

# Computational geometry

Olivier Devillers  
Monique Teillaud

*Inria*

# Computational geometry

Olivier Devillers  
Monique Teillaud



Gamble team in Nancy

Geometric Algorithms and Models Beyond the Linear and Euclidean realm

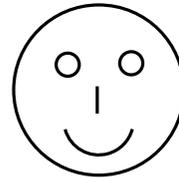
# Computational geometry

Olivier Devillers  
Monique Teillaud

*Inria*

Gamble team in Nancy

Interns welcome



Geometric Algorithms and Models Beyond the Linear and Euclidean realm

# Computational geometry

7 lectures of 3 hours

- 9-10 ● Intro: what is computational geometry.  
Convex hull: definitions, classical algorithms.
- 10-11 ● Delaunay Triangulation: definitions, motivations  
First properties and classical algorithms.
- 13-11 ● Randomized algorithms.  
Poisson Delaunay triangulation.
- 17-11 ● Numerical issues and algorithmic robustness.  
Degenerate cases and perturbation techniques.
- 20-11 ● Triangulations in the CGAL library.
- 24-11 ● Reconstruction. Meshing.
- 27-11 ● Periodic triangulations. Hyperbolic triangulations.

# Computational geometry

## Evaluation

Your grade will be in two pieces:

- Homework: exercises after each lecture.
- – Presentation of a research paper
  - or coding project using CGAL

8-12 Defense: 20 minutes ? (how many students ?)

# Computational geometry



# Computational geometry

Design geometric algorithms

# Computational geometry

Design geometric algorithms

Study complexity

# Computational geometry

Design geometric algorithms

Study complexity

Model of computation

Worst-case or random analysis

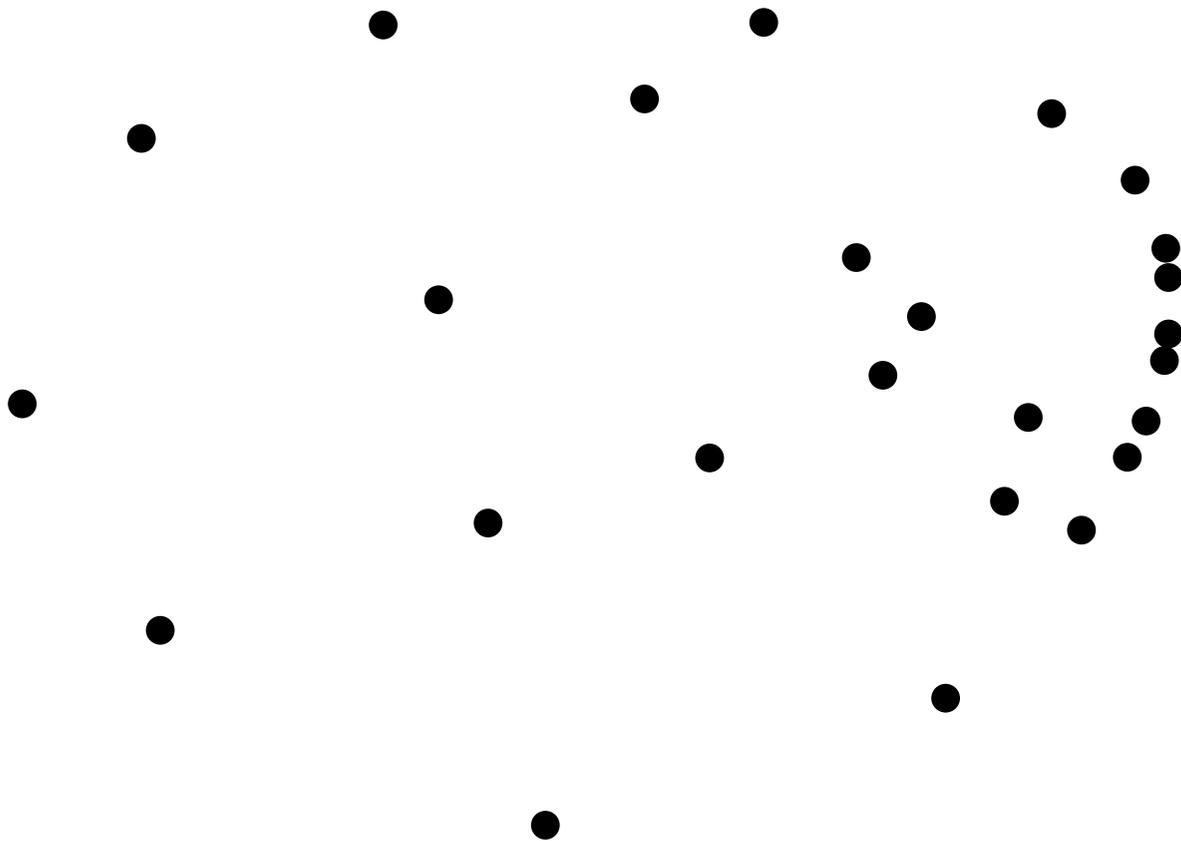
Lower bound

Asymptotic analysis

# Computational geometry problems

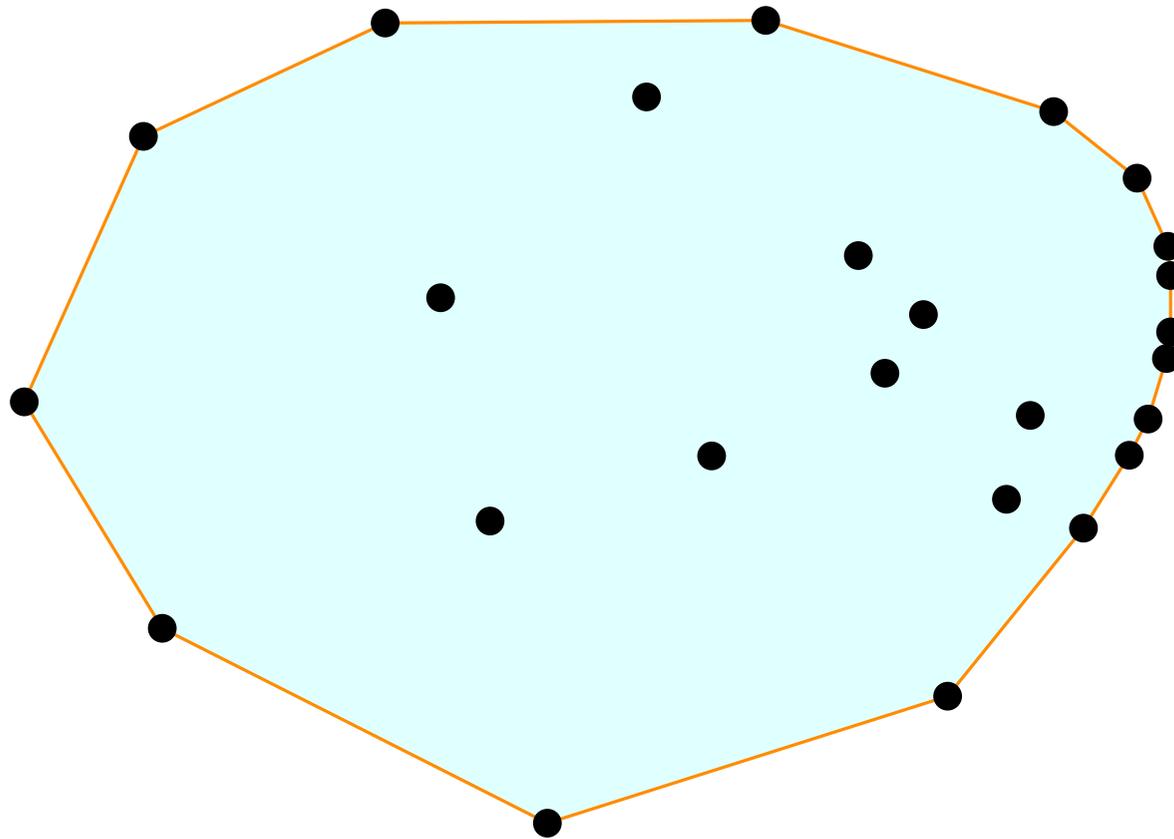
# Computational geometry problems

## Convex hull



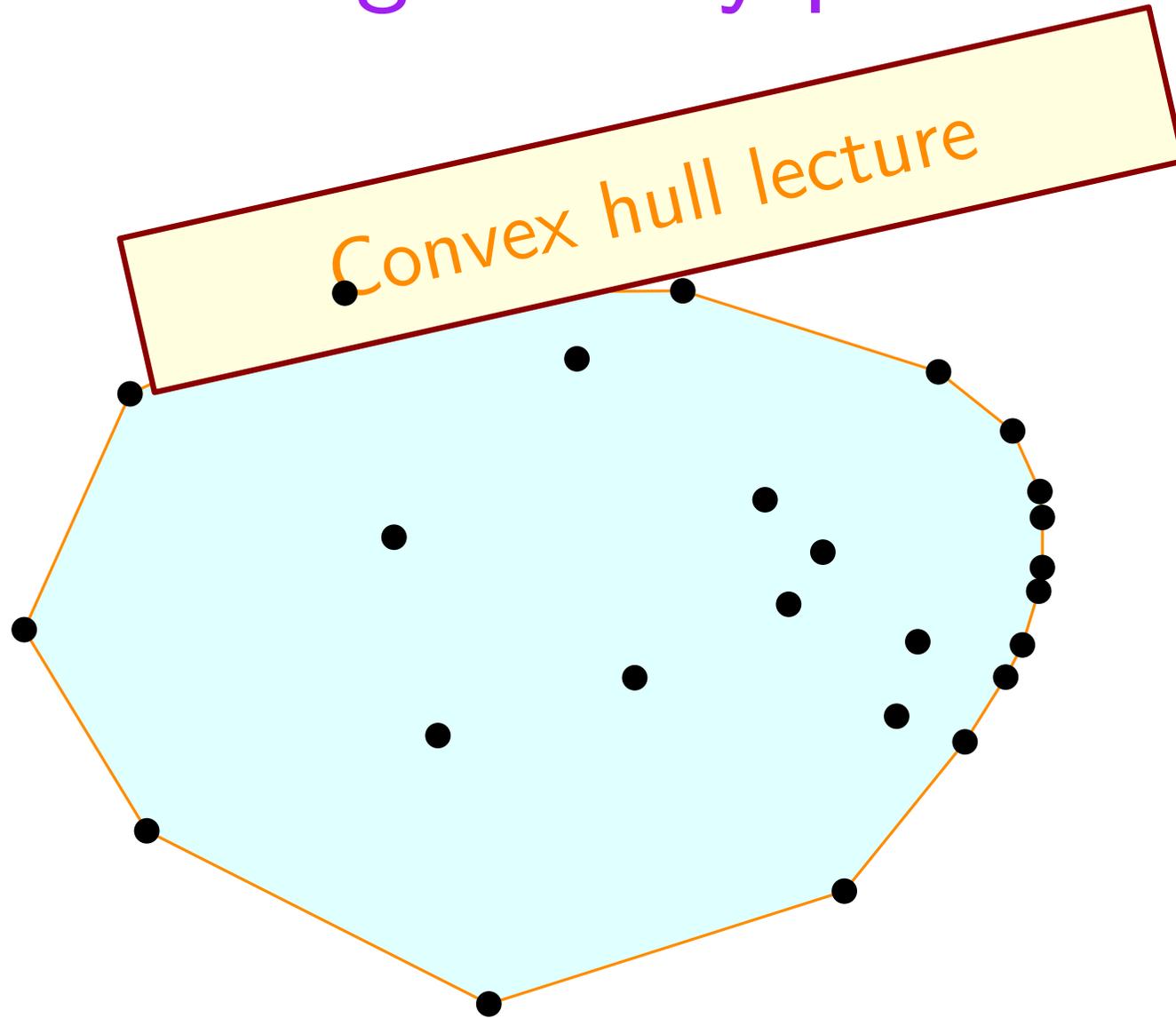
# Computational geometry problems

## Convex hull



# Computational geometry problems

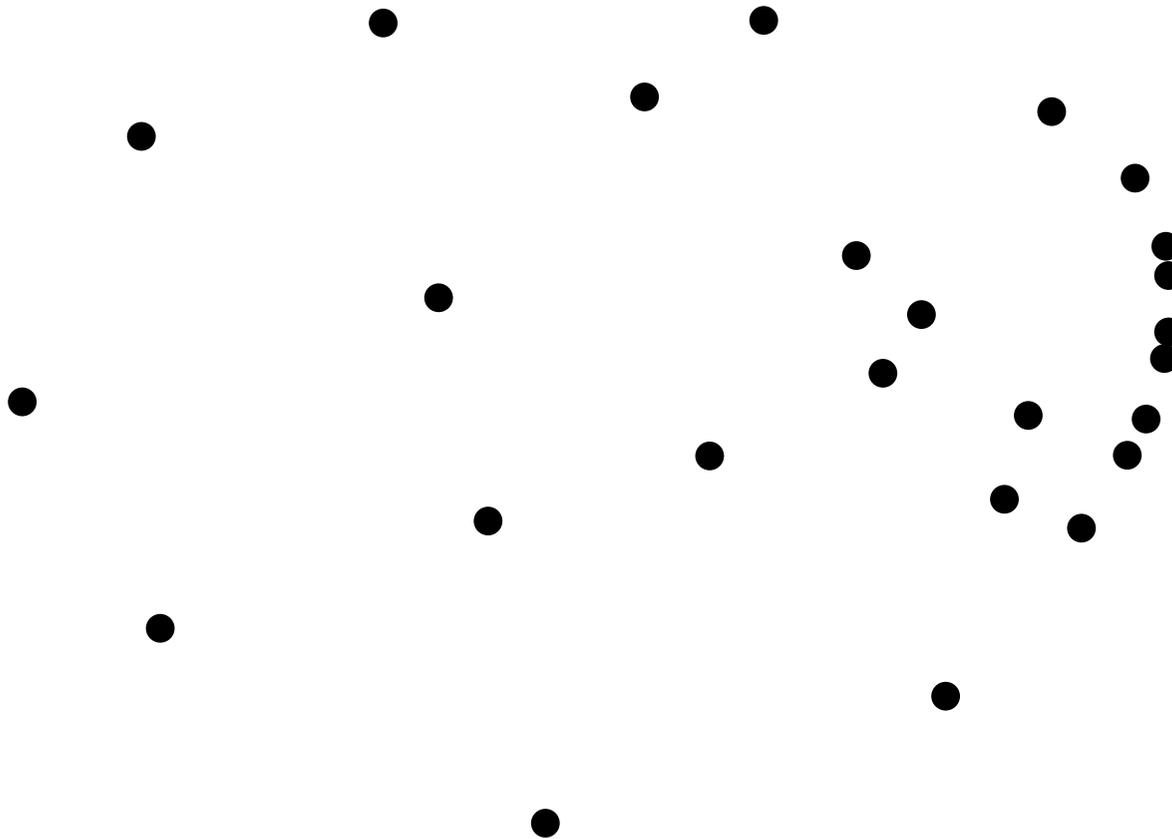
## Convex hull



# Computational geometry problems

Convex hull

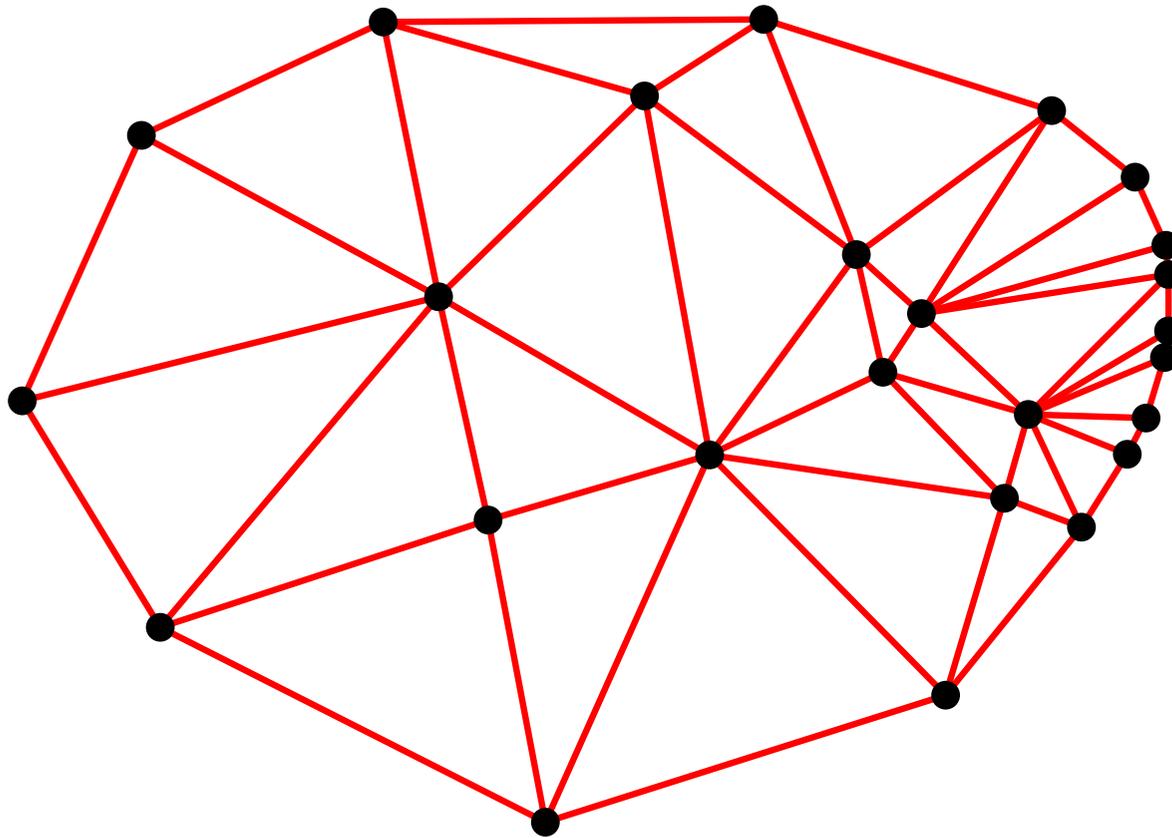
Delaunay triangulation / Voronoi diagrams



