods. Then, using existing interval arithmetic and Fast Fourier Transform library, the candidate will design and experiment new subdivision methods, and compare them with state-of-the-art approaches [Arb library[1] or Sage[2]]. This work will be first focused on univariate polynomials, and it may be extended to bivariate polynomial systems.

Profile of the candidate

The candidate should have a taste for both mathematics numerical analysis and computer science. Programming skills would be appreciated.

References

[1] Burden, R., and J. Faires. 2010. *Numerical Analysis*. Cengage Learning. http://ins.sjtu.edu.cn/people/mtang/textbook.pdf.

[2] Gathen, Joachim von zur, and Jürgen Gerhard. 2013. *Modern Computer Algebra*. 3rd ed. New York, NY, USA: Cambridge University Press.

[1] http://fredrikj.net/arb

[2] https://hal.archives-ouvertes.fr/hal-00846961v3