# Computational geometry problems

Convex hull

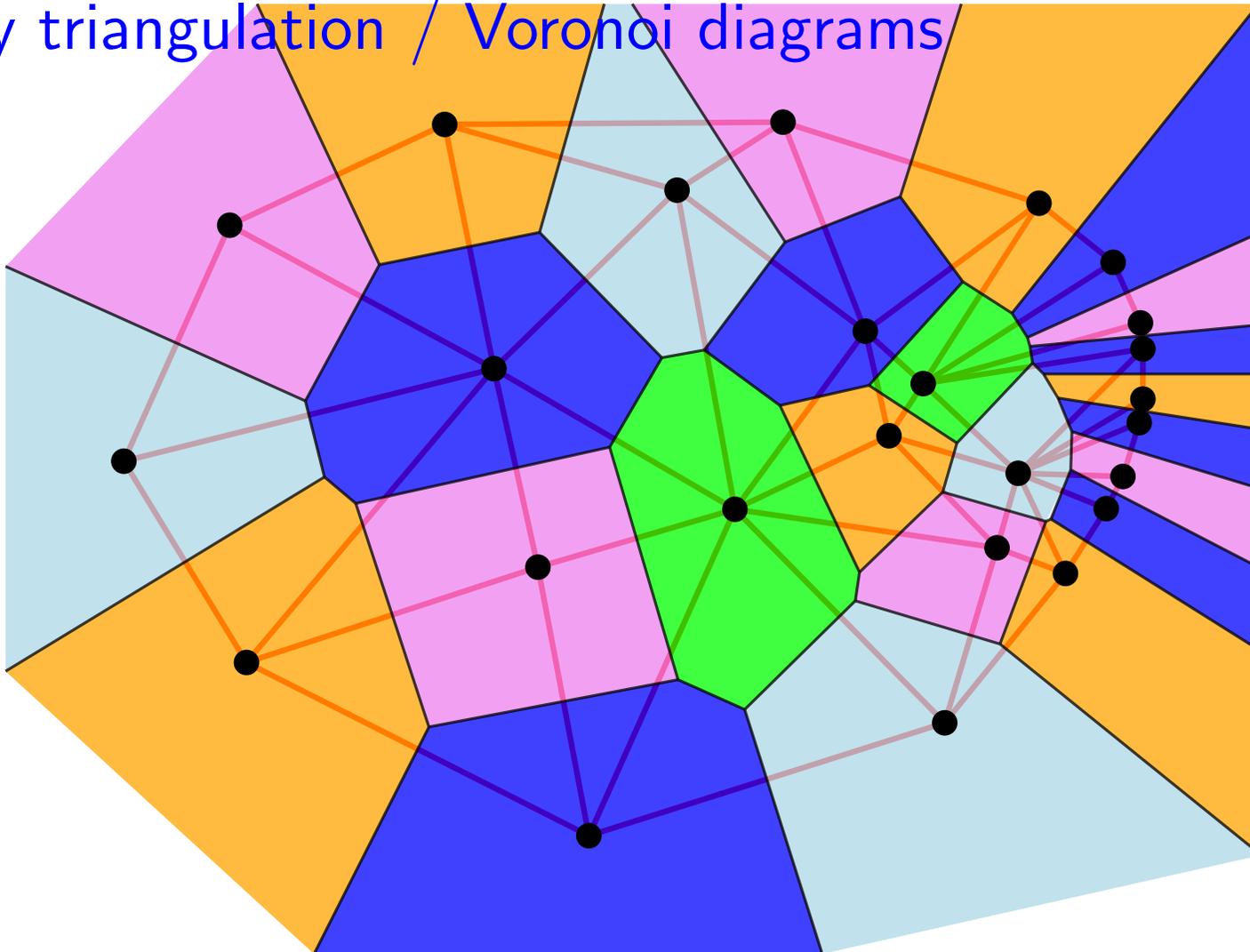
Delaunay triangulation / Voronoi diagrams



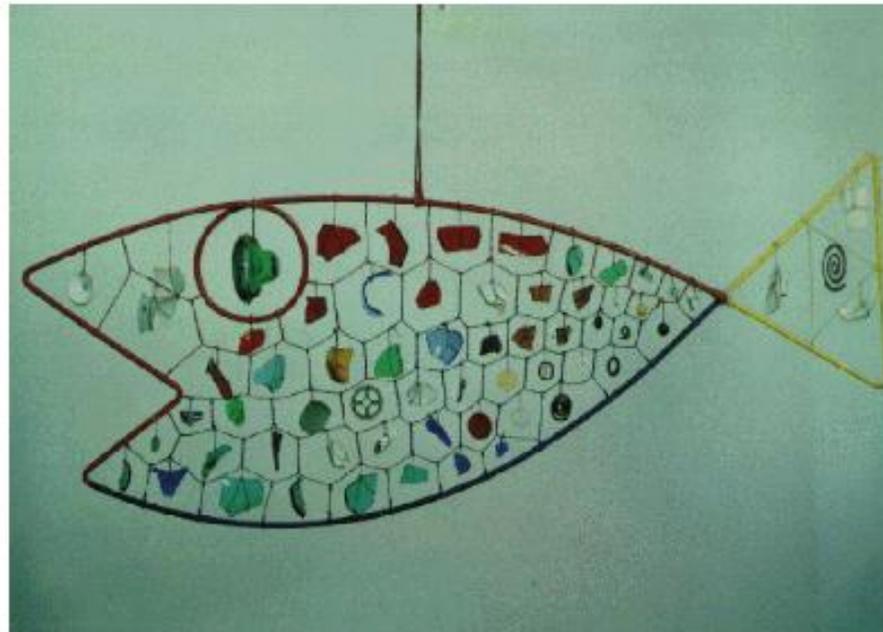
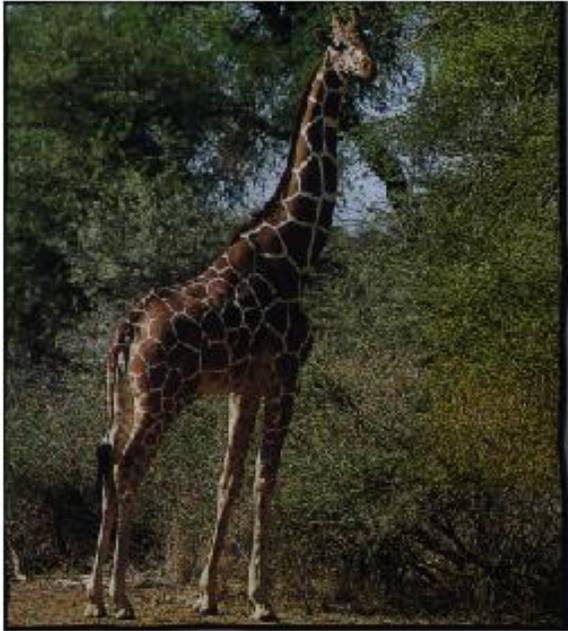
# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams



# Computational geometry problems



# Computational geometry problems



# Computational geometry problems



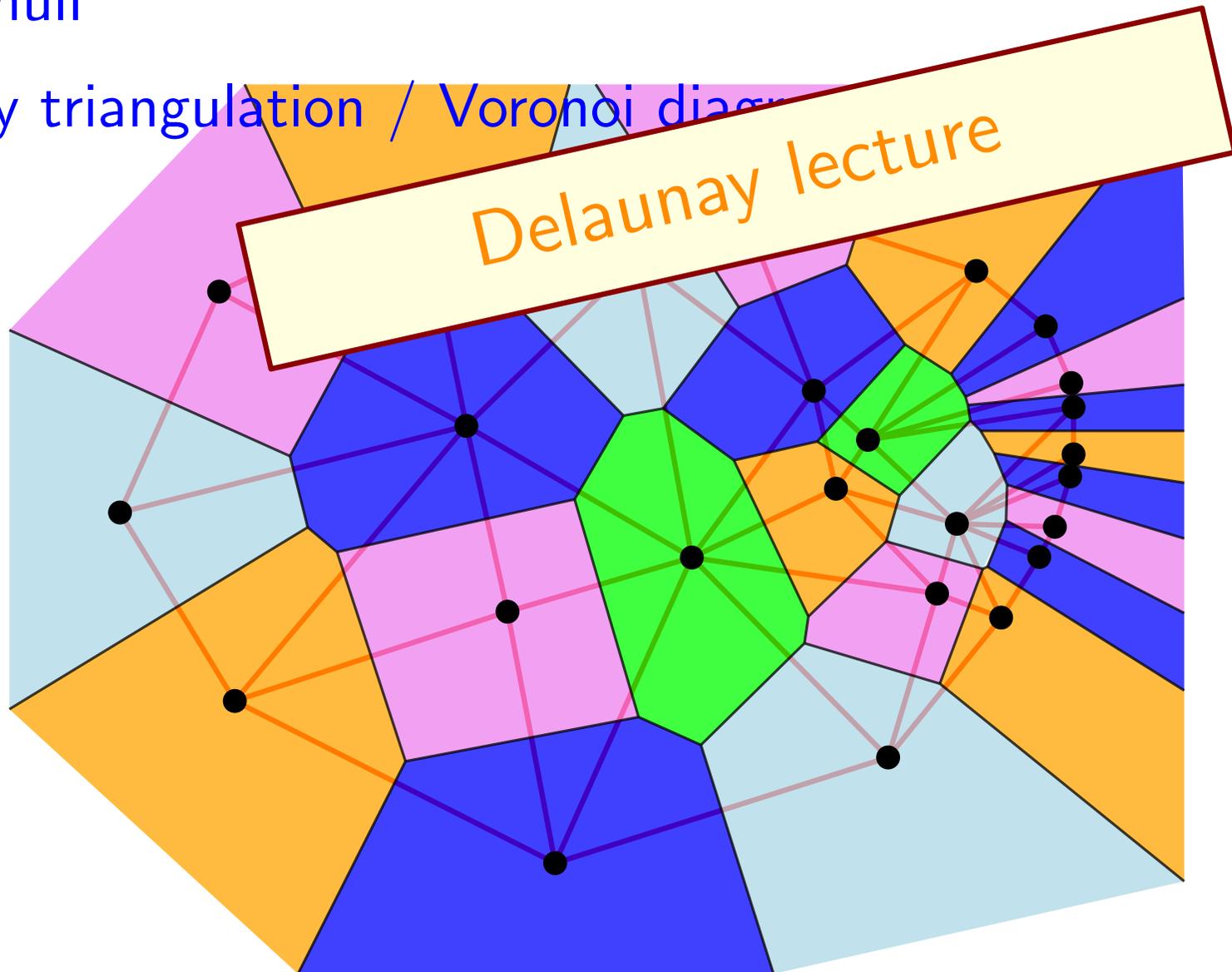
# Computational geometry problems



# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagram

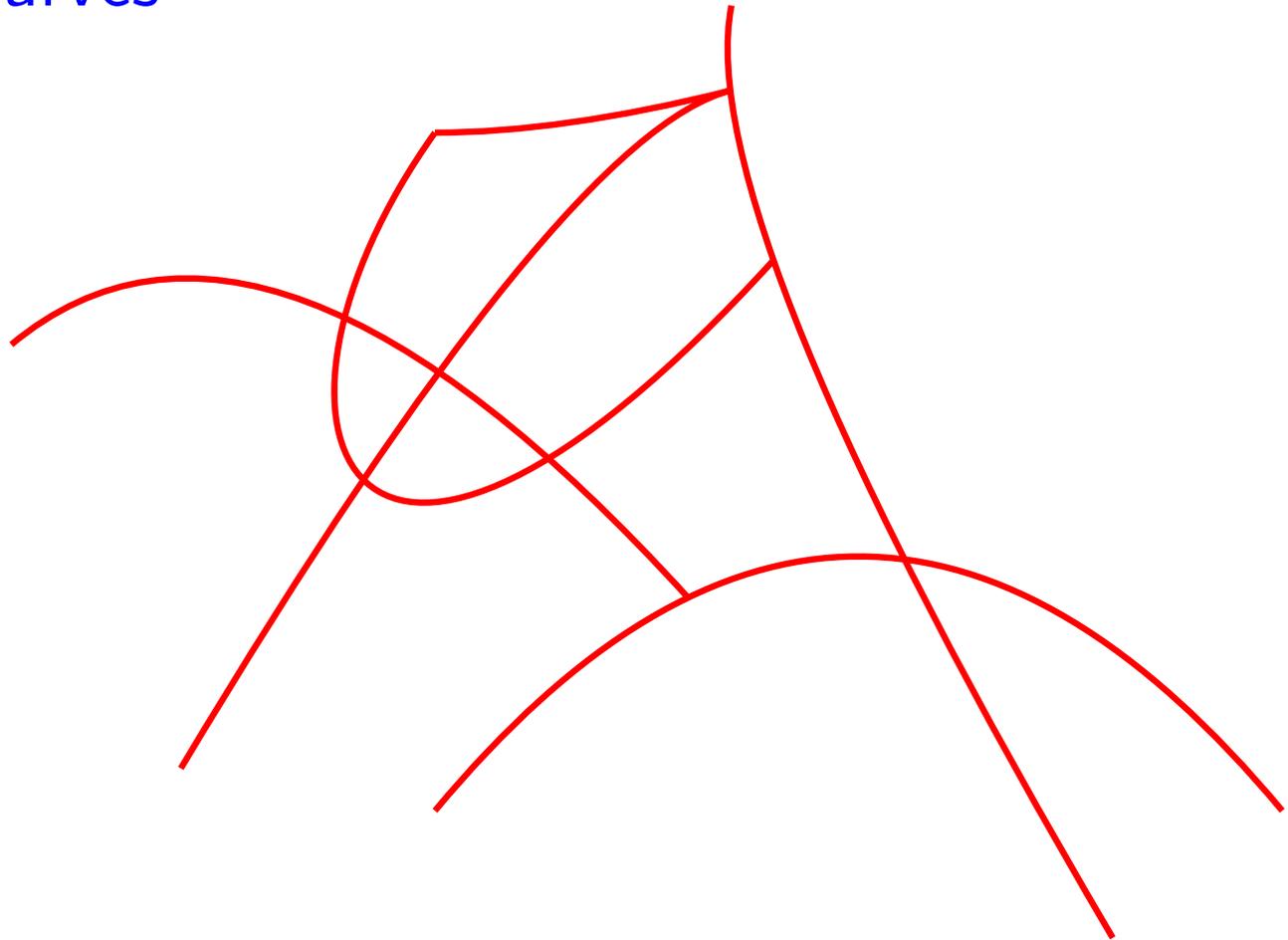


# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves

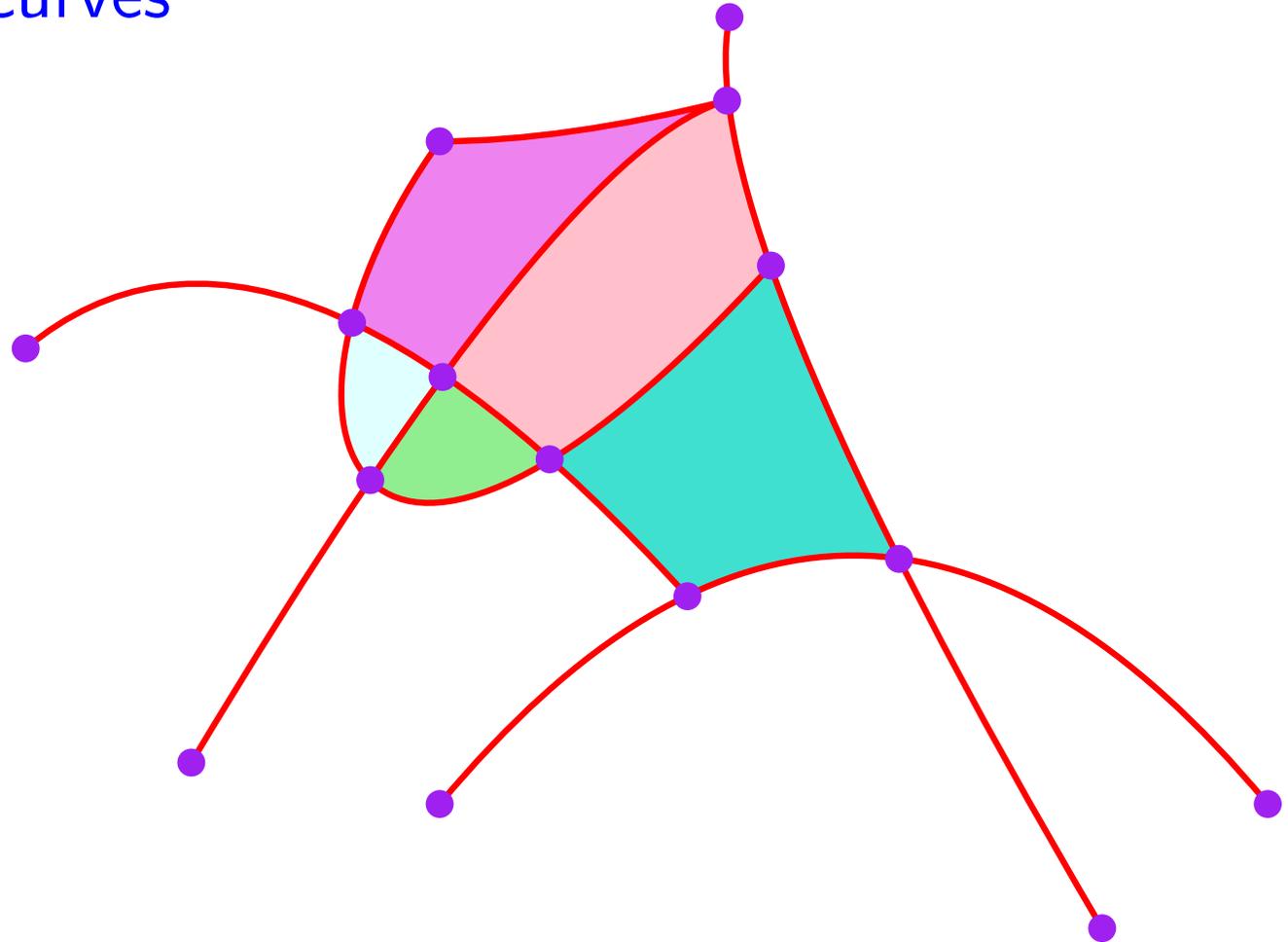


# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves



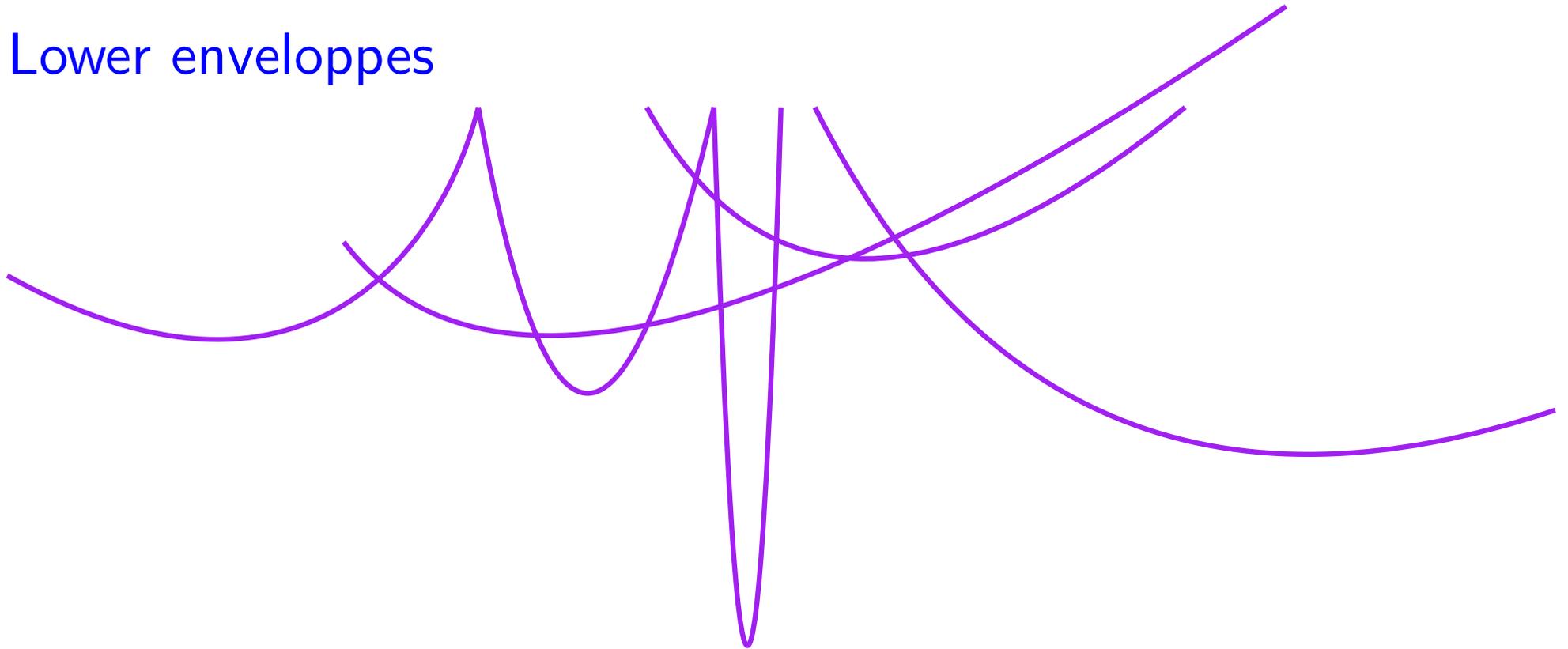
# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves

Lower envelopes



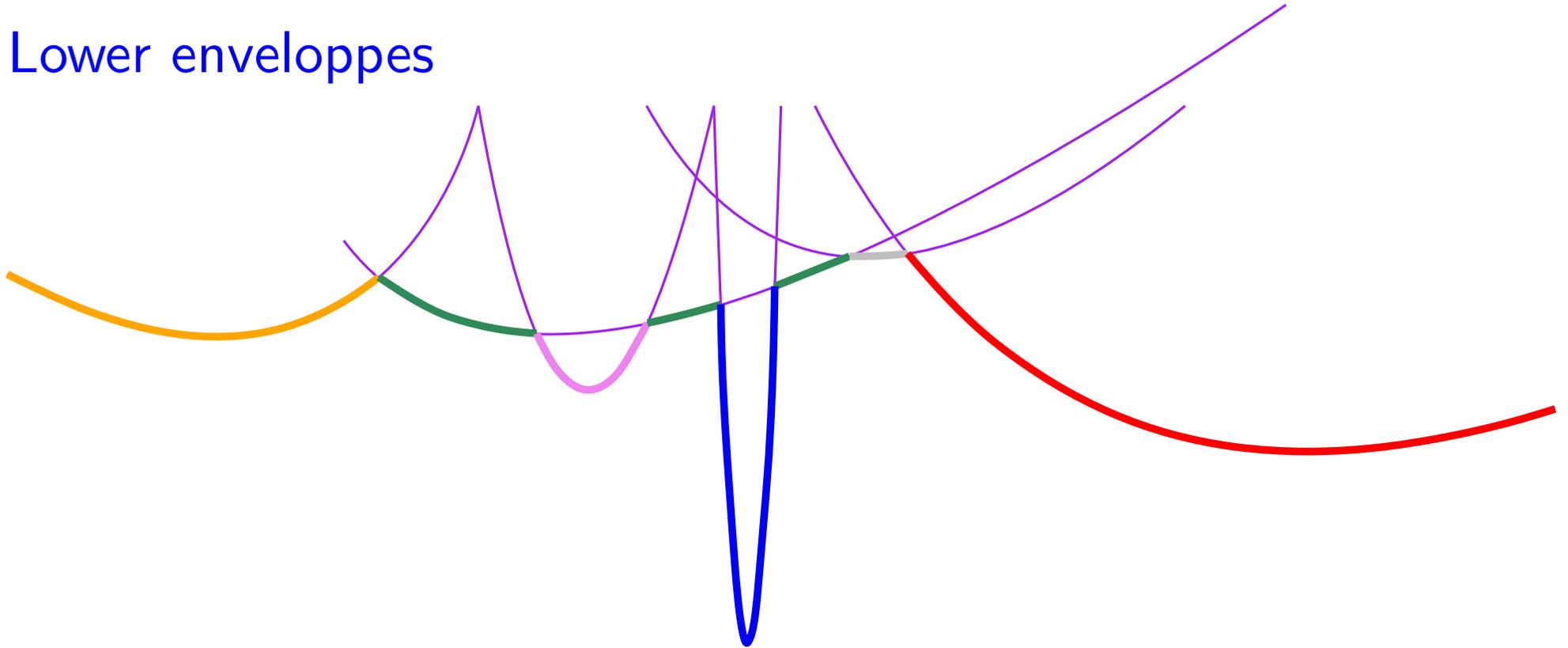
# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves

Lower envelopes



# Computational geometry problems

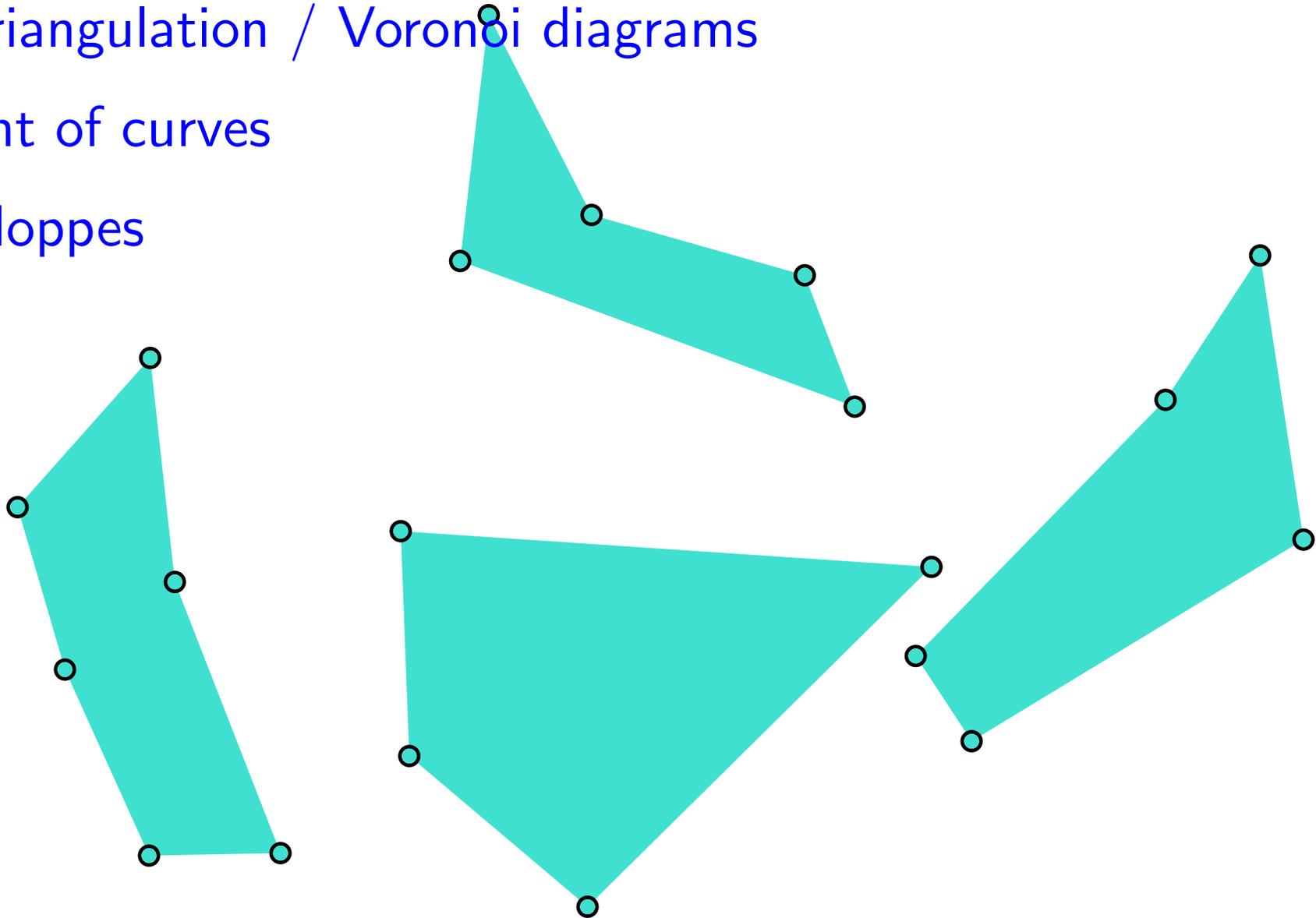
Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves

Lower envelopes

Visibility



# Computational geometry problems

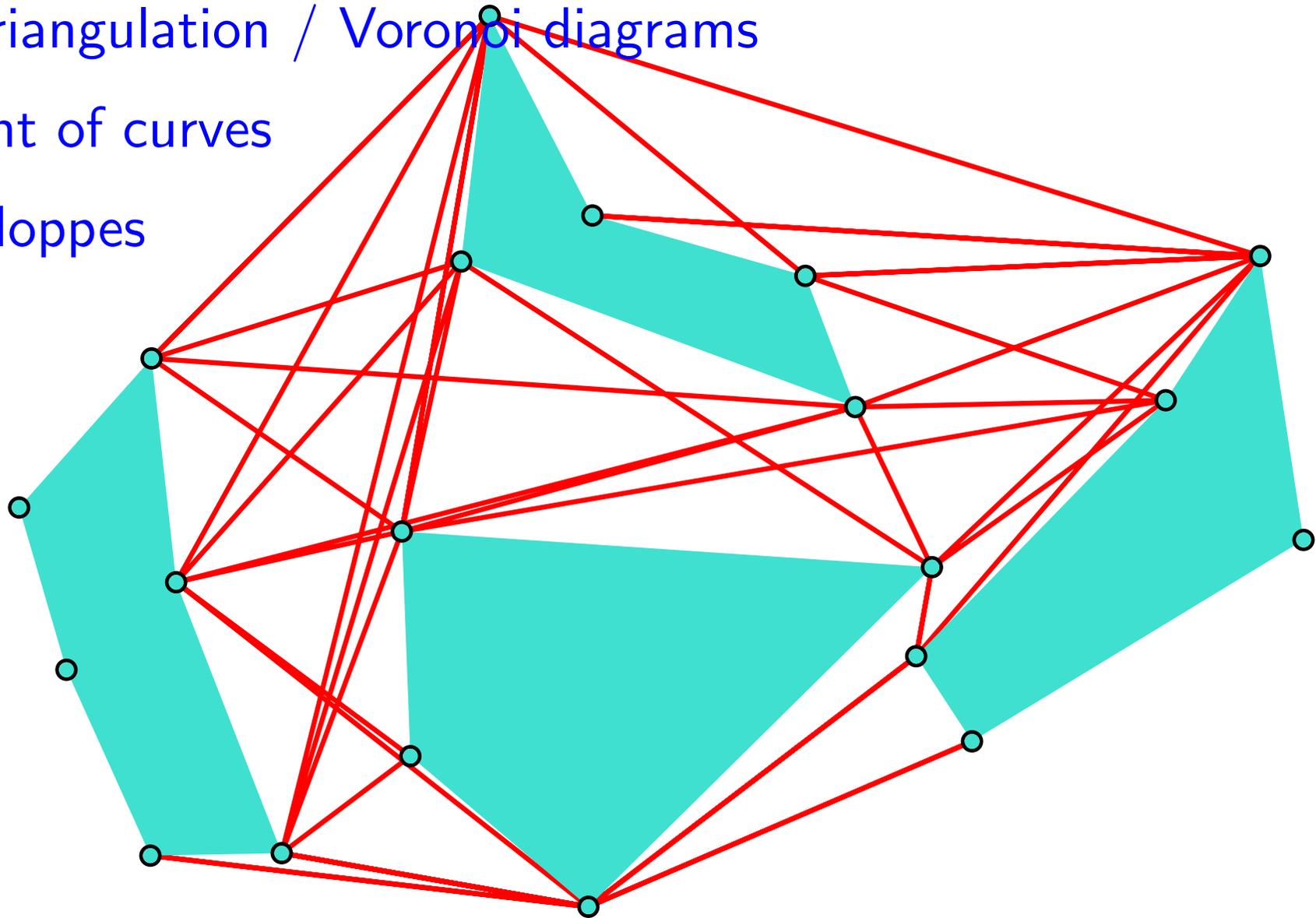
Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves

Lower envelopes

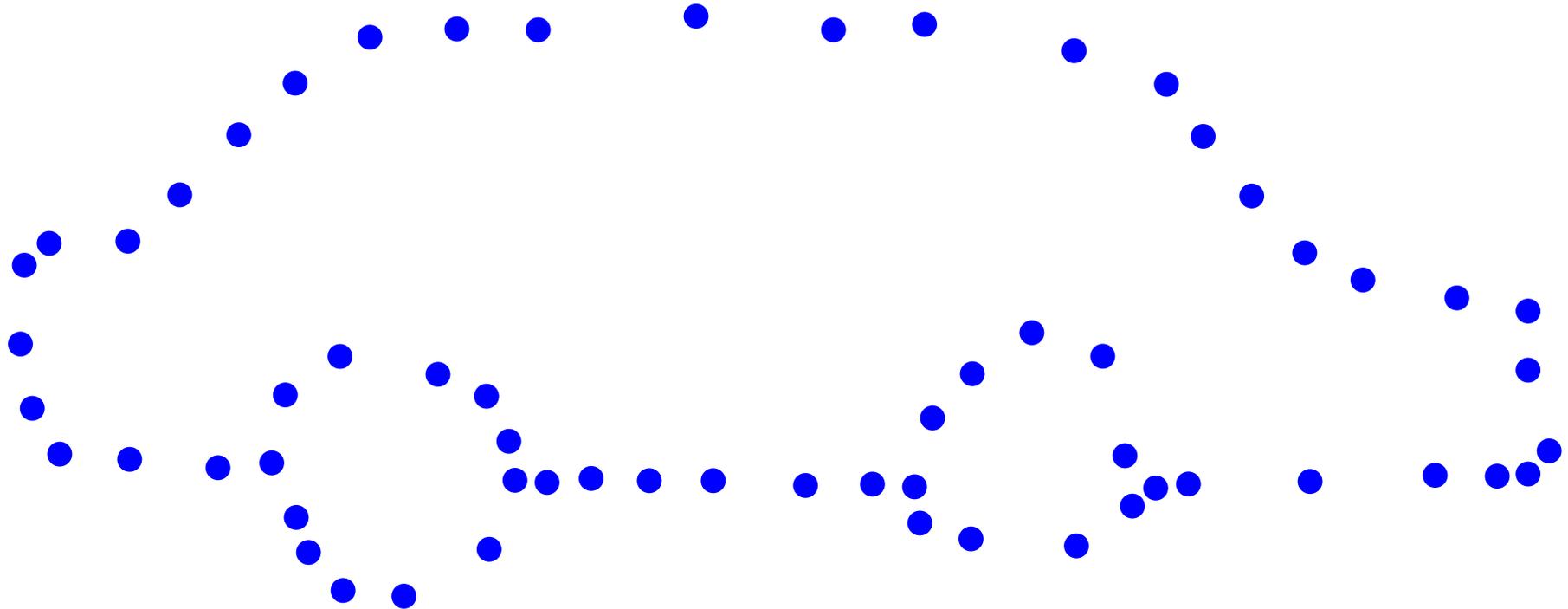
Visibility



# Computational geometry usage

# Computational geometry usage

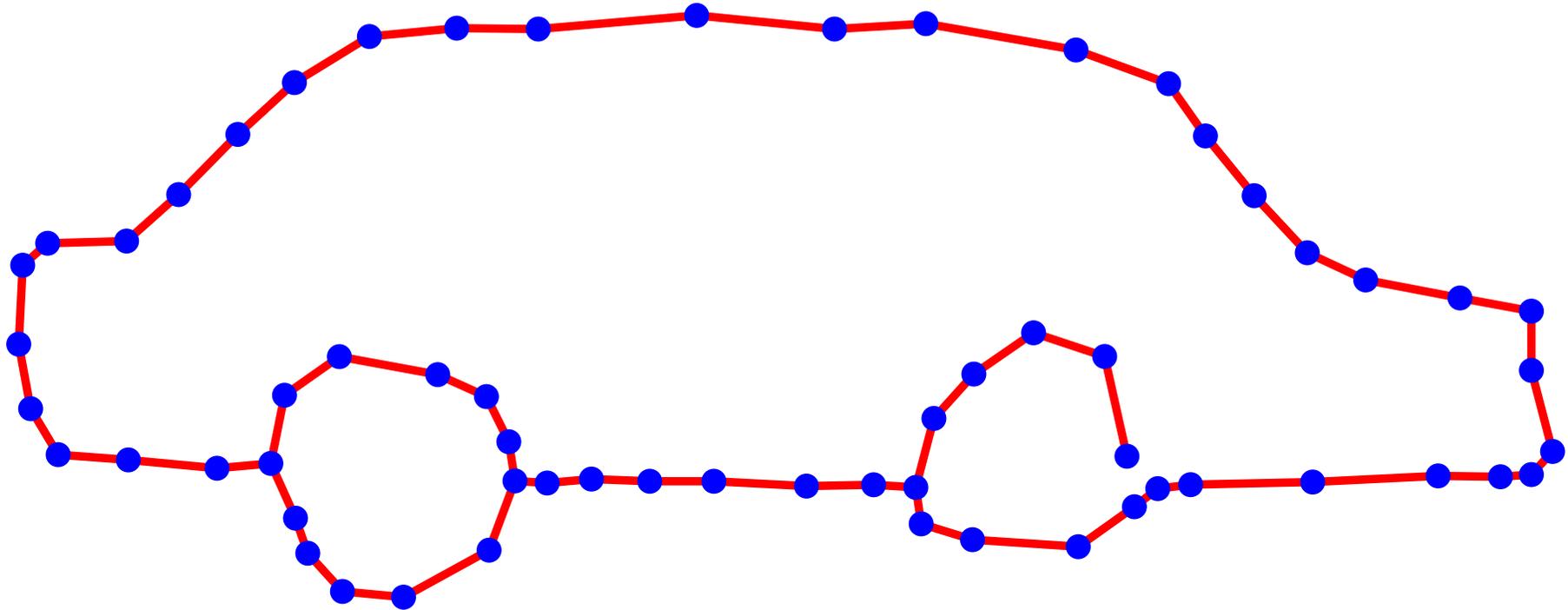
Points to shape



# Computational geometry usage

Points to shape

Reconstruction lecture

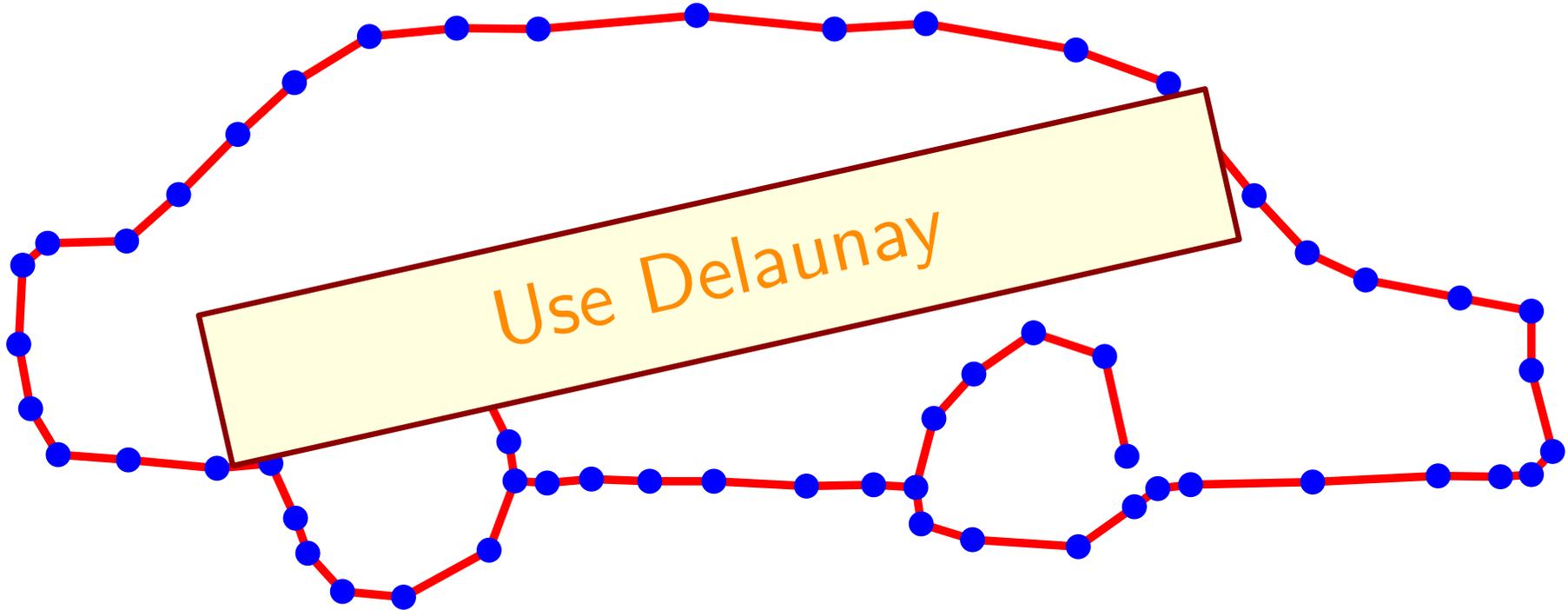


# Computational geometry usage

Points to shape

Reconstruction lecture

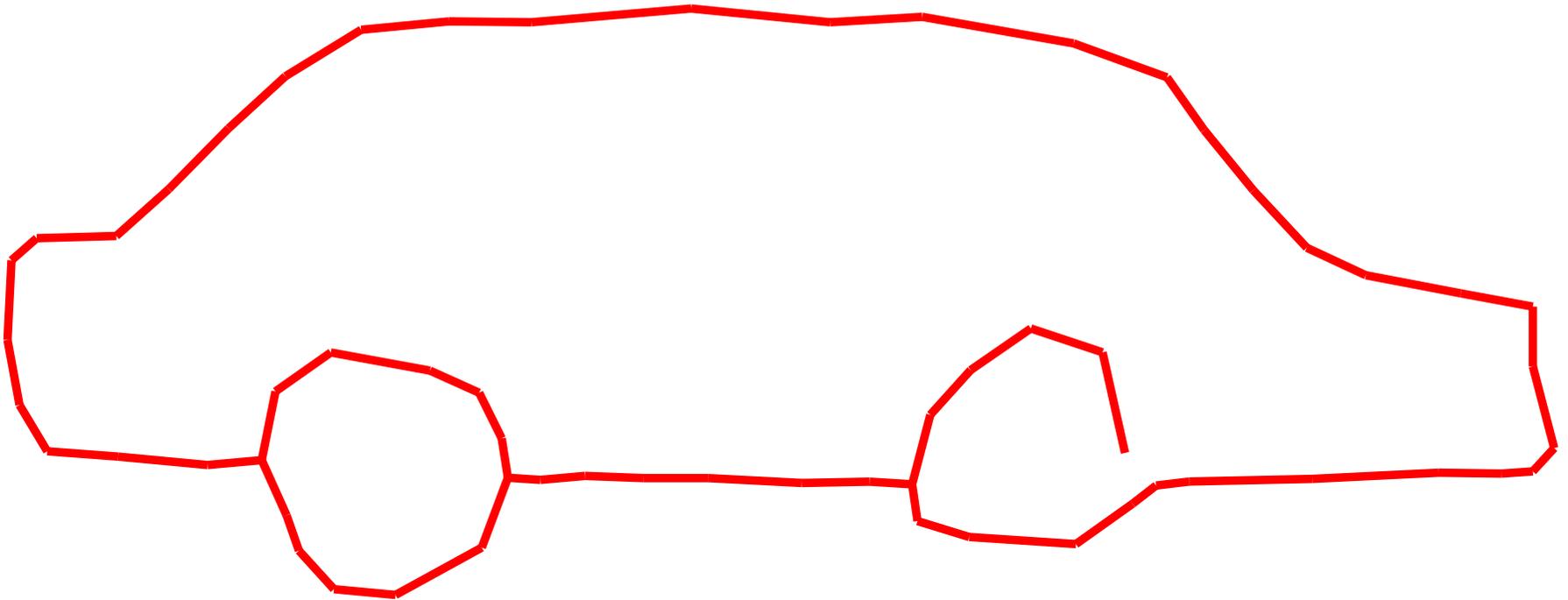
Use Delaunay



# Computational geometry usage

# Computational geometry usage

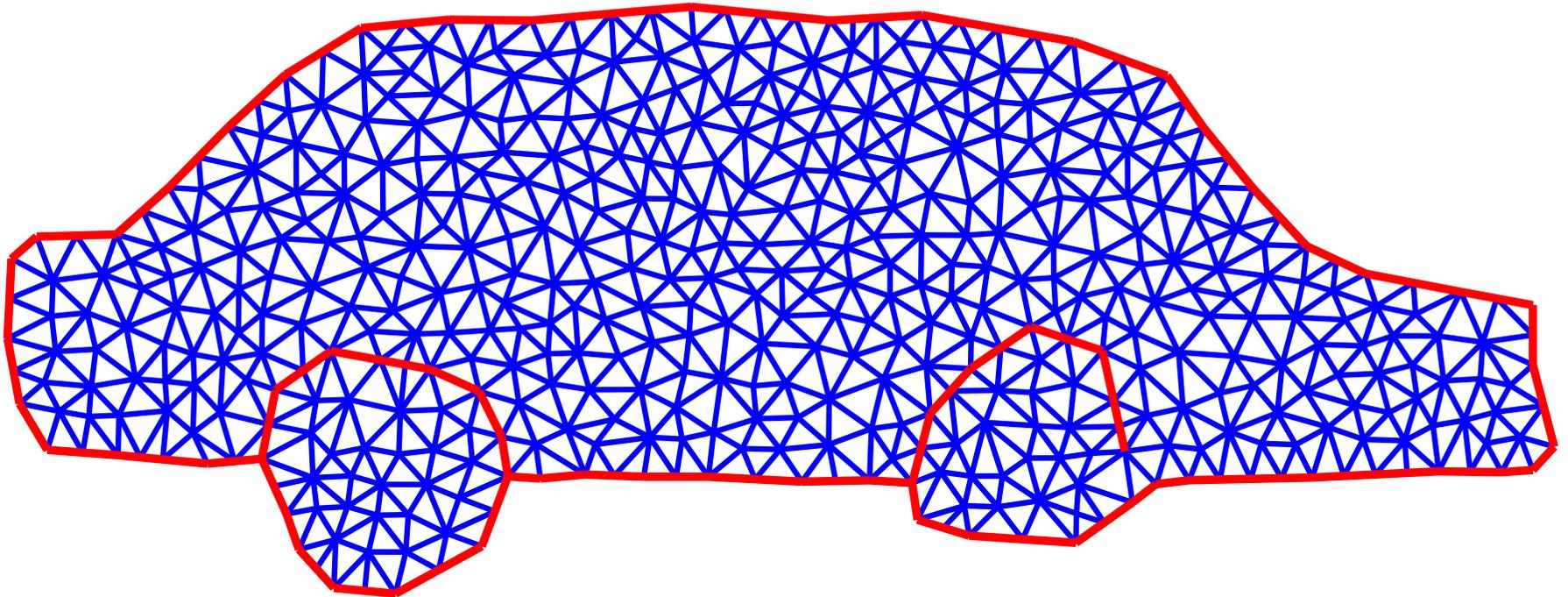
Shape to mesh



# Computational geometry usage

Shape to mesh

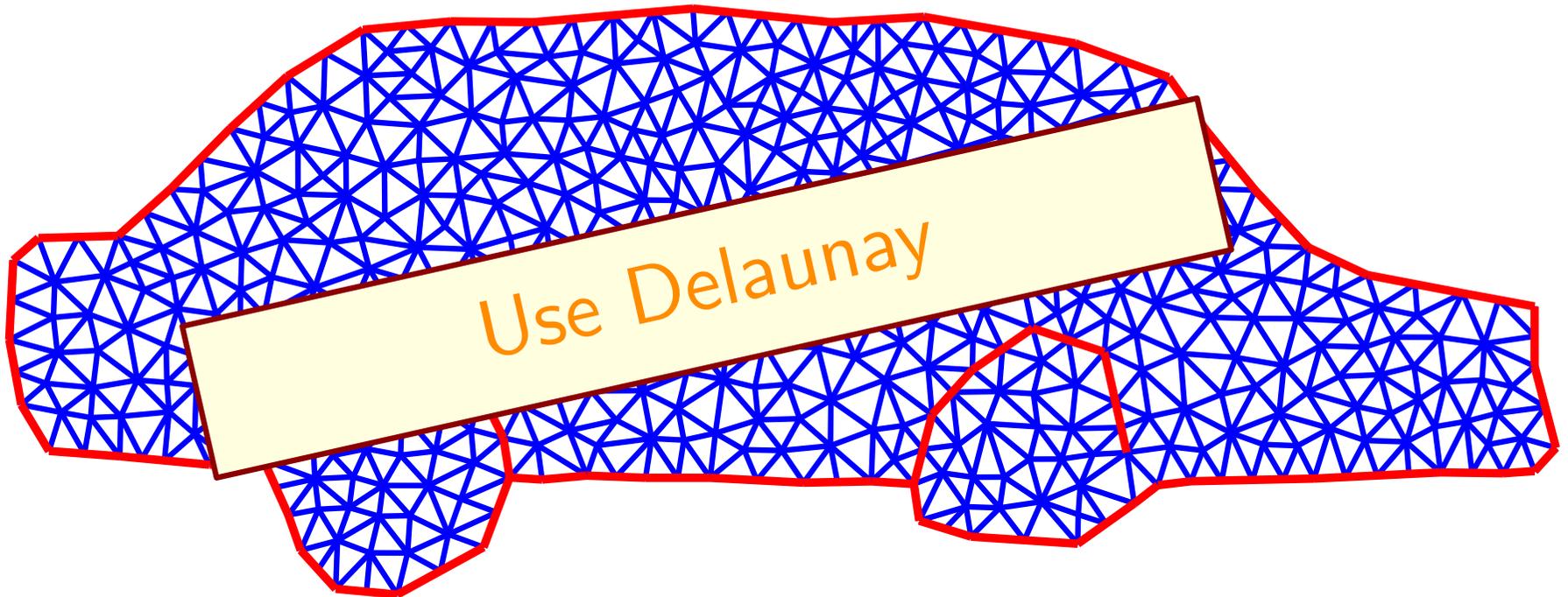
Meshing lecture



# Computational geometry usage

Shape to mesh

Meshing lecture



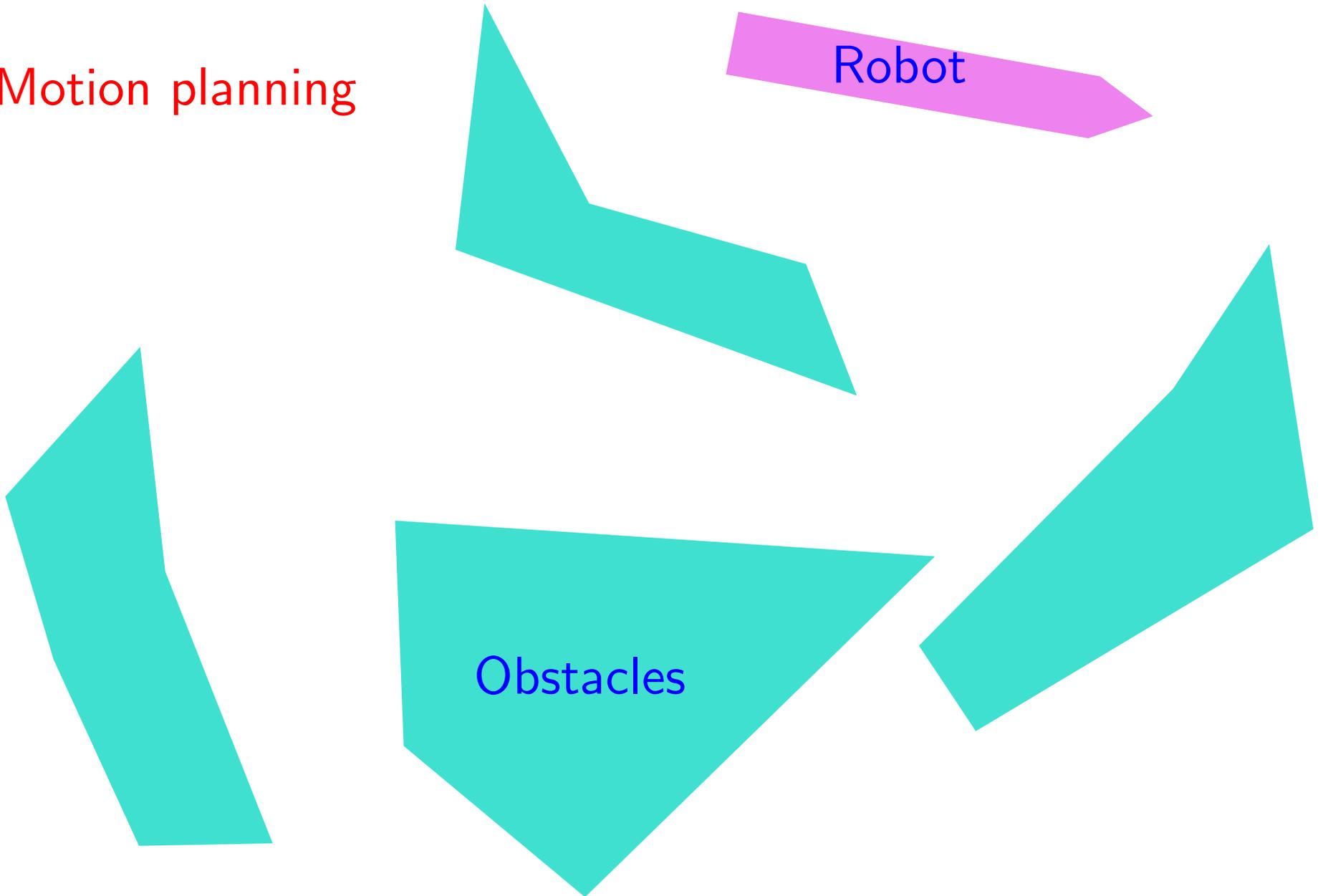
# Computational geometry usage

# Computational geometry usage

Motion planning

Robot

Obstacles

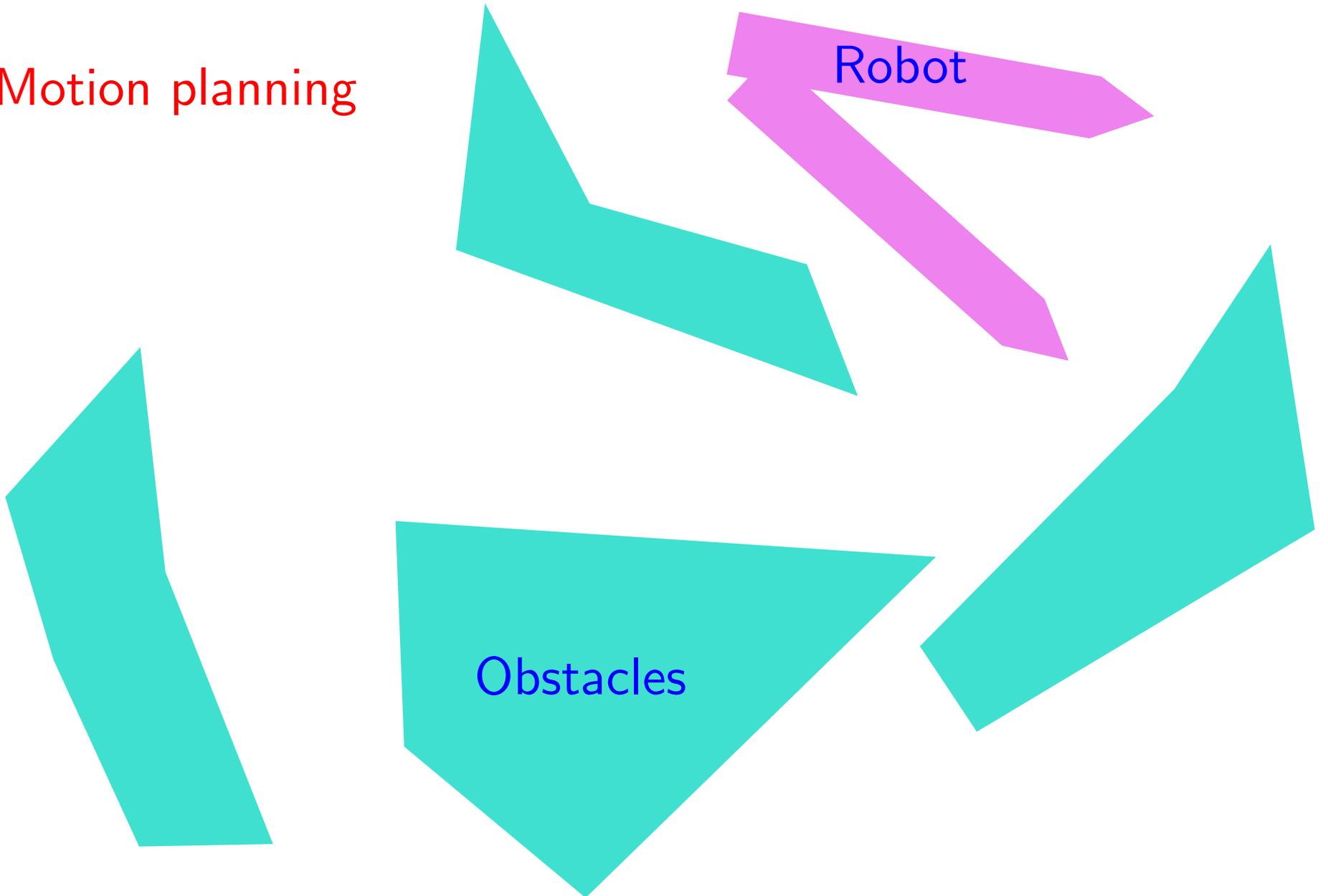


# Computational geometry usage

Motion planning

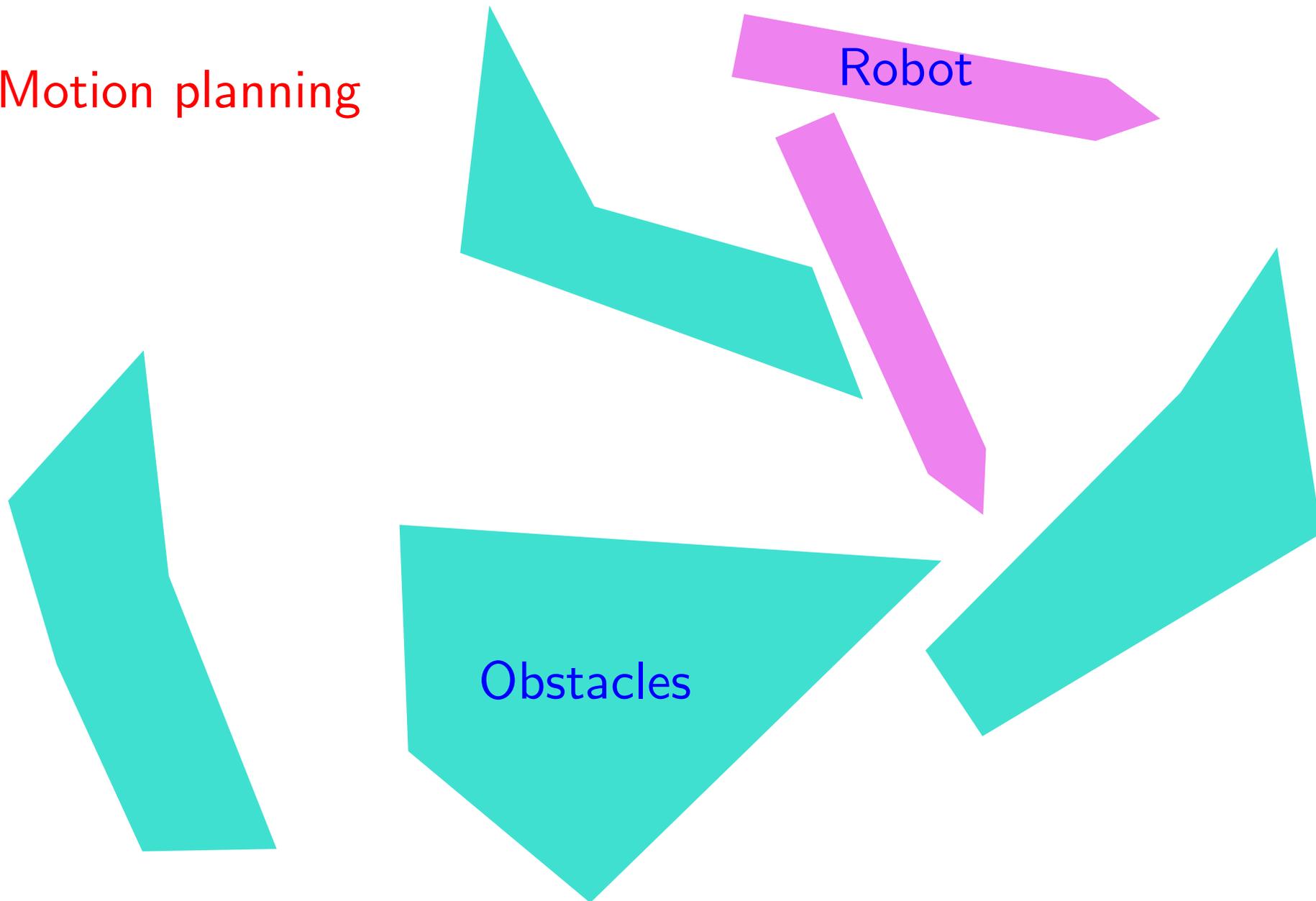
Robot

Obstacles



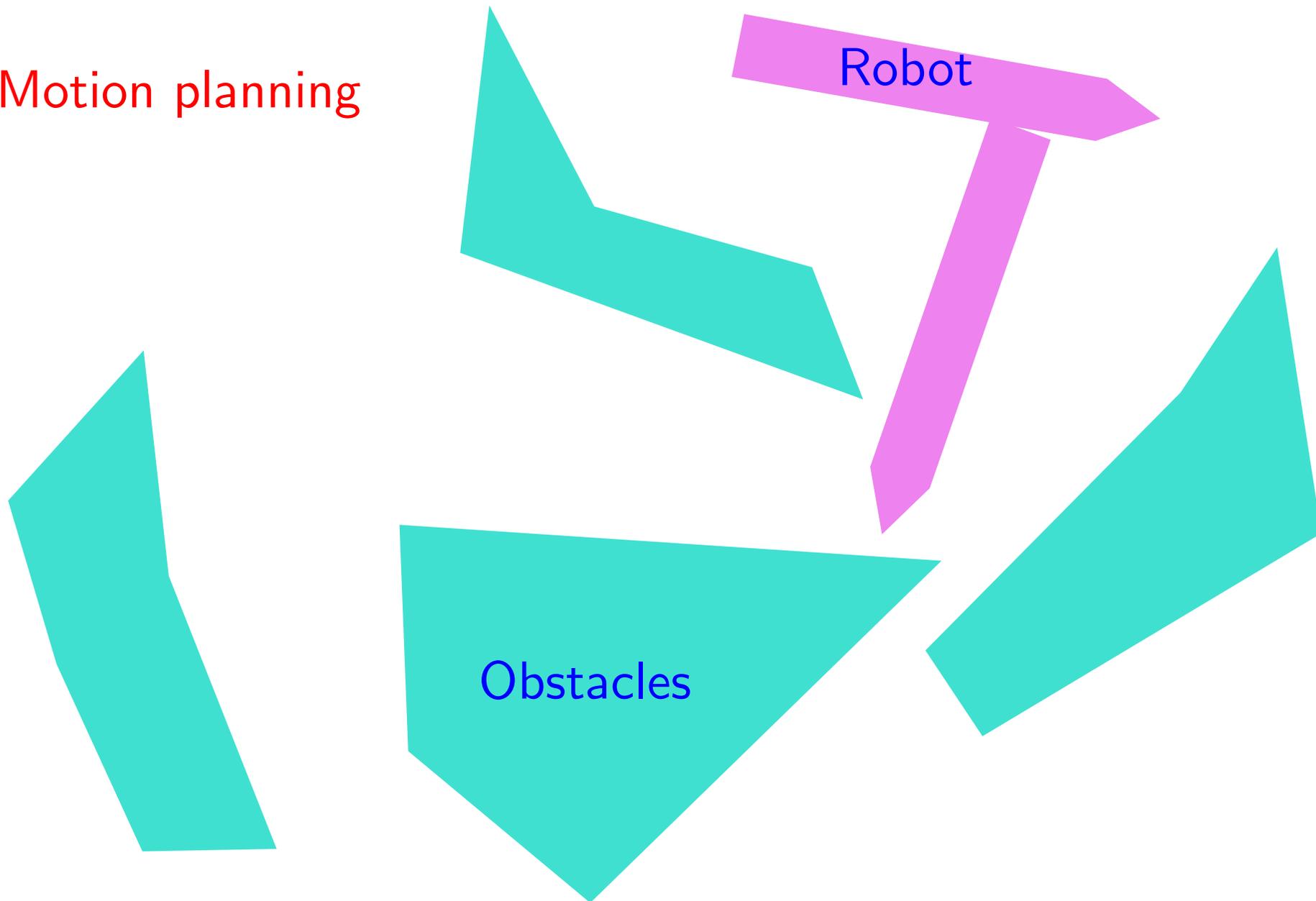
# Computational geometry usage

Motion planning



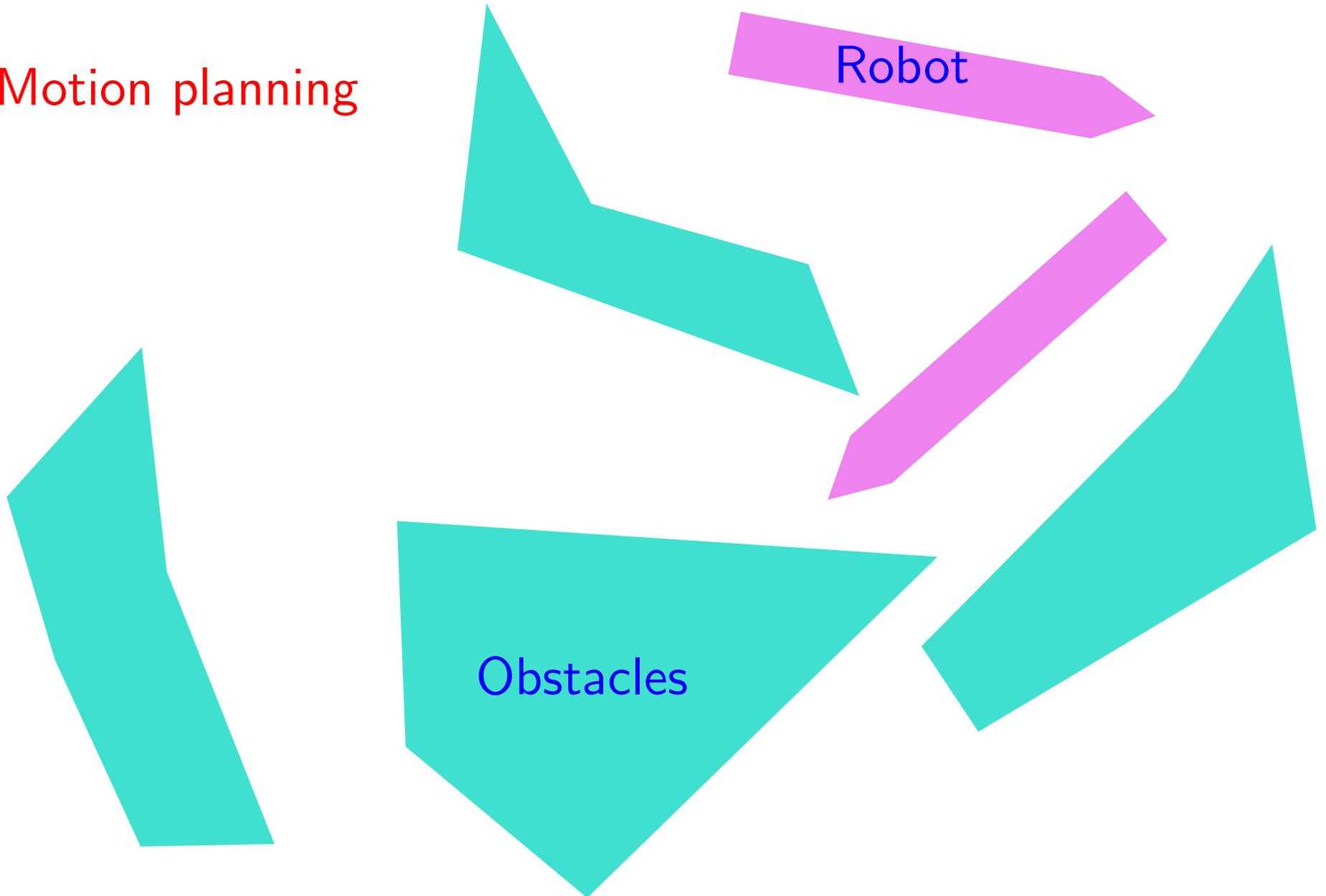
# Computational geometry usage

Motion planning



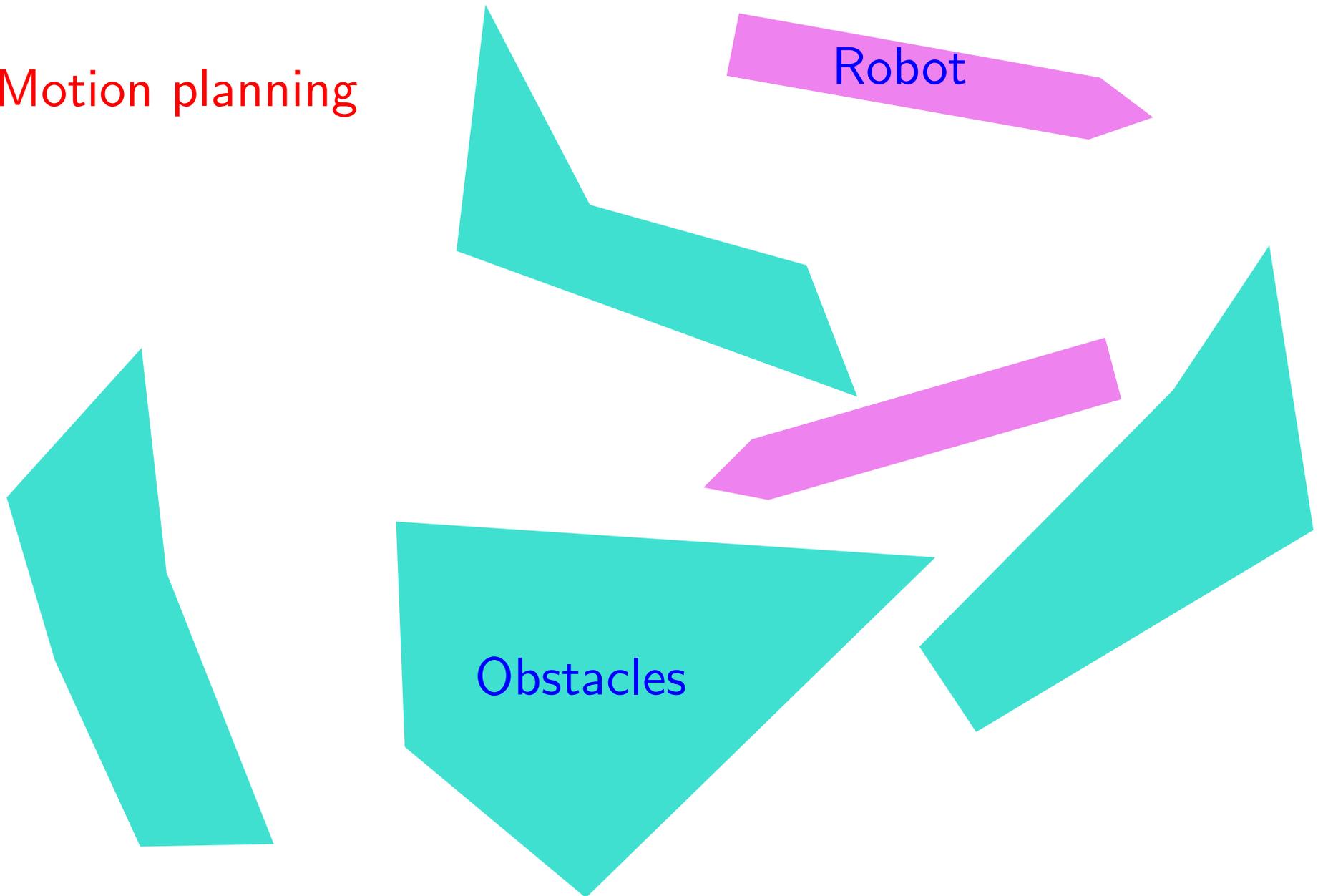
# Computational geometry usage

Motion planning



# Computational geometry usage

Motion planning

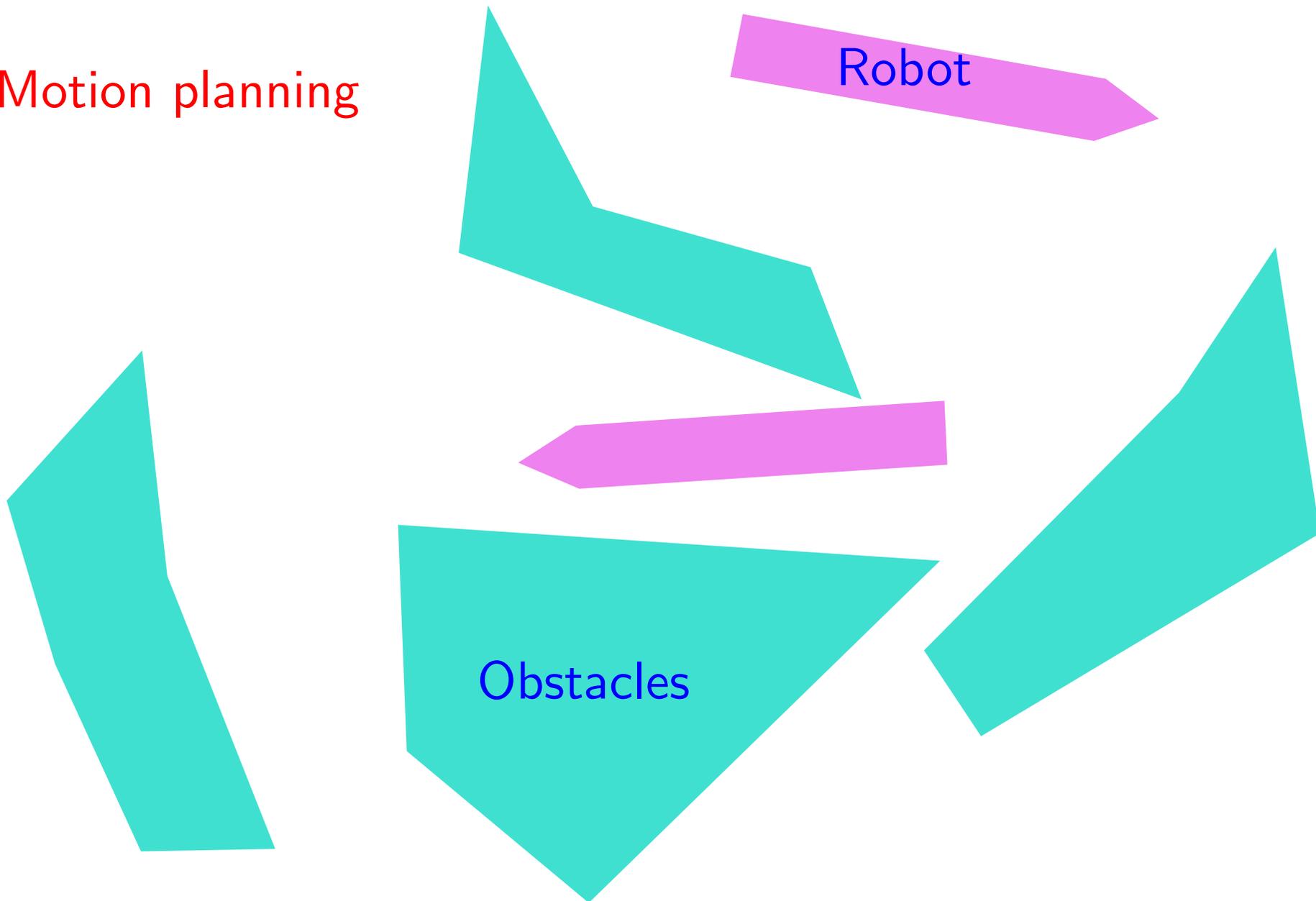


# Computational geometry usage

Motion planning

Robot

Obstacles

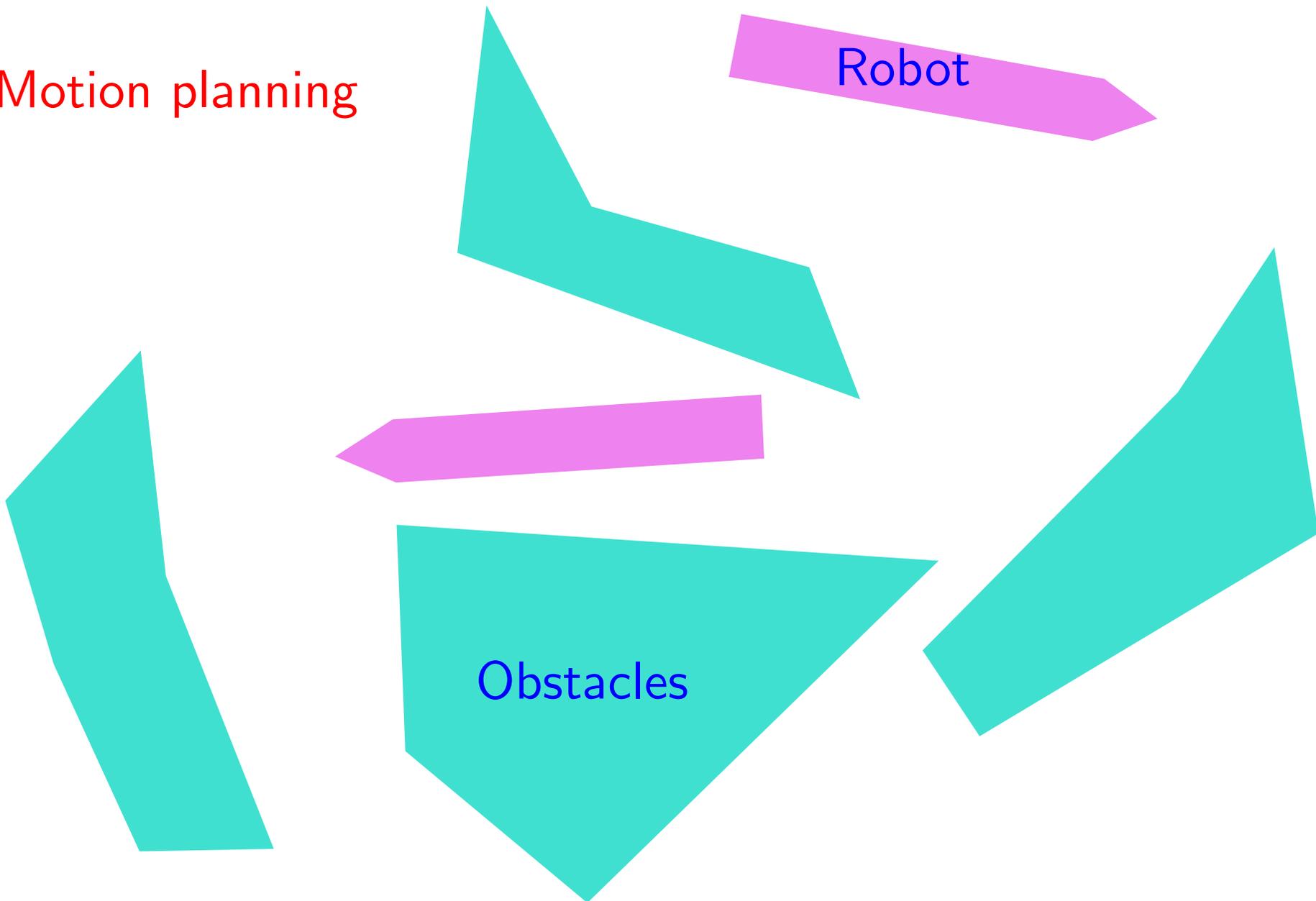


# Computational geometry usage

Motion planning

Robot

Obstacles



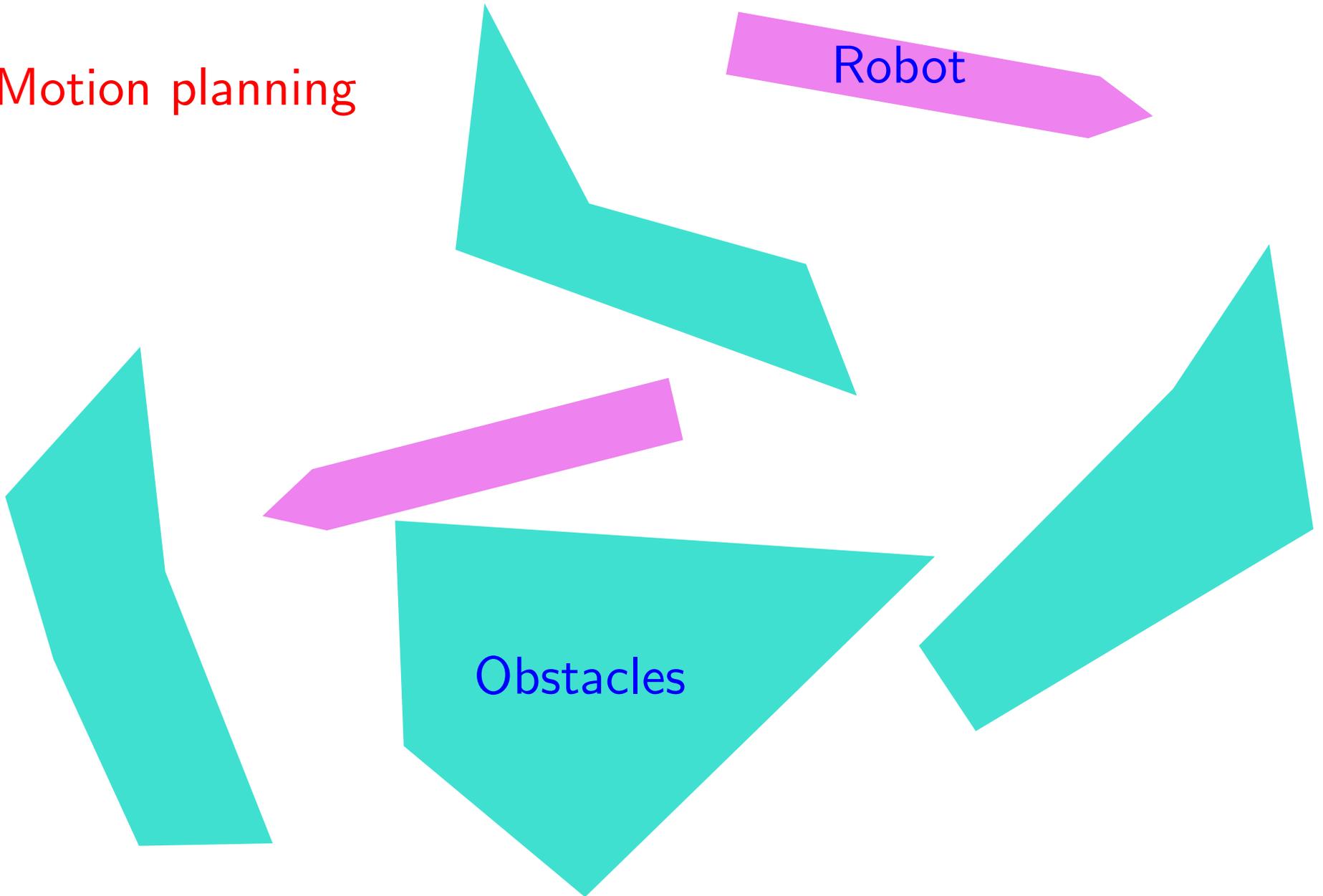
# Computational geometry usage

Motion planning

Robot

Obstacles

6 - 10

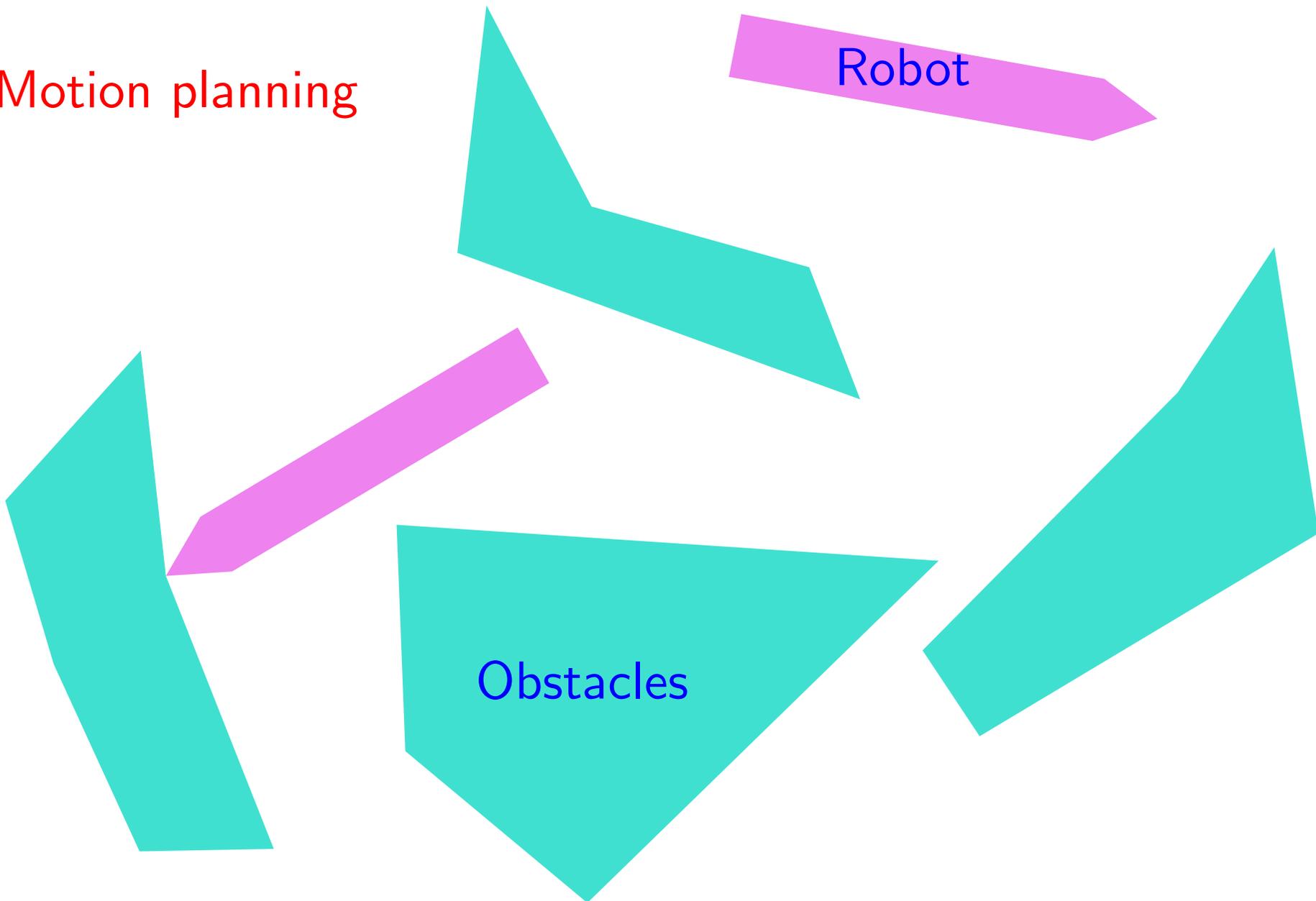


# Computational geometry usage

Motion planning

Robot

Obstacles



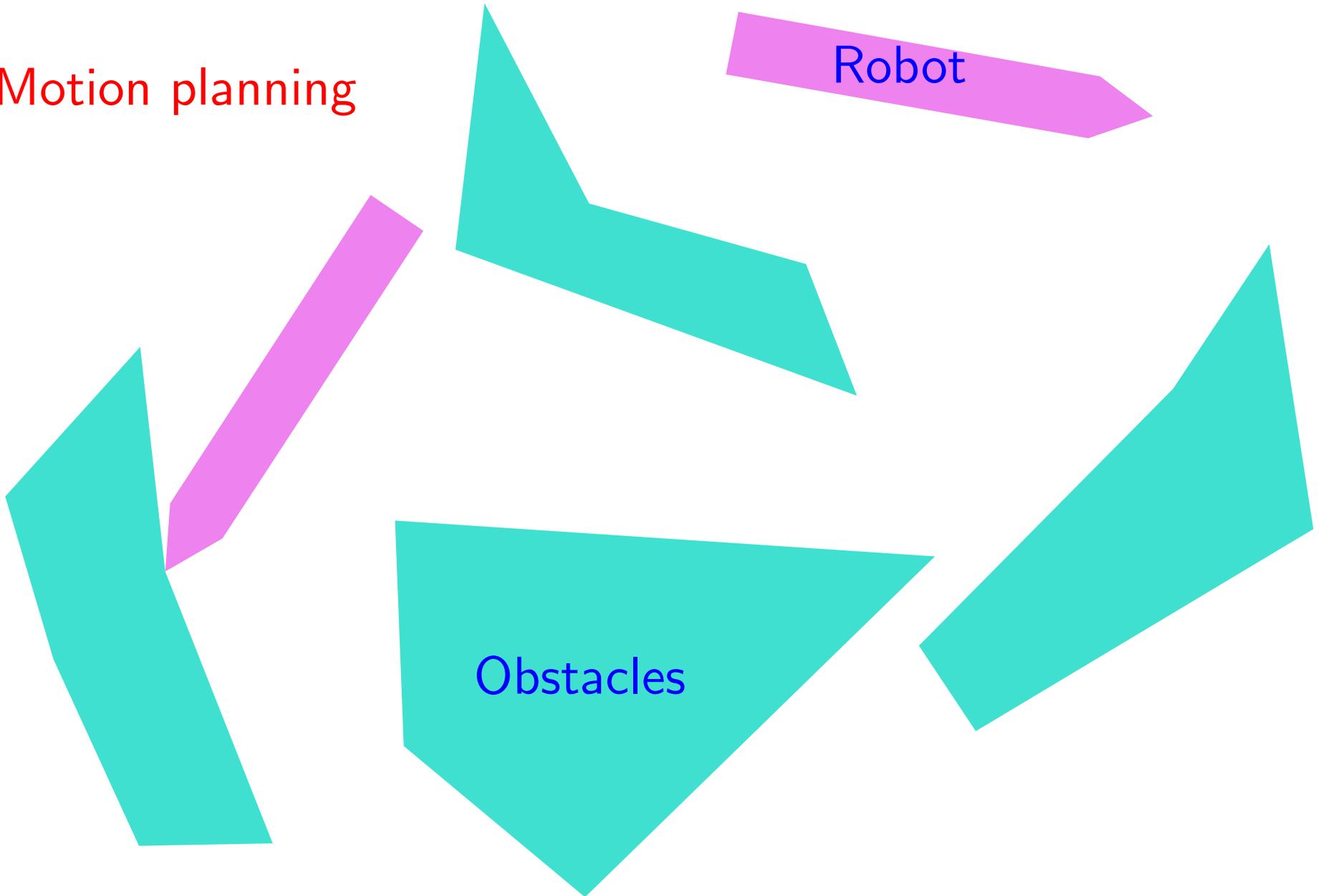


# Computational geometry usage

Motion planning

Robot

Obstacles

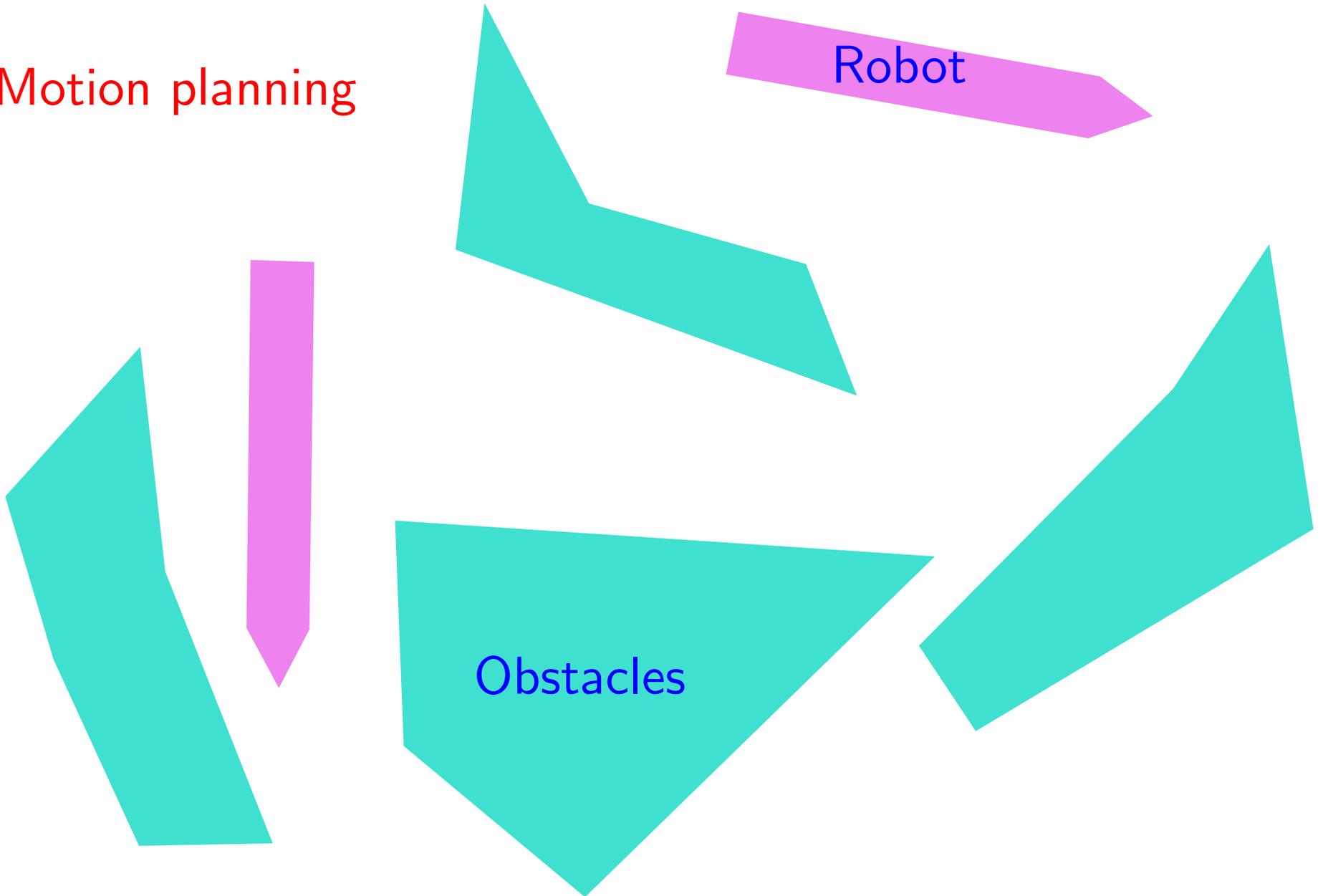


# Computational geometry usage

Motion planning

Robot

Obstacles

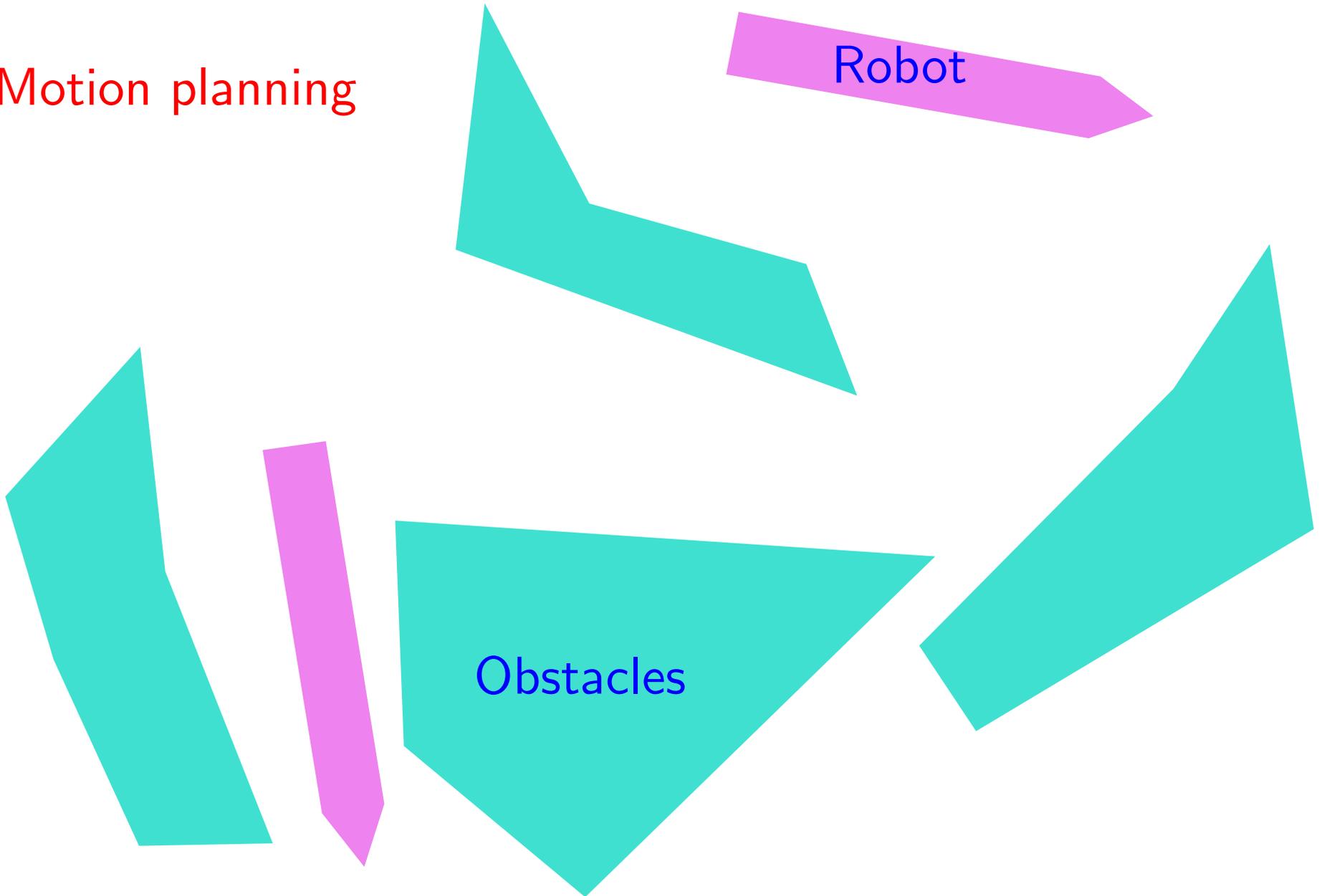


# Computational geometry usage

Motion planning

Robot

Obstacles



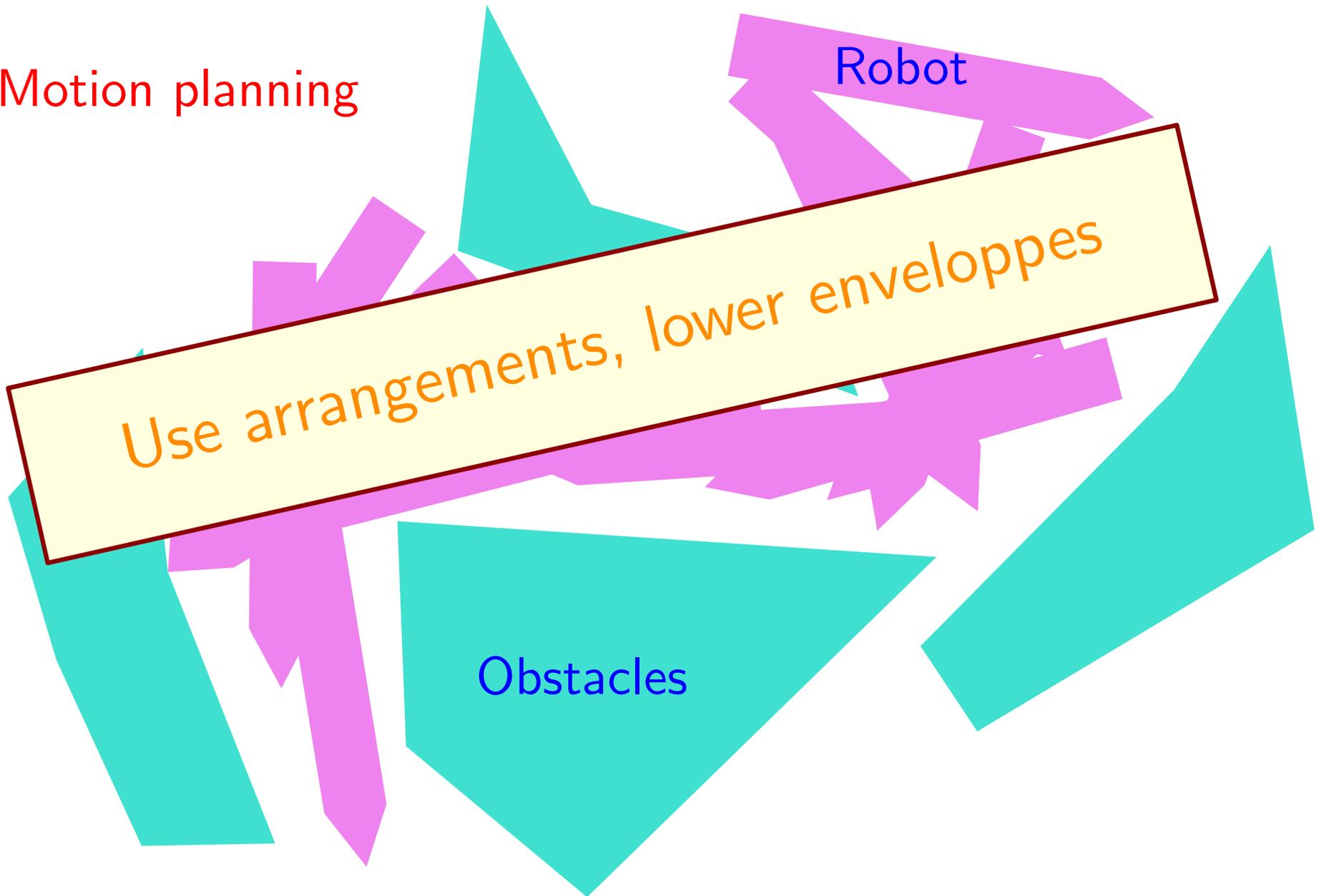
# Computational geometry usage

Motion planning

Robot

Use arrangements, lower enveloppes

Obstacles



# Computational geometry, 1975-1985

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 1975-1985

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

Fit real life data

For  $n$  big enough

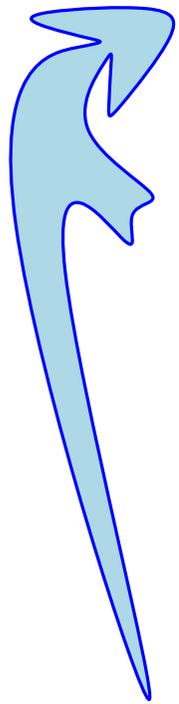
Does it exist

Real life data

Don't degeneracies exist?



# Computational geometry, 1975-1985



Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

Fit real life data ?

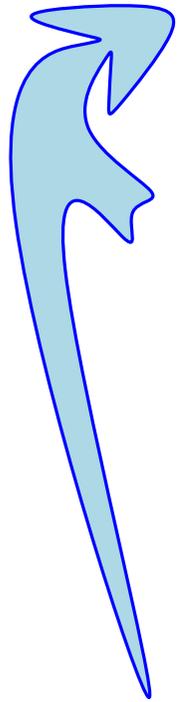
For  $n$  big enough ?

Does it exist ?

Real life data ?

Don't degeneracies exist ?

# Computational geometry, 1975-1985



Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

Not used in practice

Fit real life data ?

For  $n$  big enough ?

Does it exist ?

Real life data ?

Don't degeneracies exist ?

# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 1985-2000

Simpler

~~Complicated algorithms~~

Worst case complexities

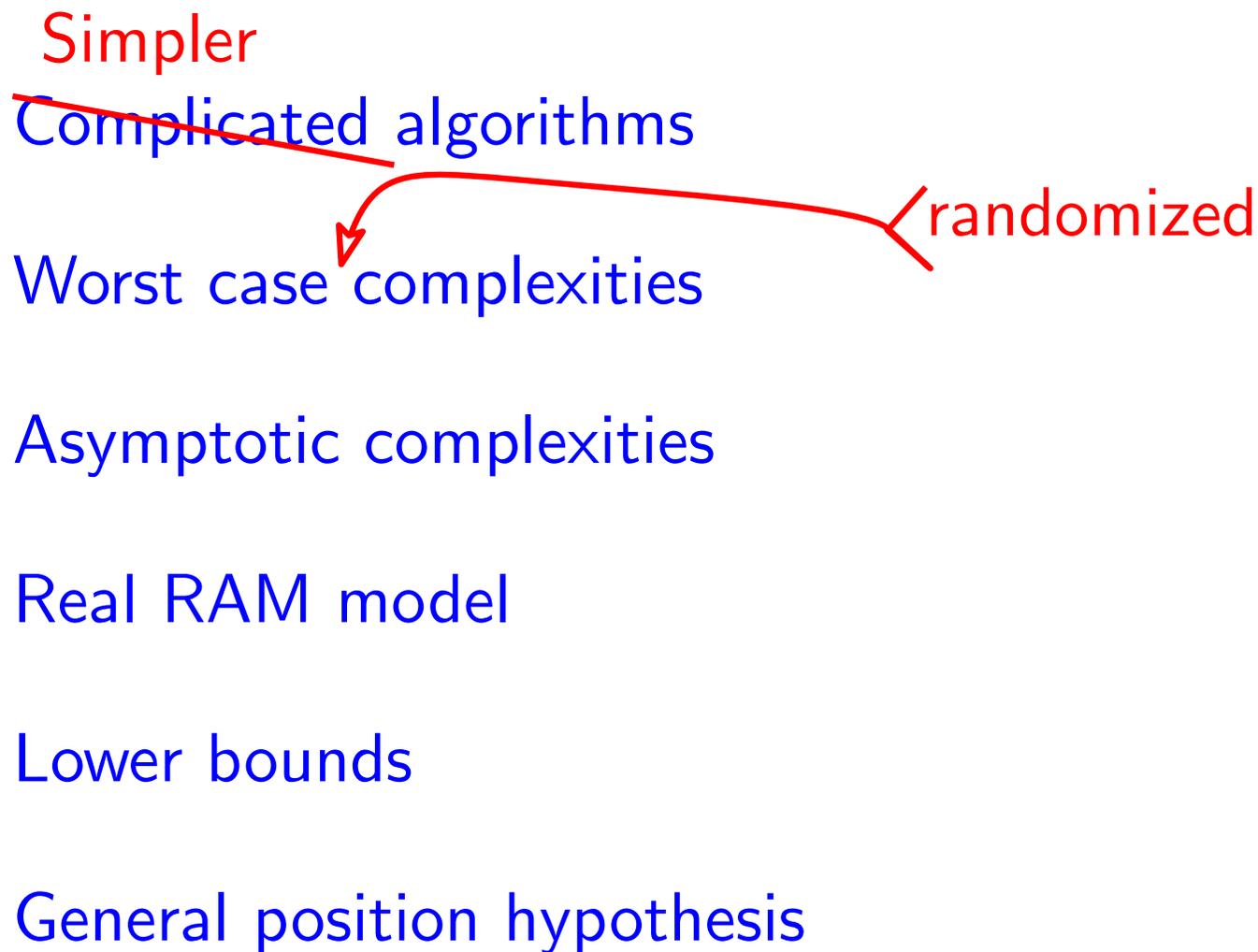
Asymptotic complexities

Real RAM model

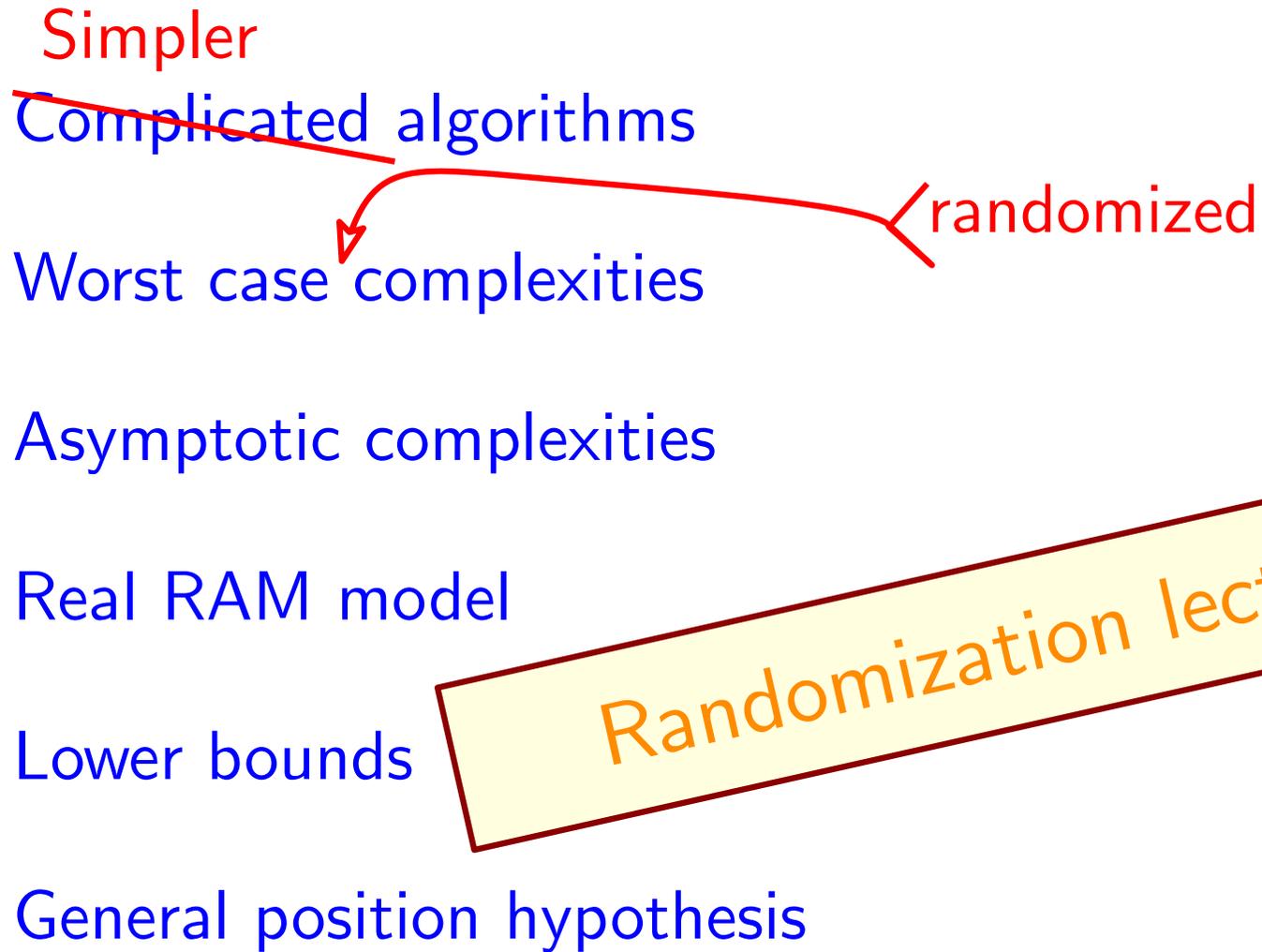
Lower bounds

General position hypothesis

# Computational geometry, 1985-2000



# Computational geometry, 1985-2000



# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model      address robustness issues

Lower bounds

General position hypothesis      solve degeneracies

# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic

Robustness lecture

Real RAM model

address robustness issues

Lower bounds

General position hypothesis

solve degeneracies

# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic complexities

Just really code it

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic complexities

Just really code it

Real RAM model

Lower bounds

General position hypothesis

CGAL lecture

# Computational geometry, 2000-

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 2000-

Complicated algorithms

Worst case complexities

Probabilistic hypotheses

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 2000-

Complicated algorithms

Worst case complexities

Probabilistic hypotheses

Old (and recent) math literature

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 2000-

Complicated algorithms

Worst case complexities

Probabilistic hypotheses

Old (and recent) math literature

Asymptotic complexities

Real RAM model

Lower bounds

Poisson Delaunay lecture

General position hypothesis

# Computational geometry, 2000-

Complicated algorithms

Beyond the Euclidean realm

Worst case complexities

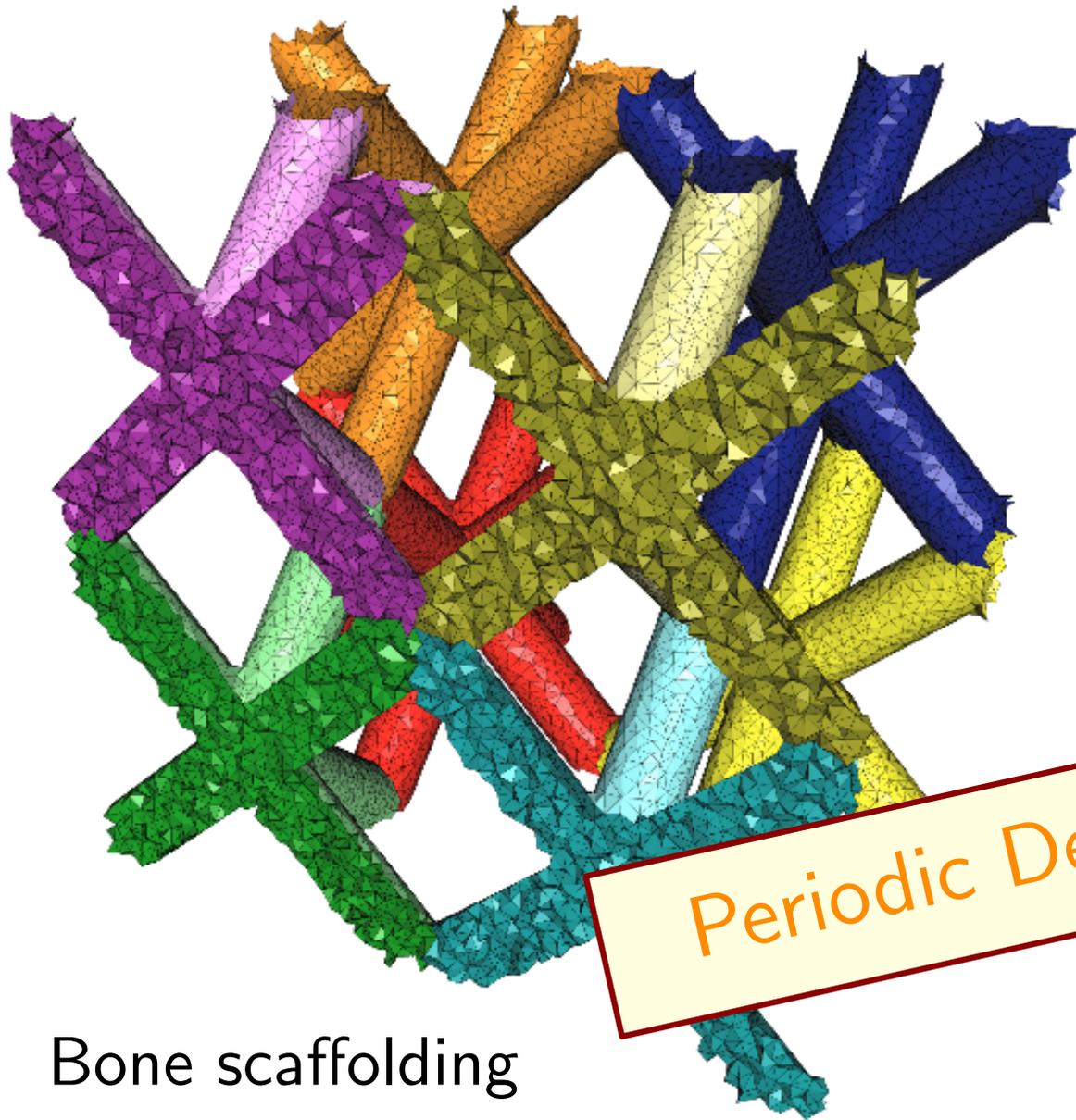
Asymptotic complexities

Real RAM model

Lower bounds

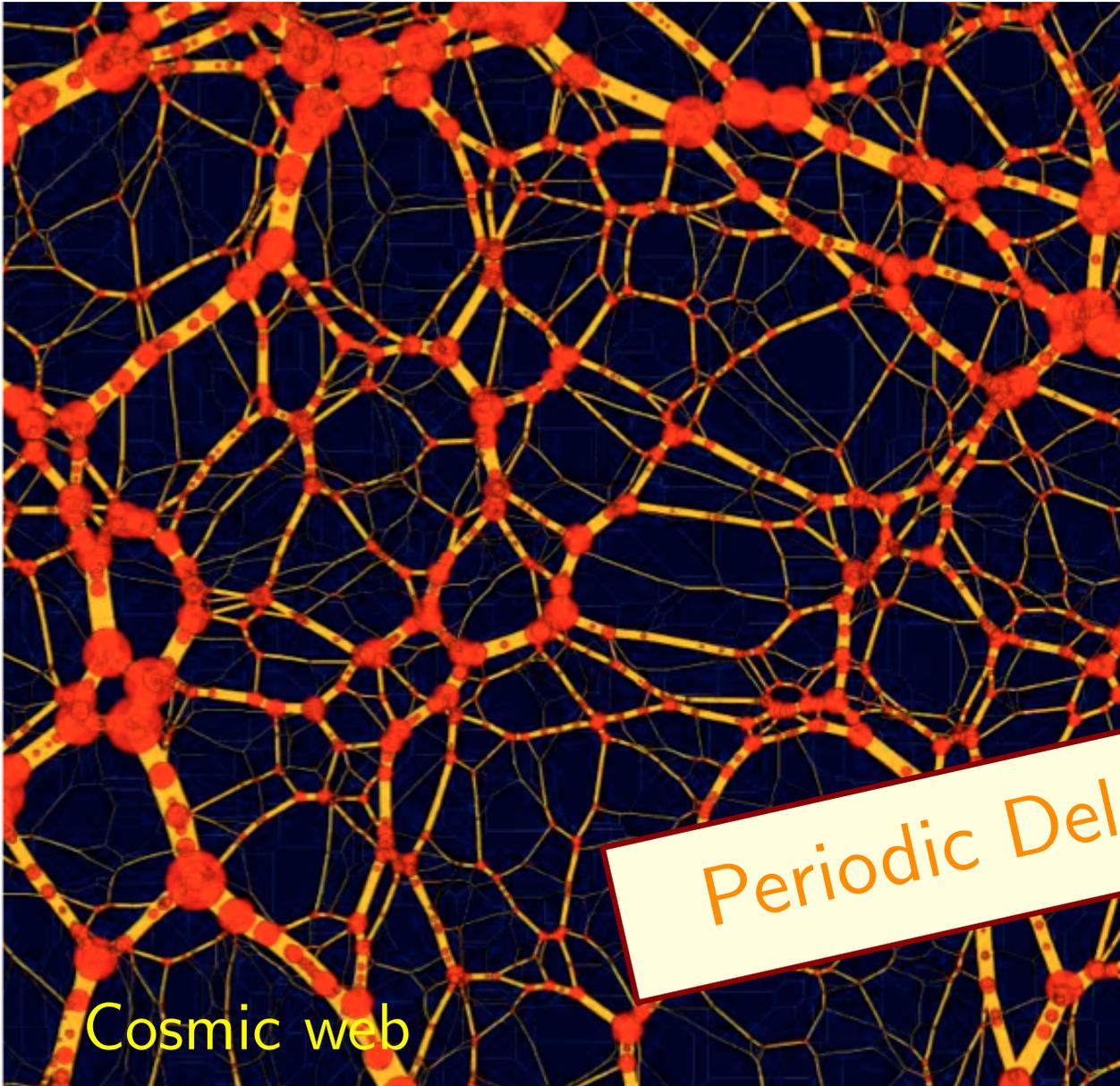
General position hypothesis

# Computational geometry, 2000-



Periodic Delaunay lecture

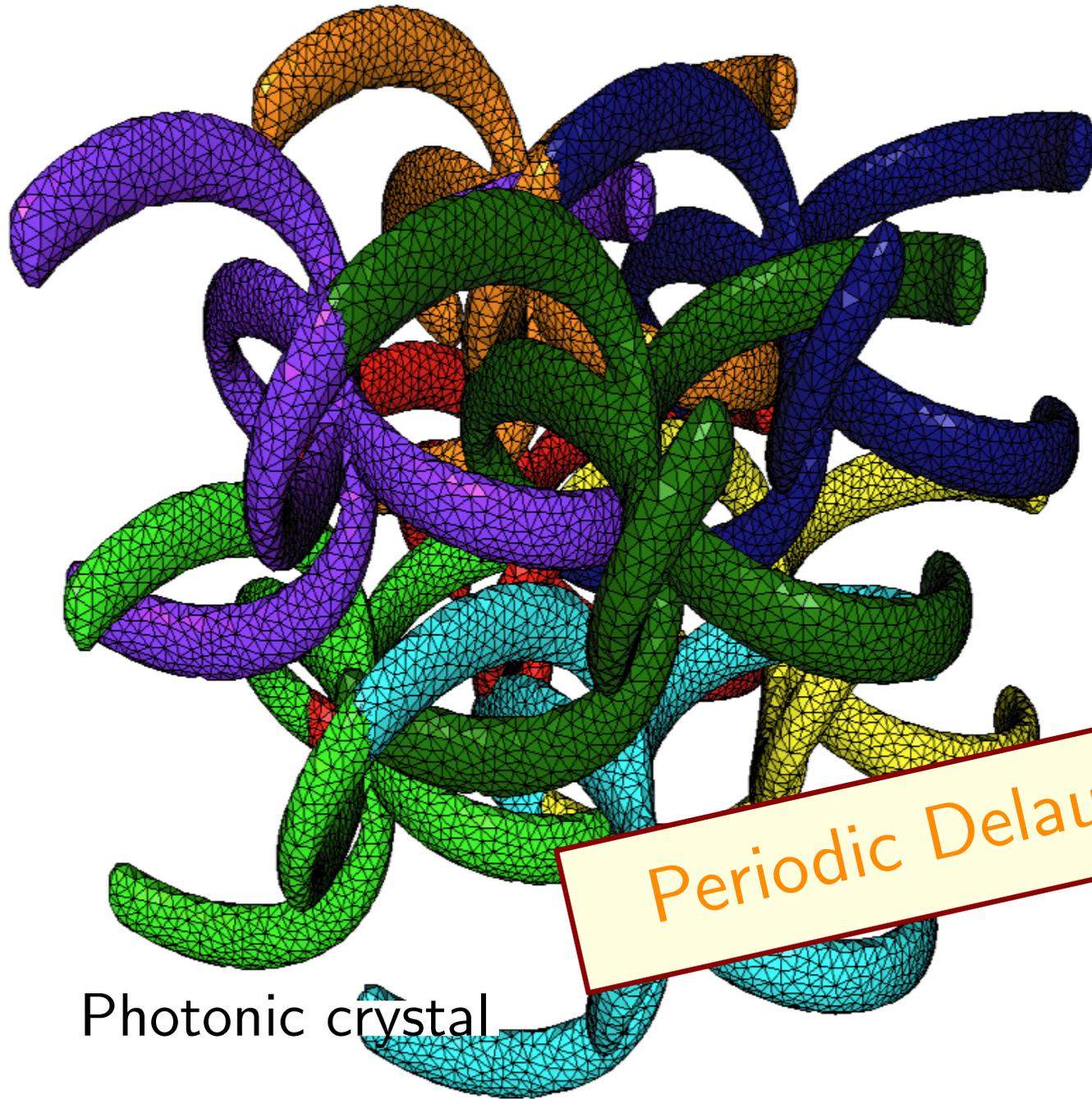
# Computational geometry, 2000-



Cosmic web

Periodic Delaunay lecture

# Computational geometry, 2000-



Periodic Delaunay lecture

Photonic crystal

# Computational geometry, 2000-

Complicated algorithms

Beyond the Euclidean realm

Worst case complexities

Asymptotic complexities

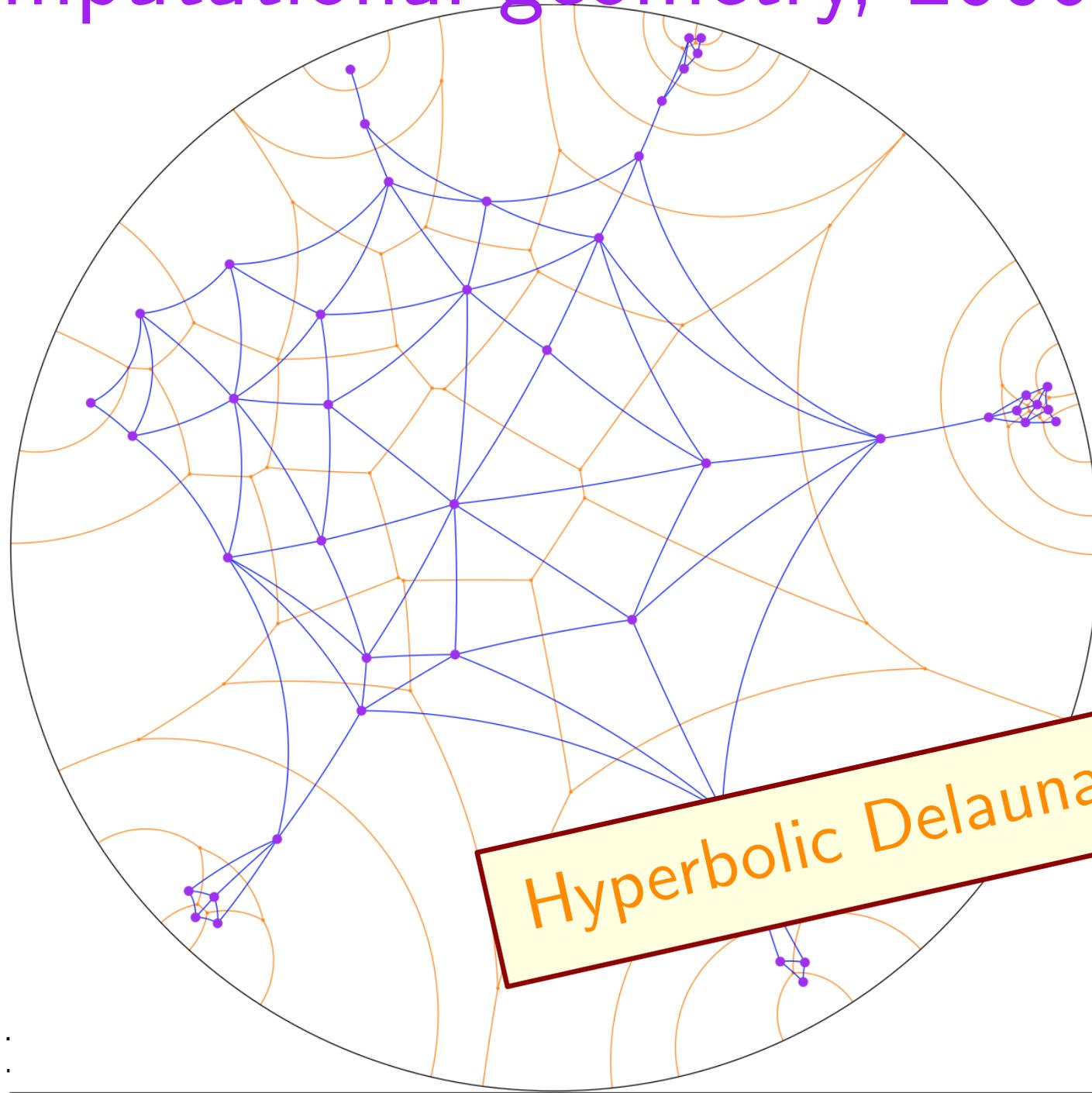
Real RAM model

Lower bounds

General position

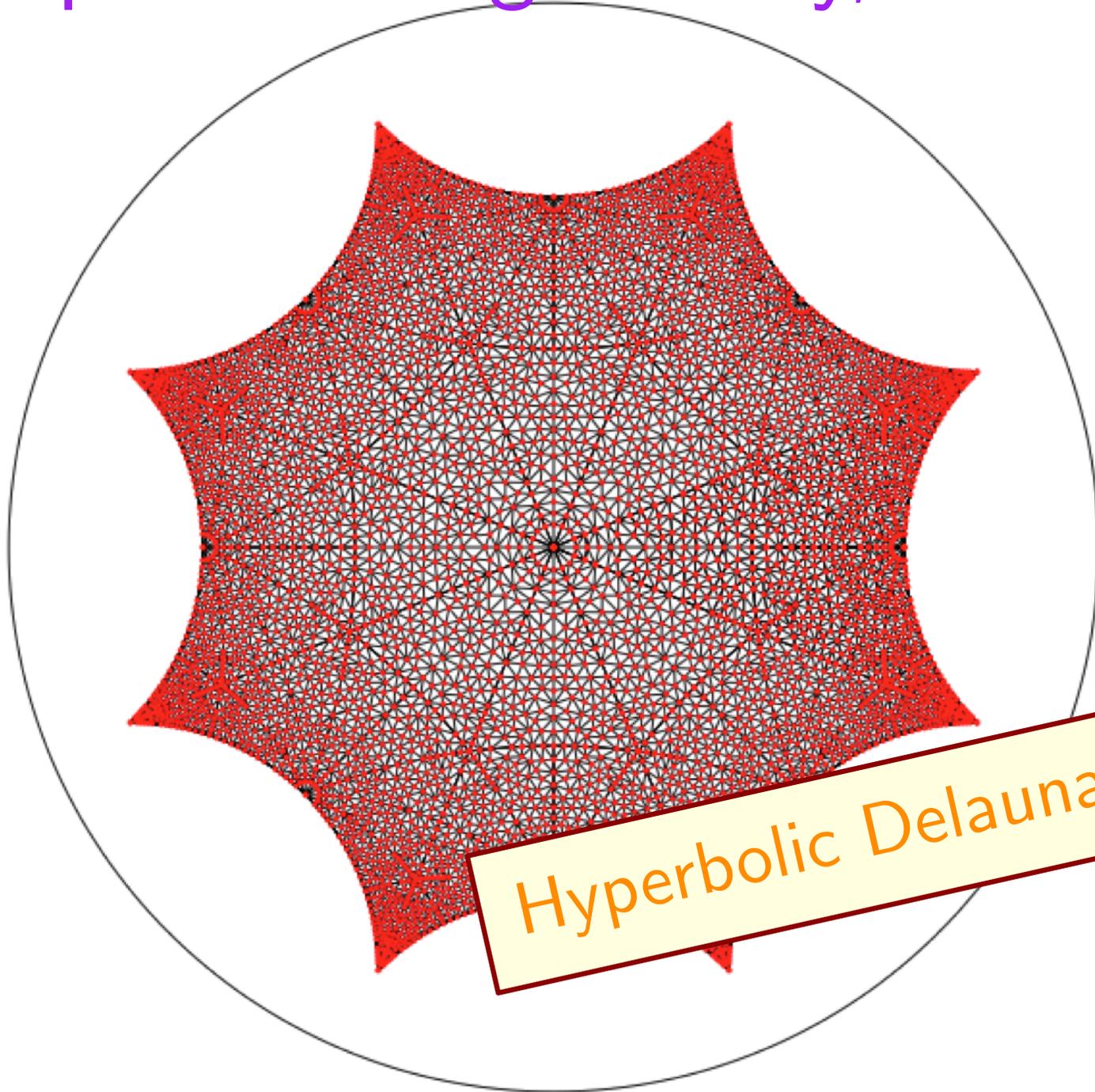
Hyperbolic Delaunay lecture

# Computational geometry, 2000-



Hyperbolic Delaunay lecture

# Computational geometry, 2000-



Hyperbolic Delaunay lecture